

## Scoring Guidelines

### Scoring Guidelines for Free-Response Question 1 (7 points)

Explanations can include figures to support or clarify the meaning of prose, but figures alone are not sufficient.

For explaining the condition for resonance in a tube closed at one end 2 points

For comparing wavelengths at low frequency to the tube lengths 1 point

For linking the above two ideas (conditions of resonance and comparing wavelengths at low frequency) to explain why only one resonance occurs at a time 1 point

For indicating that as frequency goes up, wavelength goes down 1 point

For indicating how smaller wavelengths relate to differences in tube length, explaining how both tubes can now meet boundary conditions 2 points

Example:

In order to resonate, the length of a tube must be an odd multiple of a quarter wavelength of the sound, as shown below.



For resonance at low frequencies, the wavelength of the sound is of the order of the length of the tubes. So the match can occur for only one tube at a time — the difference in tube lengths is much smaller than a half wavelength. As the frequency increases, the wavelength decreases and many more wavelengths fit inside a tube. When half the wavelength becomes of the order of the difference in tube lengths, the tubes can contain an odd multiple of quarter wavelengths for the same wavelength at the same time — for instance, one tube might contain 17 quarter wavelengths while the other contains 19 quarter wavelengths.

## Scoring Guidelines for Free-Response Question 2 (12 points)

(a) (3 points)

For a reasonable setup that would allow useful measurements	1 point
For indicating all the measurements needed to determine the velocities	1 point
For having no obviously extraneous equipment or measurements	1 point

Examples:

- Use tape to mark off two distances on the track — one for cart *A* before the collision and one for the combined carts after the collision. Push cart *A* to give it an initial speed. Use a stopwatch to measure the time it takes for the cart(s) to cross the marked distances. The speeds are the distances divided by the times.
- Place a motion detector at the left end of the track. Push cart *A* to give it an initial speed. Record position as a function of time, first for cart *A* and then for the combined carts *A* and *B*.

(b) (2 points)

For indicating a reasonable assumption about the relative size of the measurement errors before and after the collision	1 point
For correctly using the assumption in comparing the effect on the calculated value of the mass of cart <i>B</i>	1 point

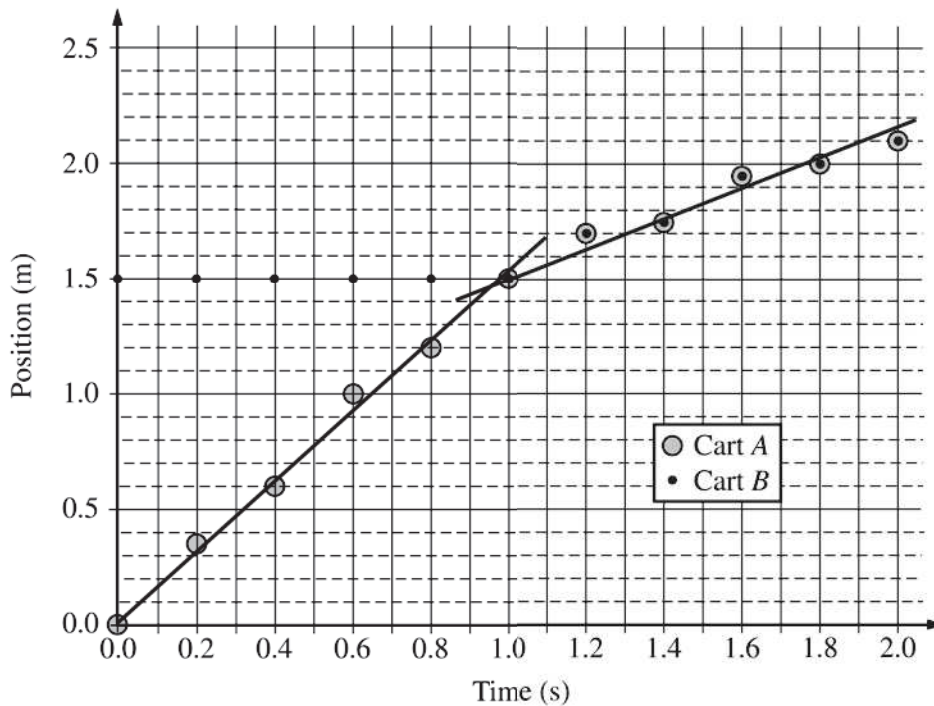
Example:

If the measurement errors are of the same magnitude, they will have a greater effect after the collision. The speed of the combined carts will be less than the initial speed of cart *A*, so errors of the same magnitude will be a greater percentage of the actual value after the collision. So the values after the collision will have a greater effect on the value of the mass of cart *B*.

A response could also argue any of the following:

- Measurement error could be greater before the collision (it could be harder to measure with the same accuracy at the greater speed). So percent error could be the same or greater.
- Measurement error could be greater before the collision (it could be harder to measure with the same accuracy at the greater speed). So the magnitude of the reported uncertainty could be the same.
- Measurement error could be the same before and after the collision if the same motion detector is used throughout.

(c) (4 points)



For providing sufficient description of the principles used in the calculation (in either a single explanation or dispersed throughout the calculations) 1 point

Conservation of momentum can be used to determine the mass of cart B:

$$m_A v_i = (m_A + m_B) v_f$$

For correctly recognizing the two regions on the graph corresponding to before and after the collision 1 point

For using data from the graph to attempt calculation of speed from slope 1 point

For indicating use of the slope of one or two drawn lines to determine one or more speeds (This point cannot be earned if calculations use data points not on the line[s].) 1 point

The speed  $v_i$  before the collision is the slope of the best-fit line for the data from 0 to 1 s.

The speed  $v_f$  after the collision is the slope of the best-fit line for the data from 1 s to 2 s.

Using the example lines drawn above:

$$v_i = \frac{(1.4 - 0) \text{ m}}{(0.9 - 0) \text{ s}} = \frac{14 \text{ m}}{9 \text{ s}}$$

$$v_f = \frac{(2.1 - 1.5) \text{ m}}{(1.9 - 1.0) \text{ s}} = \frac{0.6 \text{ m}}{0.9 \text{ s}} = \frac{2 \text{ m}}{3 \text{ s}}$$

Applying conservation of momentum:

$$(0.5 \text{ kg}) \left( \frac{14 \text{ m}}{9 \text{ s}} \right) = (0.5 \text{ kg} + m_B) \left( \frac{2 \text{ m}}{3 \text{ s}} \right)$$

$$(0.5 \text{ kg}) \left( \frac{14}{9} \right) \left( \frac{3}{2} \right) = (0.5 \text{ kg} + m_B)$$

$$(0.5 \text{ kg}) \left( \frac{7}{3} \right) = (0.5 \text{ kg} + m_B)$$

$$m_B = (0.5 \text{ kg}) \left( \frac{7}{3} - 1 \right) = (0.5 \text{ kg}) \left( \frac{4}{3} \right) = \frac{2}{3} \text{ kg}$$

(d) (3 points)

For an answer consistent with previous responses that indicates a modification of the procedure to accomplish varying the initial speed of cart A or one of the cart masses OR that indicates that the previously described procedure would provide appropriate data, so it does not need modification 1 point

For indicating that the data can be used to calculate the kinetic energy  $K$  before and after the collision 1 point

For indicating that the fraction of  $K$  lost in the various collisions should be compared 1 point

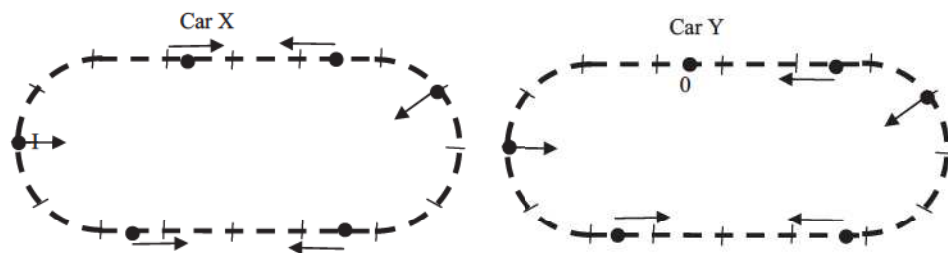
Example:

You could vary the initial speed of cart A. From the data, calculate values of kinetic energy before and after the collision using  $K = (1/2)mv^2$ . Then analyze  $((K_i - K_f)/K_i)$  to see if the changes in initial speed give different values.



## Scoring Guidelines for Free-Response Question 3 (12 points)

(a) (3 points)



For correct directions of the net forces at all the locations on the semicircular sections (i.e., all directed generally toward the center of the circle) 1 point

For correct directions of the net forces at all the locations on the bottom straightaways (i.e., directed toward the center of the segment) 1 point

For correct directions of the net forces at all locations on the top straightaway (i.e., both rightmost arrows directed toward the left, the left one for car X directed toward the right, and the left one for car Y equal to zero) 1 point

(b) (7 points total)

i) (2 points)

For realizing that the difference in time is only on the straightaways 1 point

For correct reasoning leading to Car Y taking a shorter time on the straightaways 1 point

Example:

Car X takes longer to accelerate and does not spend any time traveling at top speed. Car Y accelerates over a shorter time and spends time going at top speed. So car Y must cover the straightaways in a shorter time. Curves take the same time, so car Y must overall take a shorter time.

ii) (5 points)

The time to travel each curve is  $d/v_c$ . Answers can be expressed in terms of  $d/v_c$  or  $t_c = d/v_c$  or some other defined unit of time. The calculations below will use  $t_c = d/v_c$ .

For stating that the time to travel each curve is  $d/v_c$  1 point

For correct kinematics expressions that allow determination of the time it takes for one segment of acceleration on the straightaways 1 point

Example:  $D = v_c t_1 + \frac{1}{2} a t_1^2$ ,  $a = (2v_c - v_c)/t_1 = v_c/t_1$

For work that shows an understanding of how to determine the time that car X and car Y each spend accelerating 1 point

For work that shows an understanding of how to determine the time that car Y spends at constant speed 1 point

For correctly determining the total straightaway times for each car 1 point

Calculating the time for car X to travel one straightaway:

$$\frac{d}{2} = v_c t_1 + \frac{1}{2} a t_1^2, a = (2v_c - v_c)/t_1 = v_c/t_1$$

$$t_1 = \frac{d}{3v_c} = \frac{t_c}{3}, \text{ total time is } \frac{2t_c}{3}$$

Calculating the time for car Y to travel one straightaway:

Doing the calculation shown above using the distance of acceleration  $d/4$  gives the result that one section of acceleration takes a time  $t_c/6$ .

The time for car Y to travel one constant speed section on the straightaway is  $(d/2)/2v_c = (t_c/4)$ .

Adding three segments to get the total time for one straightaway gives  $7t_c/12$ .

The calculations show that car Y takes less time on a straightaway, and both cars take the same time on the curves, so car Y overall takes less time.

(c) (2 points)

For linking math to one aspect of qualitative reasoning that explains the difference in times 1 point

For linking math to all other qualitative reasoning that explains the difference in times 1 point

Examples:

The only difference in the calculations for the time of one segment of linear acceleration is the difference in distances. That shows that car X takes longer to accelerate. The equation  $(d/2)/2v_c = (t_c/4)$  corresponds to car Y traveling for a time at top speed.

Substituting  $a = v_c/t_1$  into the displacement equation in part (b) ii gives  $D = (3/2)v_c t_1$ . This shows that a car takes less time to reach its maximum speed when it accelerates over a shorter distance. This means Car Y reaches its maximum speed more quickly and therefore spends more time at its maximum speed than Car X does, as argued in part (b) i.