Electrical Energy, Potential and Capacitance

AP Physics B


Electric Fields and WORK

In order to bring two like charges near each other work must be done.   In order to separate two opposite charges, work must be done. Remember that whenever work gets done, energy changes form.

As the monkey does work on the positive charge, he increases the energy of that charge. The closer he brings it, the more electrical potential energy it has.   When he releases the charge, work gets done on the charge which changes its energy from electrical potential energy to kinetic energy. Every time he brings the charge back, he does work on the charge. If he brought the charge closer to the other object, it would have more electrical potential energy. If he brought 2 or 3 charges instead of one, then he would have had to do more work so he would have created more electrical potential energy. Electrical potential energy could be measured in Joules just like any other form of energy.

Electric Fields and WORK

Consider a negative charge moving in between 2 oppositely charged parallel plates initial KE=0 Final KE= 0, therefore in this case Work = ΔPE

We call this ELECTRICAL potential energy, U_E, and it is equal to the amount of work done by the ELECTRIC FORCE, caused by the ELECTRIC FIELD over distance, d, which in this case is the plate separation distance.
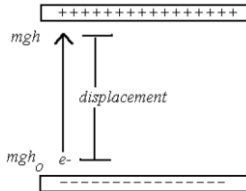
Is there a symbolic relationship with the FORMULA for gravitational potential energy?

Electric Potential

$$U_g = mgh$$

Here we see the equation for gravitational potential energy.

$$U_g \rightarrow U_E (or\ W)$$

Instead of gravitational potential energy we are talking about ELECTRIC POTENTIAL ENERGY

$$m \rightarrow q$$

A charge will be in the field instead of a mass

$$g \rightarrow E$$

The field will be an ELECTRIC FIELD instead of a gravitational field

$$h \rightarrow x \rightarrow d$$

The displacement is the same in any reference frame and use various symbols

$$U_E(W) = qEd$$

Putting it all together!

$$\boxed{\frac{W}{q}} = Ed$$

Question: What does the LEFT side of the equation mean in words?   The amount of Energy per charge!

Energy per charge

The amount of energy per charge has a specific name and it is called, VOLTAGE or ELECTRIC POTENTIAL (difference). Why the "difference"?

$$\Delta V = \frac{W}{q} = \frac{\Delta K}{q} = \frac{\frac{1}{2}mv^2}{q}$$

Gravitational Potential Energy (Joules) $GPE = mgh$

Gravity Field

$++++++++++++++++++++++++$

Electric Potential Energy (Joules) $EPE = qV$

Electric Field

Understanding "Difference"

E

High PE   Low PE

High potential   a   b   Low potential

$\leftarrow d \rightarrow$

Let's say we have a proton placed between a set of charged plates. If the proton is held fixed at the positive plate, the ELECTRIC FIELD will apply a FORCE on the proton (charge). Since like charges repel, the proton is considered to have a high potential (voltage) similar to being above the ground. It moves towards the negative plate or low potential (voltage). The plates are charged using a battery source where one side is positive and the other is negative. The positive side is at 9V, for example, and the negative side is at 0V. So basically the charge travels through a "change in voltage" much like a falling mass experiences a "change in height.
(Note: The electron does the opposite)

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BEWARE!!!!!!

W is Electric Potential Energy (Joules)
is _not_
V is Electric Potential (Joules/Coulomb)
a.k.a Voltage, Potential Difference

The "other side" of that equation?

$$U_g = mgh$$

$$U_g \rightarrow U_E (or\ W)$$

$$m \rightarrow q$$

$$g \rightarrow E$$

$$h \rightarrow x \rightarrow d$$

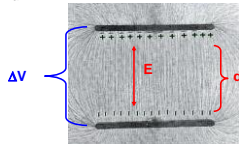$$U_E(W) = qEd$$

$$\frac{W}{q} = \boxed{Ed}$$

Since the amount of energy per charge is called Electric Potential, or Voltage, the product of the electric field and displacement is also VOLTAGE

This makes sense as it is applied usually to a set of PARALLEL PLATES.

ΔV=Ed

Example

A pair of oppositely charged, parallel plates are separated by 5.33 mm. A potential difference of 600 V exists between the plates. (a) What is the magnitude of the electric field strength between the plates? (b) What is the magnitude of the force on an electron between the plates?

$$d = 0.00533m$$

$$\Delta V = 600V$$

$$E = ?$$

$$q_{e^-} = 1.6x10^{-19}C$$

$$\Delta V = Ed$$

$$600 = E(0.0053)$$

$$E = \textbf{113,207.55 N/C}$$

$$E = \frac{F_e}{q} = \frac{F_e}{1.6x10^{-19}C}$$

$$F_e = \textbf{1.81x10}^{\textbf{-14}}\textbf{ N}$$

Example

Calculate the speed of a proton that is accelerated from rest through a potential difference of 120 V

$$q_{p^+} = 1.6x10^{-19}C$$
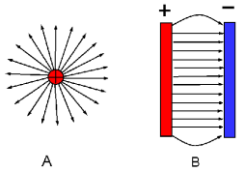$$m_{p^+} = 1.67x10^{-27}kg$$
$$V = 120V$$
$$v = ?$$

$$\Delta V = \frac{W}{q} = \frac{\Delta K}{q} = \frac{\frac{1}{2}mv^2}{q}$$

$$v = \sqrt{\frac{2q\Delta V}{m}} = \sqrt{\frac{2(1.6x10^{-19})(120)}{1.67x10^{-27}}} = \textbf{1.52x10}^5 \textbf{ m/s}$$

Electric Potential of a Point Charge

Up to this point we have focused our attention solely to that of a set of parallel plates. But those are not the ONLY thing that has an electric field. Remember, point charges have an electric field that surrounds them.

So imagine placing a TEST CHARGE out way from the point charge. Will it experience a change in electric potential energy? **YES!**

Thus is also must experience a change in electric potential as well.

Electric Potential

Let's use our "plate" analogy. Suppose we had a set of parallel plates symbolic of being "above the ground" which has potential difference of 50V and a CONSTANT Electric Field.

0.5d, V= **25 V**

0.25d, V= **12.5 V**
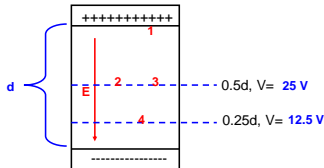
ΔV = ? From 1 to 2
25 V

ΔV = ? From 2 to 3
0 V

ΔV = ? From 3 to 4
12.5 V

ΔV = ? From 1 to 4
37.5 V

Notice that the "ELECTRIC POTENTIAL" (Voltage) DOES NOT change from 2 to 3. They are symbolically at the same height and thus at the same voltage. The line they are on is called an EQUIPOTENTIAL LINE. What do you notice about the orientation between the electric field lines and the equipotential lines?

Equipotential Lines

So let's say you had a positive charge. The electric field lines move AWAY from the charge. The equipotential lines are perpendicular to the electric field lines and thus make concentric circles around the charge. As you move AWAY from a positive charge the potential decreases. So V1>V2>V3.

Now that we have the direction or visual aspect of the equipotential line understood the question is how can we determine the potential at a certain distance away from the charge?

$\vec{E}$

V_1 V_2 V_3

r

V(r) = ?

Electric Potential of a Point Charge

$$\Delta V = \frac{W}{q} = \frac{F_E x}{q}; \quad x = r$$

$$\Delta V = \frac{F_E r}{q}; \quad F_E = k\frac{Qq}{r^2}$$

$$\Delta V = k\frac{Qqr}{qr^2} \rightarrow k\sum\frac{Q}{r}$$

Why the "sum" sign?

Voltage, unlike Electric Field, is NOT a vector! So if you have MORE than one charge you don't need to use vectors. Simply add up all the voltages that each charge contributes since voltage is a SCALAR.

WARNING! You must use the "sign" of the charge in this case.

Potential of a point charge

+q   d   −3q

$d\sqrt{2}$

d

+2q   +5q

Suppose we had 4 charges each at the corners of a square with sides equal to **d**.

If I wanted to find the potential at the CENTER I would SUM up all of the individual potentials.
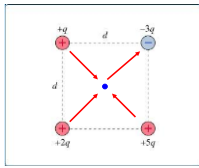
Thus the distance from a corner to the center will equal :

$$r = \frac{d\sqrt{2}}{2}$$

$$V_{center} = k\sum\frac{Q}{r}$$

$$V_{center} = k\sum(\frac{q}{r} + \frac{2q}{r} + \frac{-3q}{r} + \frac{5q}{r})$$

$$V_{center} = k(\frac{5q}{r}) \rightarrow k(\frac{10q}{d\sqrt{2}}) \rightarrow k\frac{5\sqrt{2}}{d}$$

## Electric field at the center? (Not so easy)

If they had asked us to find the electric field, we first would have to figure out the visual direction, use vectors to break individual electric fields into components and use the Pythagorean Theorem to find the resultant and inverse tangent to find the angle

So, yea….Electric Potentials are NICE to deal with!

$E_{resultant}$

Example

An electric dipole consists of two charges q_1 = +12nC and q_2 = -12nC, placed 10 cm apart as shown in the figure. Compute the potential at points a, b, and c.
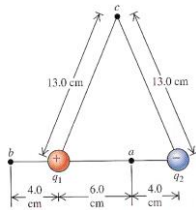
$$V_a = k \sum (\frac{q_1}{r_a} + \frac{q_2}{r_a})$$

$$V_a = 8.99x10^9 (\frac{12x10^{-9}}{0.06} + \frac{-12x10^{-9}}{0.04})$$

$$V_a = \quad \textbf{-899 V}$$

Example cont'
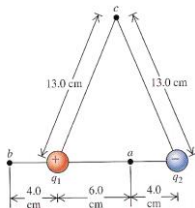
$$V_b = k \sum (\frac{q_1}{r_b} + \frac{q_2}{r_b})$$

$$V_b = 8.99x10^9 (\frac{12x10^{-9}}{0.04} + \frac{-12x10^{-9}}{0.14})$$

$$V_b = \quad \textbf{1926.4 V}$$

$$V_c = \quad \textbf{0 V}$$

Since direction isn't important, the electric potential at "c" is zero. The electric field however is NOT. The electric field would point to the right.

Applications of Electric Potential

Is there any way we can use a set of plates with an electric field? YES! We can make what is called a Parallel Plate Capacitor and Store Charges between the plates!

Dielectric capacitor
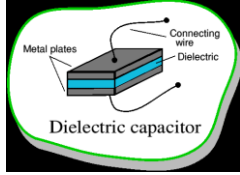
Storing Charges- Capacitors
A capacitor consists of 2 conductors of any shape placed near one another without touching. It is common; to fill up the region between these 2 conductors with an insulating material called a dielectric. We charge these plates with opposing charges to set up an electric field.

Capacitors in Kodak Cameras

Capacitors can be easily purchased at a local Radio Shack and are commonly found in disposable Kodak Cameras. When a voltage is applied to an empty capacitor, current flows through the capacitor and each side of the capacitor becomes charged. The two sides have equal and opposite charges. When the capacitor is fully charged, the current stops flowing. The collected charge is then ready to be discharged and when you press the flash it discharges very quickly released it in the form of light.
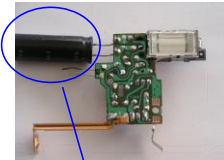
Cylindrical Capacitor

Capacitance

In the picture below, the capacitor is symbolized by a set of parallel lines. Once it's charged, the capacitor has the same voltage as the battery (1.5 volts on the battery means 1.5 volts on the capacitor) The difference between a capacitor and a battery is that a capacitor can dump its entire charge in a tiny fraction of a second, where a battery would take minutes to completely discharge itself. That's why the electronic flash on a camera uses a capacitor -- the battery charges up the flash's capacitor over several seconds, and then the capacitor dumps the full charge into the flash tube almost instantly
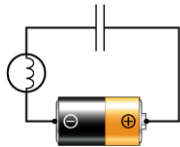
Measuring Capacitance

Let's go back to thinking about plates!

$\Delta V = Ed,$

$\Delta V \alpha E, if \ d = constant$ The unit for capacitance is the **FARAD, F.**

$E \alpha Q \quad Therefore$

$Q \alpha \Delta V$

$C = contant \ of \ proportiomlity$

$C = Capacitance$

$Q = CV$

$C = \dfrac{Q}{V}$
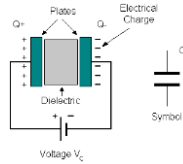
Capacitor Geometry

The capacitance of a capacitor depends on HOW you make it.

$C \alpha A \quad C \alpha \dfrac{1}{d}$

$A = area \ of \ plate$

$d = distance \ beteween \ plates$

$C \alpha \dfrac{A}{d}$

$\varepsilon_o = constant \ of \ proportiomlity$

$\varepsilon_o = vacuum \ permittivity \ constant$

$\varepsilon_o = 8.85x10^{-12} \dfrac{C^2}{Nm^2}$

$C = \dfrac{\varepsilon_o A}{d}$
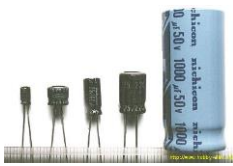
Capacitor Problems

What is the AREA of a 1F capacitor that has a plate separation of 1 mm?

$C = \varepsilon_o \dfrac{A}{D}$

$1 = 8.85x10^{-12} \dfrac{A}{0.001}$

$A = \quad \text{1.13x10}^8 \text{ m}^2$

$Sides = \quad \text{10629 m}$

Is this a practical capacitor to build?

NO! – How can you build this then?

The answer lies in REDUCING the AREA. But you must have a CAPACITANCE of 1 F. **How can you keep the capacitance at 1 F and reduce the Area at the same time?**

Add a DIELECTRIC!!!

Dielectric

Remember, the dielectric is an insulating material placed between the conductors to help store the charge. In the previous example we assumed there was NO dielectric and thus a vacuum between the plates.

Dielectric capacitor
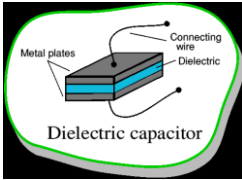
$$C = k\varepsilon_o \frac{A}{d}$$

$$k = \text{Dielectric}$$

All insulating materials have a dielectric constant associated with it. Here now you can reduce the AREA and use a LARGE dielectric to establish the capacitance at 1 F.

Using MORE than 1 capacitor

Let's say you decide that 1 capacitor will not be enough to build what you need to build. You may need to use more than 1. There are 2 basic ways to assemble them together

- Series – One after another
- Parallel – between a set of junctions and parallel to each other.
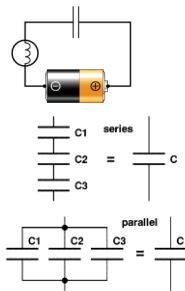
Capacitors in Series

Capacitors in series each charge each other by INDUCTION. So they each have the SAME charge. The electric potential on the other hand is divided up amongst them. In other words, the sum of the individual voltages will equal the total voltage of the battery or power source.
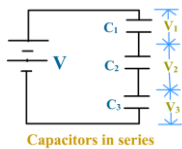
Capacitors in series

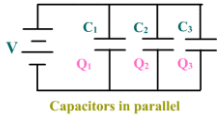$$V_{Total} = V_1 + V_2 + V_3 \ldots$$

$$V = \frac{Q}{C}$$

$$\frac{Q_T}{C_T} = \frac{Q_1}{C_1} + \frac{Q_2}{C_2} + \frac{Q_3}{C_3}$$

$$\frac{1}{C_{Series}} = \Sigma \frac{1}{C_i}$$

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Capacitors in Parallel

In a parallel configuration, the **voltage is the same** because ALL THREE capacitors touch BOTH ends of the battery. As a result, they split up the charge amongst them.

Capacitors in parallel
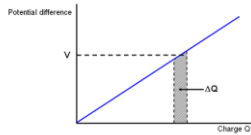
$$Q_{Total} = Q_1 + Q_2 + Q_3 \ldots$$
$$Q = CV$$
$$C_T V_T = C_1 V_1 + C_2 V_2 + C_3 V_3$$
$$V_T = V_1 = V_2 = V_3$$
$$C_{Parallel} = \Sigma C_i$$

Capacitors "STORE" energy

Anytime you have a situation where energy is "STORED" it is called POTENTIAL. In this case we have capacitor potential energy, U_c

Suppose we plot a V vs. Q graph. If we wanted to find the AREA we would MULTIPLY the 2 variables according to the equation for Area.
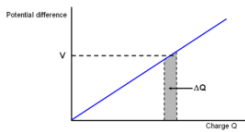
A = bh

When we do this we get Area = VQ

Let's do a unit check!

Voltage = Joules/Coulomb
Charge = Coulombs
Area = **ENERGY**

Potential Energy of a Capacitor

Since the AREA under the line is a triangle, the ENERGY(area) =1/2VQ

$$U_C = \frac{1}{2}VQ \quad C = \frac{Q}{V}$$

$$\boxed{U_C = \frac{1}{2}V(VC) \rightarrow \frac{1}{2}CV^2}$$

$$U_C = \frac{1}{2}(\frac{Q}{C})Q \rightarrow \frac{Q^2}{2C}$$

This energy or area is referred as the **potential energy stored inside a capacitor**.

Note: The slope of the line is the inverse of the capacitance.

most common form