2 Mechanics

2.1 Kinematics

Assessment statements

2.1.1 Define displacement, velocity, speed and acceleration.
2.1.2 Explain the difference between instantaneous and average values of speed, velocity and acceleration.
2.1.3 Outline the conditions under which the equations for uniformly accelerated motion may be applied.
2.1.9 Determine relative velocity in one and in two dimensions.

In Chapter 1, we observed that things move and now we are going to mathematically model that movement. Before we do that, we must define some quantities that we are going to use.

Displacement and distance

It is important to understand the difference between distance travelled and displacement. To explain this, consider the route marked out on the map shown in Figure 2.1

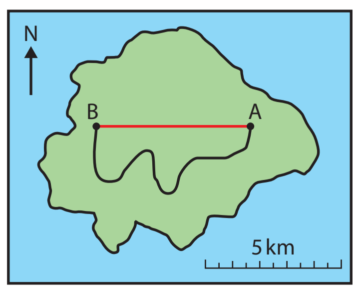


Figure 2.1

Displacement is the distance moved in a particular direction.

The unit of displacement is the metre (m).

Displacement is a vector quantity.

On the map, the displacement is the length of the straight line from A to B, a distance of 5 km west. (Note: since displacement is a vector you should always say what the direction is.)

Distance is how far you have travelled from A to B.

The unit of distance is also the metre.

Distance is a scalar quantity.

In this example, the distance travelled is the length of the path taken, which is about 10 km.

Sometimes this difference leads to a surprising result. For example, if you run all the way round a running track you will have travelled a distance of 400 m but your displacement will be 0 m.

In everyday life, it is often more important to know the distance travelled. For example, if you are going to travel from Paris to Lyon by road you will want to know that the distance by road is 450 km, not that your final displacement will be

336 km SE. However, in physics, we break everything down into its simplest units, so we start by considering motion in a straight line only. In this case it is more useful to know the displacement, since that also has information about which direction you have moved.

Velocity and speed

Both speed and velocity are a measure of how fast a body is moving, but velocity is a vector quantity and speed is a scalar.

Velocity is defined as the displacement per unit time.

$$\text{velocity} = \frac{\text{displacement}}{\text{time}}$$

The unit of velocity is m s^{-1}.

Velocity is a vector quantity.

Speed is defined as the distance travelled per unit time.

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

The unit of speed is also m s^{-1}.

Speed is a scalar quantity.

Exercise

1 Convert the following speeds into m s^{-1}.
- **(a)** A car travelling at $100\ \text{km h}^{-1}$
- **(b)** A runner running at $20\ \text{km h}^{-1}$

Figure 2.2 It's not possible to take this route across Bangkok with a constant velocity.

Average velocity and instantaneous velocity

Consider travelling by car from the north of Bangkok to the south – a distance of about 16 km. If the journey takes 4 hours, you can calculate your velocity to be $\frac{16}{4} = 4\ \text{km h}^{-1}$ in a southwards direction. This doesn't tell you anything about the journey, just the difference between the beginning and the end (unless you managed to travel at a constant speed in a straight line). The value calculated is the **average velocity** and in this example it is quite useless. If we broke the trip down into lots of small pieces, each lasting only one second, then for each second the car could be considered to be travelling in a straight line at a constant speed. For these short stages we could quote the car's **instantaneous velocity** – that's how fast it's going at that moment in time and in which direction.

The bus in the photo has a constant velocity for a very short time.

Measuring velocity

You can measure velocity with a photogate connected to a timer or computer. When a card passes through the gate it is sensed by the timer, switching it on or off.

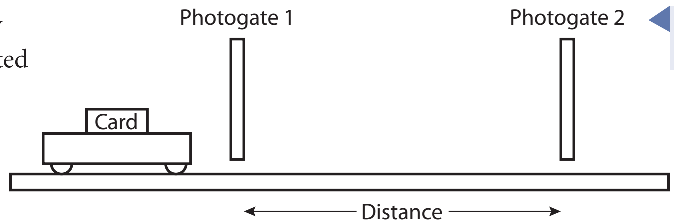


Figure 2.3 Experimental set up for measuring velocity.

$$\text{average velocity} = \frac{\text{distance}}{\text{time taken to travel between photogates}}$$

$$\text{instantaneous velocity} = \frac{\text{length of card}}{\text{time for card to pass through gate}}$$

Velocity is relative

When quoting the velocity of a body, it is important to say what the velocity is measured relative to. Consider the people in Figure 2.4

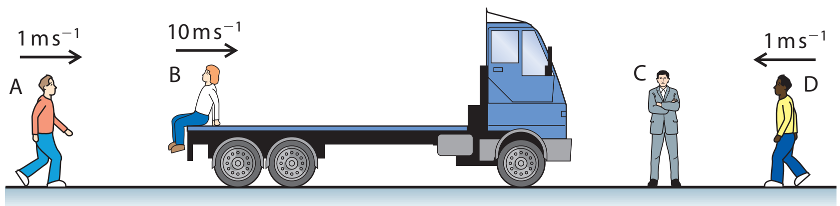


Figure 2.4 Two observers measuring the same velocity.

C measures the velocity of *A* to be 1 m s^{-1} but to B (moving on the truck towards *C*) the velocity of *A* is -9 m s^{-1} (*B* will see *A* moving away in a negative direction). You might think that *A* can't have two velocities, but he can – velocity is relative. In this example there are two observers, *B* and *C*. Each observer has a different 'frame of reference'. To convert a velocity, to *B*'s frame of reference, we must subtract the velocity of *B* relative to *C*; this is 10 m s^{-1}.

So the velocity of *A* relative to $B = 1 - 10 = -9$ m s^{-1}

We can try the same with *D* who has a velocity of -1 m s^{-1} measured by *C* and $-1 - 10 = -11$ m s^{-1} measured by *B*.

This also works in two dimensions as follows:

A now walks across the road as illustrated by the aerial view in Figure 2.5. The velocity of *A* relative to *C* is 1 m s^{-1} north.

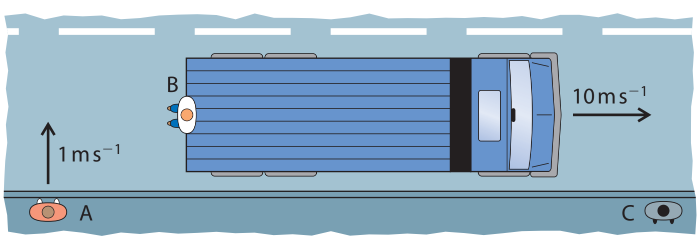


Figure 2.5 *A* now walks across the road..

The velocity of *A* relative to *B* can now be found by subtracting the vectors as shown in Figure 2.6.

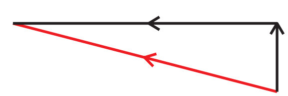


Figure 2.6 Subtracting vectors gives the relative velocity.

● **Examiner's hint:** Velocity is a vector. If the motion is in one dimension, the direction of velocity is given by its sign. Generally, right is positive and left negative.

Exercise

2 An observer standing on a road watches a bird flying east at a velocity of 10 m s^{-1}. A second observer, driving a car along the road northwards at 20 m s^{-1} sees the bird. What is the velocity of the bird relative to the driver?

Acceleration

In everyday usage, the word *accelerate* means to go faster. However in physics: acceleration is defined as the rate of change of velocity.

$$\text{acceleration} = \frac{\text{change of velocity}}{\text{time}}$$

The unit of acceleration is m s^{-2}.

Acceleration is a vector quantity.

This means that whenever a body changes its velocity it accelerates. This could be because it is getting faster, slower or just changing direction. In the example of the journey across Bangkok, the car would have been slowing down, speeding up and going round corners almost the whole time, so it would have had many different accelerations. However, this example is far too complicated for us to consider in this course (and probably any physics course). For most of this chapter we will only consider the simplest example of accelerated motion, constant acceleration.

Bodies
When we refer to a body in physics we generally mean a ball not a human body.

Constant acceleration in one dimension

In one-dimensional motion, the acceleration, velocity and displacement are all in the same direction. This means they can simply be added without having to draw triangles. Figure 2.7 shows a body that is starting from an initial velocity u and accelerating at a constant rate to velocity v in t seconds. The distance travelled in this time is s. Since the motion is in a straight line, this is also the displacement.

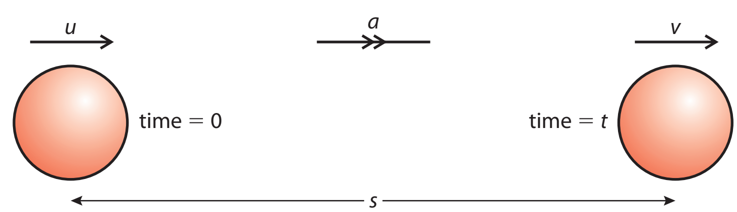


Figure 2.7 A red ball is accelerated at a constant rate.

Using the definitions already stated, we can write equations related to this example.

Average velocity

From the definition, the average velocity $= \dfrac{\text{displacement}}{\text{time}}$

So average velocity $= \dfrac{s}{t}$ (1)

Since the velocity changes at a constant rate from the beginning to the end, we can also calculate the average velocity by adding the velocities and dividing by two.

Average velocity $= \dfrac{(u + v)}{2}$ (2)

Acceleration

Acceleration is defined as the rate of change of velocity.

So $a = \dfrac{(v - u)}{t}$ (3)

We can use these equations to solve any problem involving constant acceleration. However, to make problem solving easier, we can derive two more equations by substituting from one into the other.

Equating equations (1) and(2)

$$\frac{s}{t} = \frac{(u + v)}{2}$$

so $s = \dfrac{(u + v)\,t}{2}$ (4)

Rearranging (3) gives $v = u + at$

If we substitute for v in equation (4) we get $s = ut + \frac{1}{2}at^2$ (5)

Rearranging (3) again gives $t = \dfrac{(v - u)}{a}$

If t is now substituted in equation (4) we get $v^2 = u^2 + 2as$ (6)

These equations are sometimes known as the *suvat* equations. If you know any 3 of *suva* and *t* you can find either of the other two in one step.

suvat equations

$a = \dfrac{(v - u)}{t}$

$s = \dfrac{(u + v)\,t}{2}$

$s = ut + \frac{1}{2}at^2$

$v^2 = u^2 + 2as$

Worked example

1 A car travelling at $10\,\text{m s}^{-1}$ accelerates at $2\,\text{m s}^{-2}$ for 5 s. What is its displacement?

Solution

The first thing to do is draw a simple diagram like Figure 2.8.

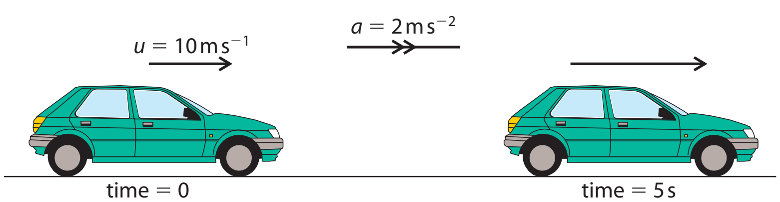


Figure 2.8 A simple diagram is always the best start.

This enables you to see what is happening at a glance rather than reading the text. The next stage is to make a list of *suvat*.

$$s = ?$$
$$u = 10\,\text{m s}^{-1}$$
$$v = ?$$
$$a = 2\,\text{m s}^{-2}$$
$$t = 5\,\text{s}$$

● **Examiner's hint:** You don't need to include units in all stages of a calculation, just the answer.

To find s you need an equation that contains *suat*. The only equation with all 4 of these quantities is $s = ut + \frac{1}{2}at^2$

Using this equation gives:

$$s = 10 \times 5 + \tfrac{1}{2} \times 2 \times 5^2$$
$$s = 75\,\text{m}$$

The sign of displacement, velocity and acceleration

We must not forget that displacement, velocity and acceleration are vectors. This means that they have direction. However, since this is a one-dimensional example, there are only two possible directions, forward and backward. We know which direction the quantity is in from the sign.

A positive displacement means that the body has moved right.

A positive velocity means the body is moving to the right.

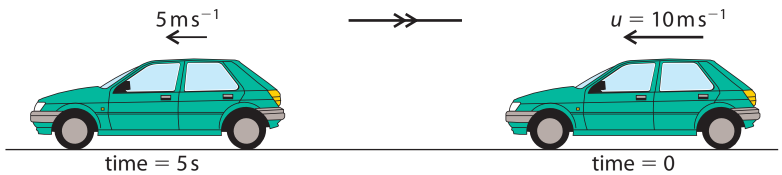


Figure 2.9

A positive acceleration means that the body is either moving to the right and getting faster or moving to the left and getting slower. This can be confusing, so consider the following example.

The car is travelling in a negative direction so the velocities are negative.

$$u = -10\ \text{m s}^{-1}$$
$$v = -5\ \text{m s}^{-1}$$
$$t = 5\ \text{s}$$

The acceleration is therefore given by

$$a = \frac{(v - u)}{t} = \frac{-5 - -10}{5} = 1\ \text{m s}^{-2}$$

The positive sign tells us that the acceleration is in a positive direction (right) even though the car is travelling in a negative direction (left).

Example

A body with a constant acceleration of $-5\ \text{m s}^{-2}$ is travelling to the right with a velocity of $20\ \text{m s}^{-1}$. What will its displacement be after 20 s?

$$s = ?$$
$$u = 20\ \text{m s}^{-1}$$
$$v = ?$$
$$a = -5\ \text{m s}^{-2}$$
$$t = 20\ \text{s}$$

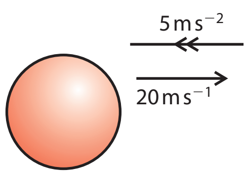


Figure 2.10 The acceleration is negative so pointing to the left.

To calculate s we can use the equation $s = ut + \frac{1}{2}at^2$

$$s = 20 \times 20 + \tfrac{1}{2}(-5) \times 20^2 = 400 - 1000 = -600\ \text{m}$$

This means that the final displacement of the body is to the left of the starting point. It has gone forward, stopped and then gone backwards.

g

The acceleration due to gravity is not constant all over the Earth. $9.81\ \text{m s}^{-2}$ is the average value. The acceleration also gets smaller the higher you go. However we ignore this change when conducting experiments in the lab since labs aren't that high.

To make the examples easier to follow, $g = 10\ \text{m s}^{-2}$ is used throughout; you should only use this approximate value in exam questions if told to do so.

Exercises

3 Calculate the final velocity of a body that starts from rest and accelerates at $5\ \text{m s}^{-2}$ for a distance of 100 m.

4 A body starts with a velocity of $20\ \text{m s}^{-1}$ and accelerates for 200 m with an acceleration of $5\ \text{m s}^{-2}$. What is the final velocity of the body?

5 A body accelerates at $10\ \text{m s}^{-2}$ reaching a final velocity of $20\ \text{m s}^{-1}$ in 5 s. What was the initial velocity of the body?

2.2 Free fall motion

Assessment statements

2.1.4 Identify the acceleration of a body falling in a vacuum near the Earth's surface with the acceleration g of free fall.

2.1.5 Solve problems involving the equations of uniformly accelerated motion.

2.1.6 Describe the effects of air resistance on falling objects.

Although a car was used in one of the previous illustrations, the acceleration of a car is not usually constant, so we shouldn't use the *suvat* equations. The only example of constant acceleration that we see in everyday life is when a body is dropped. Even then the acceleration is only constant for a short distance.

Acceleration of free fall

When a body is allowed to fall freely we say it is in free fall. Bodies falling freely on the Earth fall with an acceleration of about 9.81 m s^{-2}. (It depends where you are.) The body falls because of gravity. For that reason we use the letter g to denote this acceleration. Since the acceleration is constant, we can use the *suvat* equations to solve problems.

The effect of air resistance
If you jump out of a plane (with a parachute on) you will feel the push of the air as it rushes past you. As you fall faster and faster, the air will push upwards more and more until you can't go any faster. At this point you have reached terminal velocity. We will come back to this example after introducing forces.

Exercises

In these calculations use $g = 10$ m s^{-2}.

6 A ball is thrown upwards with a velocity of 30 m s^{-1}. What is the displacement of the ball after 2 s?

7 A ball is dropped. What will its velocity be after falling 65 cm?

8 A ball is thrown upwards with a velocity of 20 m s^{-1}. After how many seconds will the ball return to its starting point?

Measuring *g*

Measuring g by timing a ball falling from different heights is a common physics experiment that you could well perform in the practical programme of the IB course. There are various different ways of doing this but a common method is to use a timer that starts when the ball is released and stops when it hits a platform. An example of this apparatus is shown in the photo. The distance travelled by the ball and the time taken are related by the *suvat* equation $s = ut + \frac{1}{2}at^2$. This simplifies to $s = \frac{1}{2}at^2$ since the initial velocity is zero. This means that s is proportional to t^2 so if you plot a graph of s against t^2 you will get a straight line whose gradient is $\frac{1}{2}g$.

Free fall apparatus.

2.3 Graphical representation of motion

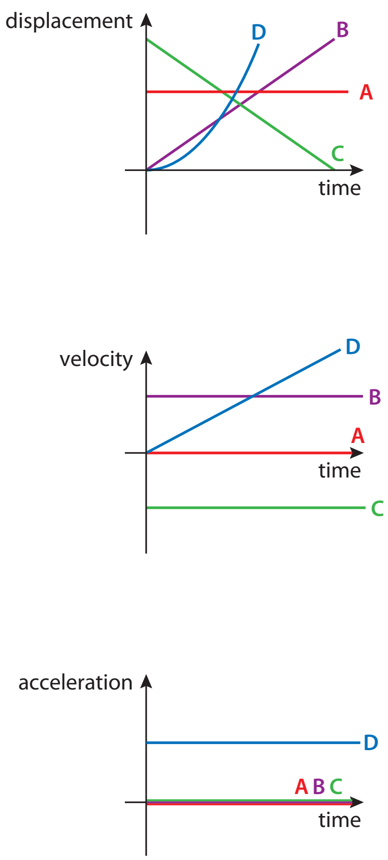
Assessment statements

2.1.7 Draw and analyse distance–time graphs, displacement–time graphs, velocity–time graphs and acceleration–time graphs.

2.1.8 Calculate and interpret the gradients of displacement–time graphs and velocity–time graphs, and the areas under velocity–time graphs and acceleration–time graphs.

Graphs are used in physics to give a visual representation of relationships. In kinematics they can be used to show how displacement, velocity and acceleration change with time. Figure 2.11 shows the graphs for four different examples of motion. They are placed vertically since they all have the same time axis.

Figure 2.11 Graphical representation of motion.



Line A

A body that is not moving.
Displacement is always the same.
Velocity is zero.
Acceleration is zero.

Line B

A body that is travelling with a constant positive velocity.
Displacement increases linearly with time.
Velocity is a constant positive value.
Acceleration is zero.

Line C

A body that has a constant negative velocity.
Displacement is decreasing linearly with time.
Velocity is a constant negative value.
Acceleration is zero.

Line D

A body that is accelerating with constant acceleration.
Displacement is increasing at a non-linear rate. The shape of this line is a parabola since displacement is proportional to t^2 ($s = ut + \frac{1}{2}at^2$).
Velocity is increasing linearly with time.
Acceleration is a constant positive value.

• **Examiner's hint:** You need to be able to
- figure out what kind of motion a body has by looking at the graphs
- sketch graphs for a given motion.

The best way to go about sketching graphs is to split the motion into sections then plot where the body is at different times; joining these points will give the displacement–time graph. Once you have done that you can work out the v–t and a–t graphs by looking at the s–t graph rather than the motion.

Gradient of displacement–time

$$\text{The gradient of a graph is } \frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x}$$

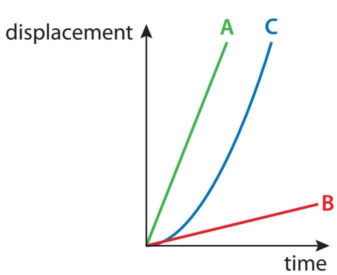
In the case of the displacement–time graph this will give

$$\text{gradient} = \frac{\Delta s}{\Delta t}$$

This is the same as velocity.

So the gradient of the displacement–time graph equals the velocity. Using this information, we can see that line A in Figure 2.12 represents a body with greater velocity than line B and that since the gradient of line C is increasing, this must be the graph for an accelerating body.

Figure 2.12



Instantaneous velocity

When a body accelerates its velocity is constantly changing. The displacement–time graph for this motion is therefore a curve. To find the instantaneous velocity from the graph we can draw a tangent to the curve and find the gradient of the tangent as shown in Figure 2.13.

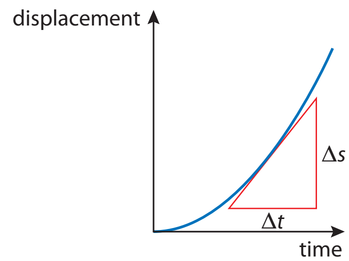


Figure 2.13

Area under velocity–time graph

The area under the velocity–time graph for the body travelling at constant velocity v shown in Figure 2.14 is given by

$$\text{area} = v\Delta t$$

But we know from the definition of velocity that $v = \frac{\Delta s}{\Delta t}$

Rearranging gives $\Delta s = v\Delta t$ so the area under a velocity–time graph gives the displacement.

This is true not only for simple cases such as this but for all examples.

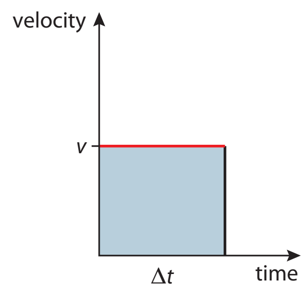


Figure 2.14

Gradient of velocity–time graph

The gradient of the velocity–time graph is given by $\frac{\Delta v}{\Delta t}$. This is the same as acceleration.

Area under acceleration–time graph

The area under an acceleration–time graph in Figure 2.15 is given by $a\Delta t$. But we know from the definition of acceleration that $a = \frac{(v - u)}{t}$

Rearranging this gives $v - u = a\Delta t$ so the area under the graph gives the change in velocity.

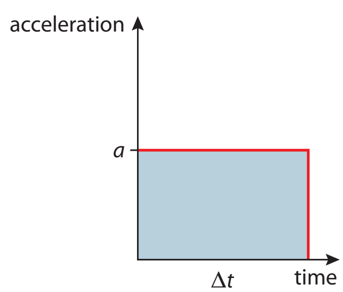


Figure 2.15

If you have covered calculus in your maths course you may recognise these equations:

$$v = \frac{ds}{dt} a = \frac{dv}{dt} = \frac{d^2s}{dt^2} \text{ and } s = \int v dt,\ v = \int a dt$$

Exercises

9 Sketch a velocity–time graph for a body starting from rest and accelerating at a constant rate to a final velocity of 25 m s^{-1} in 10 seconds. Use the graph to find the distance travelled and the acceleration of the body.

10 Describe the motion of the body whose velocity–time graph is shown in Figure 2.16. What is the final displacement of the body?

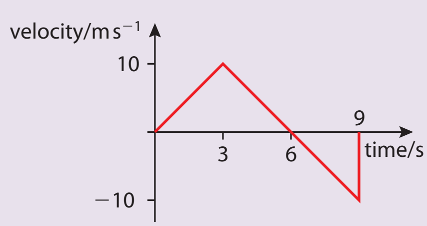
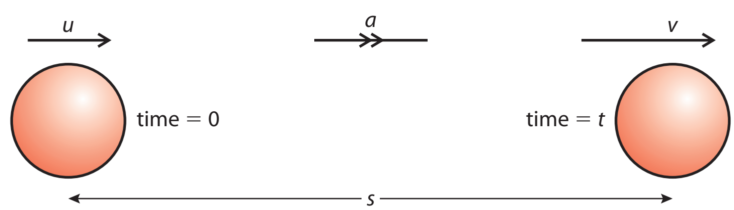


Figure 2.16

Example 1: the *suvat* example

As an example let us consider the motion we looked at when deriving the *suvat* equations.

Figure 2.17 A body with constant acceleration.

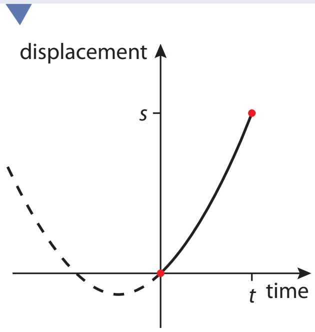


Negative time
Negative time doesn't mean going back in time – it means the time before you started the clock.

Displacement–time

The body starts with velocity u and travels to the right with constant acceleration, a for a time t. If we take the starting point to be zero displacement, then the displacement–time graph starts from zero and rises to s in t seconds. We can therefore plot the two points shown in Figure 2.18. The body is accelerating so the line joining these points is a parabola. The whole parabola has been drawn to show what it would look like – the reason it is offset is because the body is not starting from rest. The part of the curve to the left of the origin tells us what the particle was doing before we started the clock.

Figure 2.18



Velocity–time

Figure 2.19

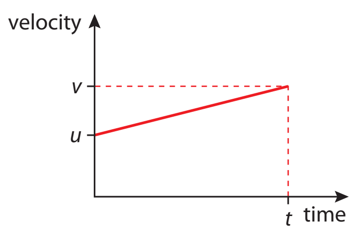


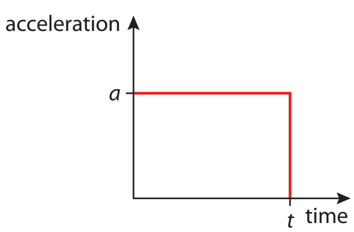
Figure 2.19 is a straight line with a positive gradient showing that the acceleration is constant. The line doesn't start from the origin since the initial velocity is u.

The gradient of this line is $\frac{(v - u)}{t}$ which we know from the *suvat* equations is acceleration.

The area under the line makes the shape of a trapezium. The area of this trapezium is $\frac{1}{2}(v + u)t$. This is the *suvat* equation for s.

Acceleration–time

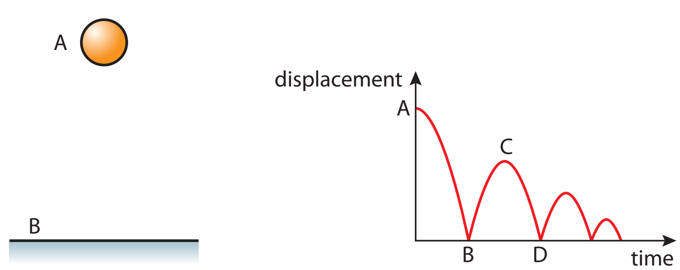
Figure 2.20



The acceleration is constant so the acceleration–time graph is simply a horizontal line as shown in Figure 2.20. The area under this line is $a \times t$ which we know from the *suvat* equations equals $(v - u)$.

Example 2: The bouncing ball

Figure 2.21



Consider a rubber ball dropped from some position above the ground A onto a hard surface B. The ball bounces up and down several times. Figure 2.21 shows the displacement–time graph for 4 bounces. From the graph we see that the ball starts above the ground then falls with increasing velocity (as deduced by the increasing negative gradient). When the ball bounces at B the velocity suddenly changes from negative to positive as the ball begins to travel back up. As the ball goes up, its velocity gets less until it stops at C and begins to fall again.

To view a simulation that enables you to plot the graphs as you watch the motion, visit www.heinemann.co.uk/hotlinks, enter the express code 4426P and click on Weblink 2.1.

Exercise

11 By considering the gradient of the displacement–time graph in Figure 2.21 plot the velocity–time graph for the motion of the bouncing ball.

Example 3: A ball falling with air resistance

Figure 2.22 represents the motion of a ball that is dropped several hundred metres through the air. It starts from rest and accelerates for some time. As the ball accelerates, the air resistance gets bigger, which prevents the ball from getting any faster. At this point the ball continues with constant velocity.

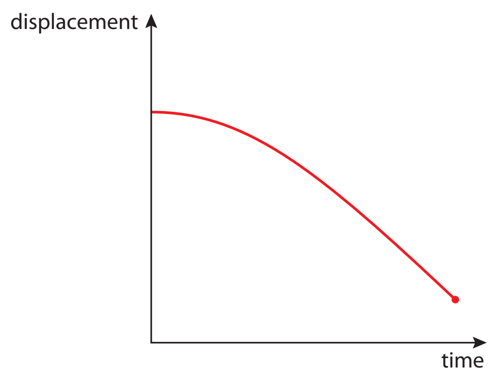


Figure 2.22

Exercise

12 By considering the gradient of the displacement–time graph plot the velocity–time graph for the motion of the falling ball.

2.4 Projectile motion

Assessment statements

- 9.1.1 State the independence of the vertical and the horizontal components of velocity for a projectile in a uniform field.
- 9.1.2 Describe and sketch the trajectory of projectile motion as parabolic in the absence of air resistance.
- 9.1.3 Describe qualitatively the effect of air resistance on the trajectory of a projectile.
- 9.1.4 Solve problems on projectile motion.