2 Mechanics

2.1 Kinematics

Assessment statements

2.1.1 Define displacement, velocity, speed and acceleration.
2.1.2 Explain the difference between instantaneous and average values of speed, velocity and acceleration.
2.1.3 Outline the conditions under which the equations for uniformly accelerated motion may be applied.
2.1.9 Determine relative velocity in one and in two dimensions.

In Chapter 1, we observed that things move and now we are going to mathematically model that movement. Before we do that, we must define some quantities that we are going to use.

Displacement and distance

It is important to understand the difference between distance travelled and displacement. To explain this, consider the route marked out on the map shown in Figure 2.1

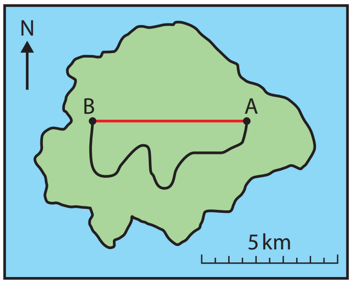


Figure 2.1

Displacement is the distance moved in a particular direction.

The unit of displacement is the metre (m).

Displacement is a vector quantity.

On the map, the displacement is the length of the straight line from A to B, a distance of 5 km west. (Note: since displacement is a vector you should always say what the direction is.)

Distance is how far you have travelled from A to B.

The unit of distance is also the metre.

Distance is a scalar quantity.

In this example, the distance travelled is the length of the path taken, which is about 10 km.

Sometimes this difference leads to a surprising result. For example, if you run all the way round a running track you will have travelled a distance of 400 m but your displacement will be 0 m.

In everyday life, it is often more important to know the distance travelled. For example, if you are going to travel from Paris to Lyon by road you will want to know that the distance by road is 450 km, not that your final displacement will be

336 km SE. However, in physics, we break everything down into its simplest units, so we start by considering motion in a straight line only. In this case it is more useful to know the displacement, since that also has information about which direction you have moved.

Velocity and speed

Both speed and velocity are a measure of how fast a body is moving, but velocity is a vector quantity and speed is a scalar.

Velocity is defined as the displacement per unit time.

$$\text{velocity} = \frac{\text{displacement}}{\text{time}}$$

The unit of velocity is m s^{-1}.

Velocity is a vector quantity.

Speed is defined as the distance travelled per unit time.

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

The unit of speed is also m s^{-1}.

Speed is a scalar quantity.

Exercise

1 Convert the following speeds into m s^{-1}.

(a) A car travelling at $100\ \text{km h}^{-1}$

(b) A runner running at $20\ \text{km h}^{-1}$

Figure 2.2 It's not possible to take this route across Bangkok with a constant velocity.

Average velocity and instantaneous velocity

Consider travelling by car from the north of Bangkok to the south – a distance of about 16 km. If the journey takes 4 hours, you can calculate your velocity to be $\frac{16}{4} = 4\ \text{km h}^{-1}$ in a southwards direction. This doesn't tell you anything about the journey, just the difference between the beginning and the end (unless you managed to travel at a constant speed in a straight line). The value calculated is the **average velocity** and in this example it is quite useless. If we broke the trip down into lots of small pieces, each lasting only one second, then for each second the car could be considered to be travelling in a straight line at a constant speed. For these short stages we could quote the car's **instantaneous velocity** – that's how fast it's going at that moment in time and in which direction.

The bus in the photo has a constant velocity for a very short time.

Measuring velocity

You can measure velocity with a photogate connected to a timer or computer. When a card passes through the gate it is sensed by the timer, switching it on or off.

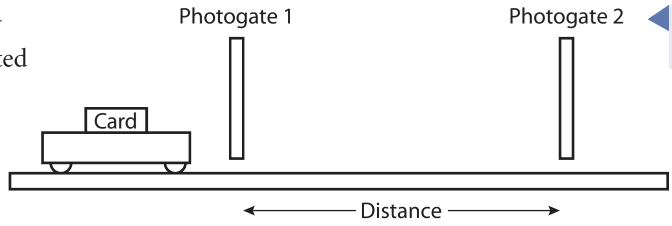


Figure 2.3 Experimental set up for measuring velocity.

$$\text{average velocity} = \frac{\text{distance}}{\text{time taken to travel between photogates}}$$

$$\text{instantaneous velocity} = \frac{\text{length of card}}{\text{time for card to pass through gate}}$$

Velocity is relative

When quoting the velocity of a body, it is important to say what the velocity is measured relative to. Consider the people in Figure 2.4

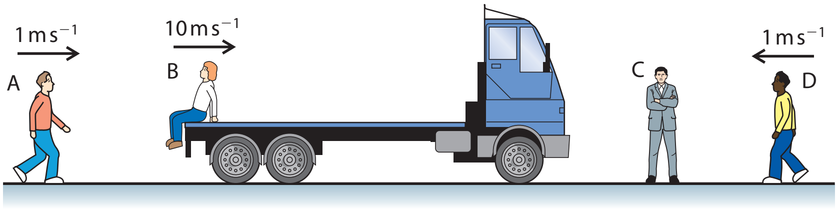


Figure 2.4 Two observers measuring the same velocity.

C measures the velocity of *A* to be 1 m s^{-1} but to B (moving on the truck towards *C*) the velocity of *A* is −9 m s^{-1} (*B* will see *A* moving away in a negative direction). You might think that *A* can't have two velocities, but he can – velocity is relative. In this example there are two observers, *B* and *C*. Each observer has a different 'frame of reference'. To convert a velocity, to *B*'s frame of reference, we must subtract the velocity of *B* relative to *C*; this is 10 m s^{-1}.

● **Examiner's hint:** Velocity is a vector. If the motion is in one dimension, the direction of velocity is given by its sign. Generally, right is positive and left negative.

So the velocity of *A* relative to *B* = 1 − 10 = −9 m s^{-1}

We can try the same with *D* who has a velocity of −1 m s^{-1} measured by *C* and −1 − 10 = −11 m s^{-1} measured by *B*.

This also works in two dimensions as follows:

A now walks across the road as illustrated by the aerial view in Figure 2.5. The velocity of *A* relative to *C* is 1 m s^{-1} north.

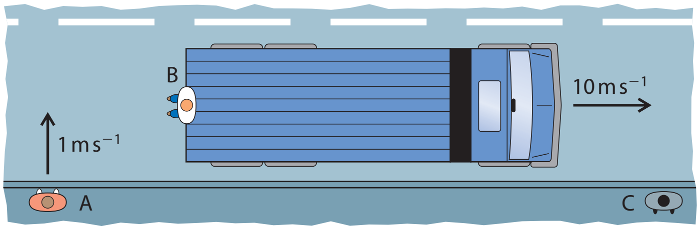


Figure 2.5 *A* now walks across the road..

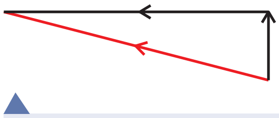


Figure 2.6 Subtracting vectors gives the relative velocity.

The velocity of *A* relative to *B* can now be found by subtracting the vectors as shown in Figure 2.6.

Exercise

2 An observer standing on a road watches a bird flying east at a velocity of 10 m s^{-1}. A second observer, driving a car along the road northwards at 20 m s^{-1} sees the bird. What is the velocity of the bird relative to the driver?

Acceleration

In everyday usage, the word *accelerate* means to go faster. However in physics: acceleration is defined as the rate of change of velocity.

$$\text{acceleration} = \frac{\text{change of velocity}}{\text{time}}$$

The unit of acceleration is $\mathrm{m\,s^{-2}}$.

Acceleration is a vector quantity.

This means that whenever a body changes its velocity it accelerates. This could be because it is getting faster, slower or just changing direction. In the example of the journey across Bangkok, the car would have been slowing down, speeding up and going round corners almost the whole time, so it would have had many different accelerations. However, this example is far too complicated for us to consider in this course (and probably any physics course). For most of this chapter we will only consider the simplest example of accelerated motion, constant acceleration.

Bodies
When we refer to a body in physics we generally mean a ball not a human body.

Constant acceleration in one dimension

In one-dimensional motion, the acceleration, velocity and displacement are all in the same direction. This means they can simply be added without having to draw triangles. Figure 2.7 shows a body that is starting from an initial velocity u and accelerating at a constant rate to velocity v in t seconds. The distance travelled in this time is s. Since the motion is in a straight line, this is also the displacement.

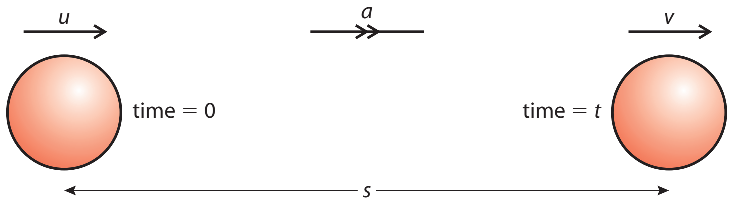


Figure 2.7 A red ball is accelerated at a constant rate.

Using the definitions already stated, we can write equations related to this example.

Average velocity

From the definition, the average velocity $= \dfrac{\text{displacement}}{\text{time}}$

So average velocity $= \dfrac{s}{t}$ (1)

Since the velocity changes at a constant rate from the beginning to the end, we can also calculate the average velocity by adding the velocities and dividing by two.

Average velocity $= \dfrac{(u + v)}{2}$ (2)

Acceleration

Acceleration is defined as the rate of change of velocity.

So $a = \dfrac{(v - u)}{t}$ (3)

We can use these equations to solve any problem involving constant acceleration. However, to make problem solving easier, we can derive two more equations by substituting from one into the other.

Equating equations (1) and(2)

$$\frac{s}{t} = \frac{(u + v)}{2}$$

so $s = \frac{(u + v)\,t}{2}$ (4)

Rearranging (3) gives $v = u + at$

If we substitute for v in equation (4) we get $s = ut + \frac{1}{2}at^2$ (5)

Rearranging (3) again gives $t = \frac{(v - u)}{a}$

If t is now substituted in equation (4) we get $v^2 = u^2 + 2as$ (6)

These equations are sometimes known as the *suvat* equations. If you know any 3 of *suva* and *t* you can find either of the other two in one step.

suvat equations

$a = \frac{(v - u)}{t}$

$s = \frac{(u + v)\,t}{2}$

$s = ut + \frac{1}{2}at^2$

$v^2 = u^2 + 2as$

Worked example

1 A car travelling at 10 m s^{-1} accelerates at 2 m s^{-2} for 5 s. What is its displacement?

Solution

The first thing to do is draw a simple diagram like Figure 2.8.

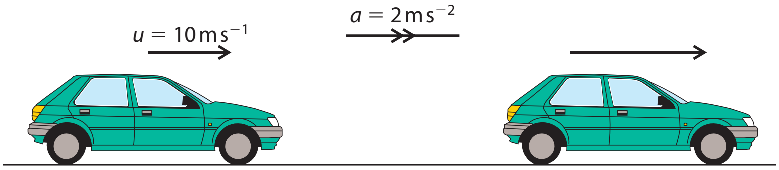


Figure 2.8 A simple diagram is always the best start.

This enables you to see what is happening at a glance rather than reading the text. The next stage is to make a list of *suvat*.

$$s = ?$$
$$u = 10\text{ m s}^{-1}$$
$$v = ?$$
$$a = 2\text{ m s}^{-2}$$
$$t = 5\text{ s}$$

● **Examiner's hint:** You don't need to include units in all stages of a calculation, just the answer.

To find s you need an equation that contains *suat*. The only equation with all 4 of these quantities is $s = ut + \frac{1}{2}at^2$

Using this equation gives:

$$s = 10 \times 5 + \frac{1}{2} \times 2 \times 5^2$$
$$s = 75\text{ m}$$

The sign of displacement, velocity and acceleration

We must not forget that displacement, velocity and acceleration are vectors. This means that they have direction. However, since this is a one-dimensional example, there are only two possible directions, forward and backward. We know which direction the quantity is in from the sign.

A positive displacement means that the body has moved right.

A positive velocity means the body is moving to the right.

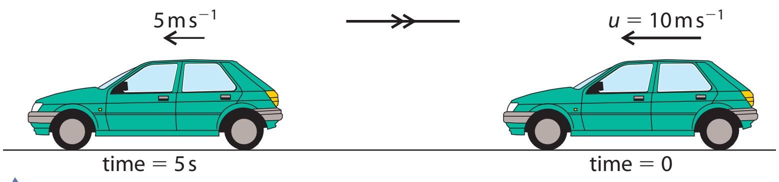


Figure 2.9

A positive acceleration means that the body is either moving to the right and getting faster or moving to the left and getting slower. This can be confusing, so consider the following example.

The car is travelling in a negative direction so the velocities are negative.

$$u = -10\ \text{m s}^{-1}$$
$$v = -5\ \text{m s}^{-1}$$
$$t = 5\ \text{s}$$

The acceleration is therefore given by

$$a = \frac{(v - u)}{t} = \frac{-5 - -10}{5} = 1\ \text{m s}^{-2}$$

The positive sign tells us that the acceleration is in a positive direction (right) even though the car is travelling in a negative direction (left).

Example

A body with a constant acceleration of $-5\ \text{m s}^{-2}$ is travelling to the right with a velocity of $20\ \text{m s}^{-1}$. What will its displacement be after 20 s?

$$s = ?$$
$$u = 20\ \text{m s}^{-1}$$
$$v = ?$$
$$a = -5\ \text{m s}^{-2}$$
$$t = 20\ \text{s}$$

To calculate s we can use the equation $s = ut + \frac{1}{2}at^2$

$$s = 20 \times 20 + \tfrac{1}{2}(-5) \times 20^2 = 400 - 1000 = -600\ \text{m}$$

This means that the final displacement of the body is to the left of the starting point. It has gone forward, stopped and then gone backwards.

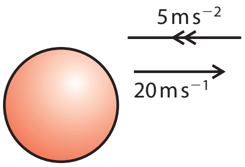


Figure 2.10 The acceleration is negative so pointing to the left.

g

The acceleration due to gravity is not constant all over the Earth. $9.81\ \text{m s}^{-2}$ is the average value. The acceleration also gets smaller the higher you go. However we ignore this change when conducting experiments in the lab since labs aren't that high.

To make the examples easier to follow, $g = 10\ \text{m s}^{-2}$ is used throughout; you should only use this approximate value in exam questions if told to do so.

Exercises

3 Calculate the final velocity of a body that starts from rest and accelerates at $5\ \text{m s}^{-2}$ for a distance of 100 m.

4 A body starts with a velocity of $20\ \text{m s}^{-1}$ and accelerates for 200 m with an acceleration of $5\ \text{m s}^{-2}$. What is the final velocity of the body?

5 A body accelerates at $10\ \text{m s}^{-2}$ reaching a final velocity of $20\ \text{m s}^{-1}$ in 5 s. What was the initial velocity of the body?

2.2 Free fall motion

Assessment statements

2.1.4 Identify the acceleration of a body falling in a vacuum near the Earth's surface with the acceleration g of free fall.

2.1.5 Solve problems involving the equations of uniformly accelerated motion.

2.1.6 Describe the effects of air resistance on falling objects.

Although a car was used in one of the previous illustrations, the acceleration of a car is not usually constant, so we shouldn't use the *suvat* equations. The only example of constant acceleration that we see in everyday life is when a body is dropped. Even then the acceleration is only constant for a short distance.

Acceleration of free fall

When a body is allowed to fall freely we say it is in free fall. Bodies falling freely on the Earth fall with an acceleration of about $9.81\ \mathrm{m\,s^{-2}}$. (It depends where you are.) The body falls because of gravity. For that reason we use the letter g to denote this acceleration. Since the acceleration is constant, we can use the *suvat* equations to solve problems.

The effect of air resistance

If you jump out of a plane (with a parachute on) you will feel the push of the air as it rushes past you. As you fall faster and faster, the air will push upwards more and more until you can't go any faster. At this point you have reached terminal velocity. We will come back to this example after introducing forces.

Exercises

In these calculations use $g = 10\ \mathrm{m\,s^{-2}}$.

6 A ball is thrown upwards with a velocity of $30\ \mathrm{m\,s^{-1}}$. What is the displacement of the ball after 2 s?

7 A ball is dropped. What will its velocity be after falling 65 cm?

8 A ball is thrown upwards with a velocity of $20\ \mathrm{m\,s^{-1}}$. After how many seconds will the ball return to its starting point?

Measuring *g*

Measuring g by timing a ball falling from different heights is a common physics experiment that you could well perform in the practical programme of the IB course. There are various different ways of doing this but a common method is to use a timer that starts when the ball is released and stops when it hits a platform. An example of this apparatus is shown in the photo. The distance travelled by the ball and the time taken are related by the *suvat* equation $s = ut + \frac{1}{2}at^2$. This simplifies to $s = \frac{1}{2}at^2$ since the initial velocity is zero. This means that s is proportional to t^2 so if you plot a graph of s against t^2 you will get a straight line whose gradient is $\frac{1}{2}g$.

Free fall apparatus.

2.3 Graphical representation of motion

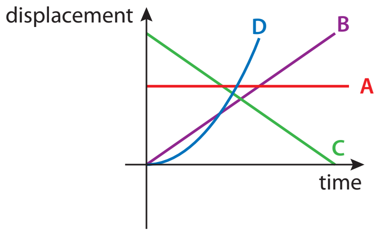
Assessment statements

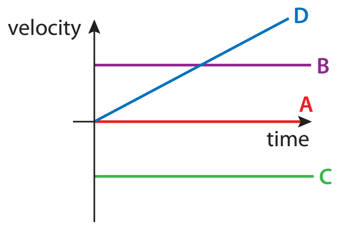
2.1.7 Draw and analyse distance–time graphs, displacement–time graphs, velocity–time graphs and acceleration–time graphs.

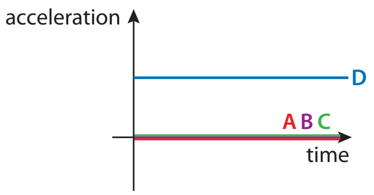
2.1.8 Calculate and interpret the gradients of displacement–time graphs and velocity–time graphs, and the areas under velocity–time graphs and acceleration–time graphs.

Graphs are used in physics to give a visual representation of relationships. In kinematics they can be used to show how displacement, velocity and acceleration change with time. Figure 2.11 shows the graphs for four different examples of motion. They are placed vertically since they all have the same time axis.

Figure 2.11 Graphical representation of motion.







Line A

A body that is not moving.
Displacement is always the same.
Velocity is zero.
Acceleration is zero.

Line B

A body that is travelling with a constant positive velocity.
Displacement increases linearly with time.
Velocity is a constant positive value.
Acceleration is zero.

Line C

A body that has a constant negative velocity.
Displacement is decreasing linearly with time.
Velocity is a constant negative value.
Acceleration is zero.

Line D

A body that is accelerating with constant acceleration.
Displacement is increasing at a non-linear rate. The shape of this line is a parabola since displacement is proportional to t^2 ($s = ut + \frac{1}{2}at^2$).
Velocity is increasing linearly with time.
Acceleration is a constant positive value.

Examiner's hint: You need to be able to

- figure out what kind of motion a body has by looking at the graphs
- sketch graphs for a given motion.

The best way to go about sketching graphs is to split the motion into sections then plot where the body is at different times; joining these points will give the displacement–time graph. Once you have done that you can work out the v–t and a–t graphs by looking at the s–t graph rather than the motion.

Gradient of displacement–time

The gradient of a graph is $\dfrac{\text{change in } y}{\text{change in } x}$

$$= \frac{\Delta y}{\Delta x}$$

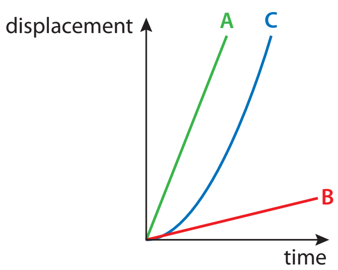
In the case of the displacement–time graph this will give

$$\text{gradient} = \frac{\Delta s}{\Delta t}$$

This is the same as velocity.

So the gradient of the displacement–time graph equals the velocity. Using this information, we can see that line A in Figure 2.12 represents a body with greater velocity than line B and that since the gradient of line C is increasing, this must be the graph for an accelerating body.

Figure 2.12



Instantaneous velocity

When a body accelerates its velocity is constantly changing. The displacement–time graph for this motion is therefore a curve. To find the instantaneous velocity from the graph we can draw a tangent to the curve and find the gradient of the tangent as shown in Figure 2.13.

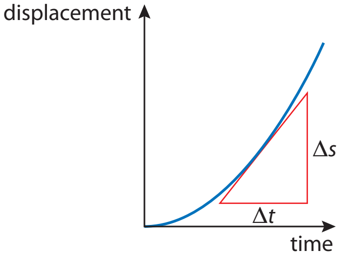


Figure 2.13

Area under velocity–time graph

The area under the velocity–time graph for the body travelling at constant velocity v shown in Figure 2.14 is given by

$$\text{area} = v\Delta t$$

But we know from the definition of velocity that $v = \frac{\Delta s}{\Delta t}$

Rearranging gives $\Delta s = v\Delta t$ so the area under a velocity–time graph gives the displacement.

This is true not only for simple cases such as this but for all examples.

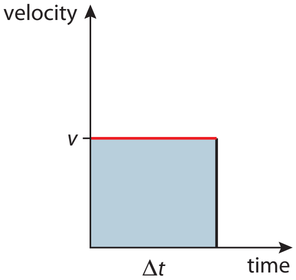


Figure 2.14

Gradient of velocity–time graph

The gradient of the velocity–time graph is given by $\frac{\Delta v}{\Delta t}$. This is the same as acceleration.

Area under acceleration–time graph

The area under an acceleration–time graph in Figure 2.15 is given by $a\Delta t$. But we know from the definition of acceleration that $a = \frac{(v - u)}{t}$

Rearranging this gives $v - u = a\Delta t$ so the area under the graph gives the change in velocity.

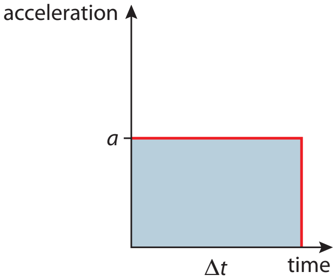


Figure 2.15

If you have covered calculus in your maths course you may recognise these equations:

$v = \frac{ds}{dt} a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$ and $s = \int v dt$, $v = \int a dt$

Exercises

9 Sketch a velocity–time graph for a body starting from rest and accelerating at a constant rate to a final velocity of 25 $m s^{-1}$ in 10 seconds. Use the graph to find the distance travelled and the acceleration of the body.

10 Describe the motion of the body whose velocity–time graph is shown in Figure 2.16. What is the final displacement of the body?

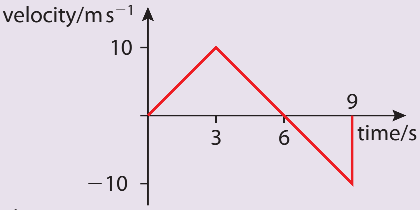


Figure 2.16

Example 1: the *suvat* example

As an example let us consider the motion we looked at when deriving the *suvat* equations.

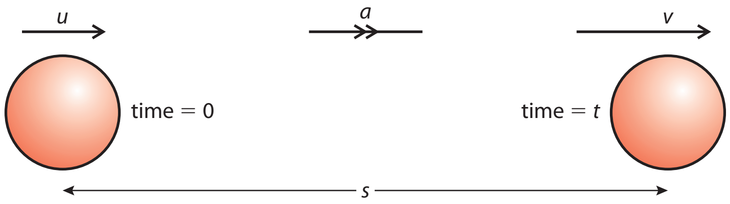


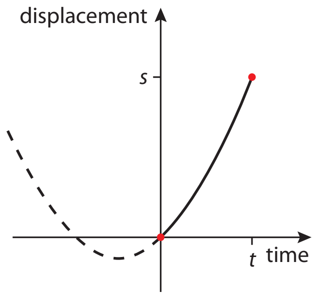
Figure 2.17 A body with constant acceleration.

Negative time
Negative time doesn't mean going back in time – it means the time before you started the clock.

Displacement–time

The body starts with velocity u and travels to the right with constant acceleration, a for a time t. If we take the starting point to be zero displacement, then the displacement–time graph starts from zero and rises to s in t seconds. We can therefore plot the two points shown in Figure 2.18. The body is accelerating so the line joining these points is a parabola. The whole parabola has been drawn to show what it would look like – the reason it is offset is because the body is not starting from rest. The part of the curve to the left of the origin tells us what the particle was doing before we started the clock.

Figure 2.18



Velocity–time

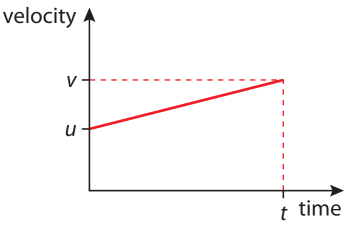


Figure 2.19

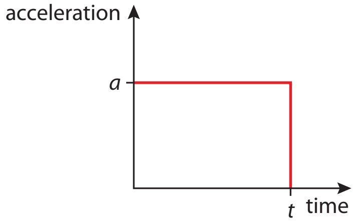
Figure 2.19 is a straight line with a positive gradient showing that the acceleration is constant. The line doesn't start from the origin since the initial velocity is u.

The gradient of this line is $\frac{(v - u)}{t}$ which we know from the *suvat* equations is acceleration.

The area under the line makes the shape of a trapezium. The area of this trapezium is $\frac{1}{2}(v + u)t$. This is the *suvat* equation for s.

Acceleration–time

Figure 2.20



The acceleration is constant so the acceleration–time graph is simply a horizontal line as shown in Figure 2.20. The area under this line is $a \times t$ which we know from the *suvat* equations equals $(v - u)$.

Example 2: The bouncing ball

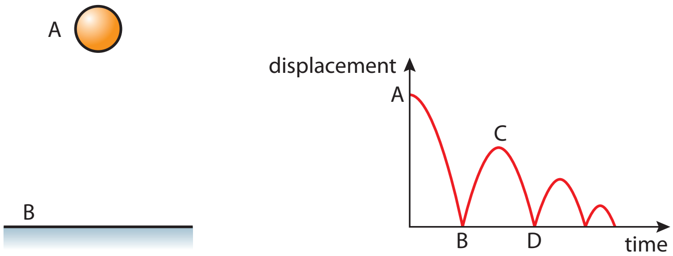


Figure 2.21

Consider a rubber ball dropped from some position above the ground A onto a hard surface B. The ball bounces up and down several times. Figure 2.21 shows the displacement–time graph for 4 bounces. From the graph we see that the ball starts above the ground then falls with increasing velocity (as deduced by the increasing negative gradient). When the ball bounces at B the velocity suddenly changes from negative to positive as the ball begins to travel back up. As the ball goes up, its velocity gets less until it stops at C and begins to fall again.

To view a simulation that enables you to plot the graphs as you watch the motion, visit www.heinemann.co.uk/hotlinks, enter the express code 4426P and click on Weblink 2.1.

Exercise

11 By considering the gradient of the displacement–time graph in Figure 2.21 plot the velocity–time graph for the motion of the bouncing ball.

Example 3: A ball falling with air resistance

Figure 2.22 represents the motion of a ball that is dropped several hundred metres through the air. It starts from rest and accelerates for some time. As the ball accelerates, the air resistance gets bigger, which prevents the ball from getting any faster. At this point the ball continues with constant velocity.

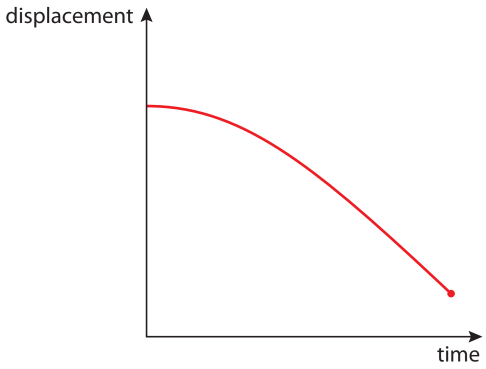


Figure 2.22

Exercise

12 By considering the gradient of the displacement–time graph plot the velocity–time graph for the motion of the falling ball.

2.4 Projectile motion

Assessment statements

9.1.1 State the independence of the vertical and the horizontal components of velocity for a projectile in a uniform field.

9.1.2 Describe and sketch the trajectory of projectile motion as parabolic in the absence of air resistance.

9.1.3 Describe qualitatively the effect of air resistance on the trajectory of a projectile.

9.1.4 Solve problems on projectile motion.

We all know what happens when a ball is thrown; it follows a curved path like the one in the photo below. We can see from this photo that the path is parabolic, and later we will show why that is the case.

A stroboscopic photograph of a projected ball.

Modelling projectile motion

All examples of motion up to this point have been in one dimension but projectile motion is two-dimensional. However, if we take components of all the vectors vertically and horizontally, we can simplify this into two simultaneous one-dimensional problems. The important thing to realise is that the vertical and horizontal components are independent of each other; you can test this by dropping a stone off a cliff and throwing one forward at the same time, they both hit the bottom together. The downward motion is not altered by the fact that one is also moving forward.

Consider a ball that is projected at an angle θ to the horizontal, as shown in Figure 2.23. We can split the motion into three parts, beginning, middle and end, and analyse the vectors representing displacement, velocity and time at each stage. Note that since the path is symmetrical, the motion on the way down is the same as the way up.

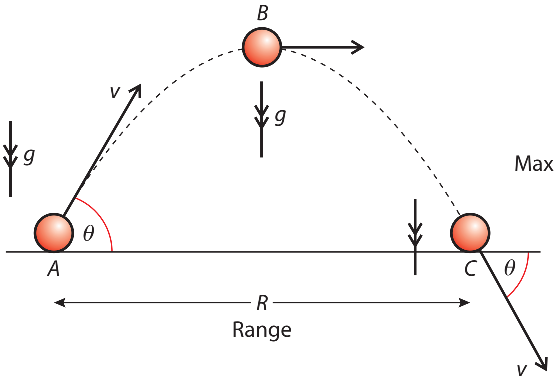


Figure 2.23 A projectile launched at an angle θ.

Horizontal components

At A (time = 0)	At B (time = $\frac{t}{2}$)	At C (time = t)
Displacement = zero Velocity = $v\cos\theta$ Acceleration = 0	Displacement = $\frac{R}{2}$ Velocity = $v\cos\theta$ Acceleration = 0	Displacement = R Velocity = $v\cos\theta$ Acceleration = 0

Vertical components

At A	At B	At C
Displacement = zero Velocity = $v\sin\theta$ Acceleration = $-g$	Displacement = h Velocity = zero Acceleration = $-g$	Displacement = zero Velocity = $-v\sin\theta$ Acceleration = $-g$

We can see that the vertical motion is constant acceleration and the horizontal motion is constant velocity. We can therefore use the *suvat* equations.

suvat for horizontal motion

Since acceleration is zero there is only one equation needed to define the motion

suvat	A to C
Velocity = $v = \frac{s}{t}$	$R = v\cos\theta\, t$

suvat for vertical motion

When considering the vertical motion it is worth splitting the motion into two parts.

suvat	At B	At C
$s = \frac{1}{2}(u+v)t$	$h = \frac{1}{2}(v\sin\theta)\frac{t}{2}$	$0 = \frac{1}{2}(v\sin\theta - v\sin\theta)t$
$v^2 = u^2 + 2as$	$0 = v^2\sin^2\theta - 2gh$	$(-v\sin\theta)^2 = (v\sin\theta)^2 - 0$
$s = ut + \frac{1}{2}at^2$	$h = v\sin\theta\, t - \frac{1}{2}g\left(\frac{t}{2}\right)^2$	$0 = v\sin\theta\, t - \frac{1}{2}gt^2$
$a = \frac{v-u}{t}$	$g = \frac{v\sin\theta - 0}{\frac{t}{2}}$	$g = \frac{v\sin\theta - -v\sin\theta}{t}$

Some of these equations are not very useful since they simply state that 0 = 0. However we do end up with three useful ones (highlighted):

$R = v\cos\theta\, t$ (1)

$0 = v^2\sin^2\theta - 2gh$ or $h = \frac{v^2\sin^2\theta}{2g}$ (2)

$0 = v\sin\theta\, t - \frac{1}{2}gt^2$ or $t = \frac{2v\sin\theta}{g}$ (3)

Solving problems

In a typical problem you will be given the magnitude and direction of the initial velocity and asked to find either the maximum height or range. To calculate h you can use equation (2) but to calculate R you need to find the time of flight so must use (3) first (you could also substitute for t into equation (1) to give a fourth equation but maybe we have enough equations already).

You do not have to remember a lot of equations to solve a projectile problem. If you understand how to apply the *suvat* equations to the two components of the projectile motion, you only have to remember the *suvat* equations (and they are in the databook).

Parabolic path

Since the horizontal displacement is proportional to t the path has the same shape as a graph of vertical displacement plotted against time. This is parabolic since the vertical displacement is proportional to t^2.

Maximum range

For a given value of v the maximum range is when $v\cos\theta\, t$ is a maximum value. Now $t = \frac{2v\sin\theta}{g}$. If we substitute this for t we get

$$R = \frac{2v^2\cos\theta\sin\theta}{g}.$$

This is a maximum when $\cos\theta\sin\theta$ is maximum, which is when $\theta = 45°$.

To view a simulation of projectile motion, visit www.heinemann.co.uk/hotlinks, enter the express code 4426P and click on Weblink 2.2.

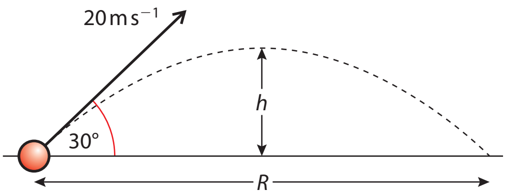
Worked example

1 A ball is thrown at an angle of 30° to the horizontal at a speed of $20\ \text{m s}^{-1}$. Calculate its range and the maximum height reached.

2 A ball is thrown horizontally from a cliff top with a horizontal speed of $10\ \text{m s}^{-1}$. If the cliff is 20 m high what is the range of the ball?

Solution

1 First, as always, draw a diagram, including labels defining all the quantities known and unknown.



Now we need to find the time of flight. If we apply $s = ut + \frac{1}{2}at^2$ to the whole flight we get

$$t = \frac{2v\sin\theta}{g} = \frac{(2 \times 20 \times \sin 30°)}{10} = 2\ \text{s}$$

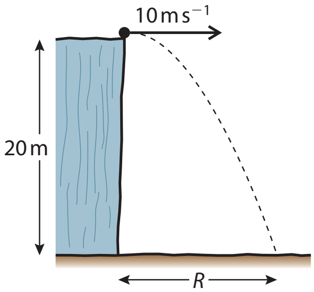
We can now apply $s = vt$ to the whole flight to find the range:

$$R = v\cos\theta t = 20 \times \cos 30° \times 2 = \mathbf{34.6\ m}$$

Finally to find the height, we use $s = ut + \frac{1}{2}at^2$ to the vertical motion, but remember, this is only half the complete flight so the time is 1 s.

$$h = v\sin\theta t - \frac{1}{2}gt^2 = 20 \times \sin 30° \times 1 - \frac{1}{2} \times 10 \times 1^2 = 10 - 5 = \mathbf{5\ m}$$

2 This is an easy one since there aren't any angles to deal with. The initial vertical component of the velocity is zero and the horizontal component is $10\ \text{m s}^{-1}$. To calculate the time of flight we apply $s = ut + \frac{1}{2}at^2$ to the vertical component. Knowing that the final displacement is −20 m this gives



$$-20\ \text{m} = 0 - \frac{1}{2}gt^2 \text{ so } t = \sqrt{\frac{(2 \times 20)}{10}} = 2\ \text{s}$$

We can now use this value to find the range by applying the formula $s = vt$ to the horizontal component: $R = 10 \times 2 = \mathbf{20\ m}$

If you have ever played golf you will know it is not true that the maximum range is achieved with an angle of 45°, it's actually much less. This is because the ball is held up by the air like an aeroplane is. In this photo Alan Shepard is playing golf on the moon. Here the maximum range will be at 45°.

Exercises

13 Calculate the range of a projectile thrown at an angle of 60° to the horizontal with velocity $30\ \text{m s}^{-1}$.

14 You throw a ball at a speed of $20\ \text{m s}^{-1}$.

(a) At what angle must you throw it so that it will just get over a wall that is 5 m high?

(b) How far away from the wall must you be standing?

15 A gun is aimed so that it points directly at the centre of a target 200 m away. If the bullet travels at $200\ \text{m s}^{-1}$ how far below the centre of the target will the bullet hit?

16 If you can throw a ball at $20\ \text{m s}^{-1}$ what is the maximum distance you can throw it?

Projectile motion with air resistance

In all the examples above we have ignored the fact that the air will resist the motion of the ball. The actual path of a ball including air resistance is likely to be as shown in Figure 2.24.

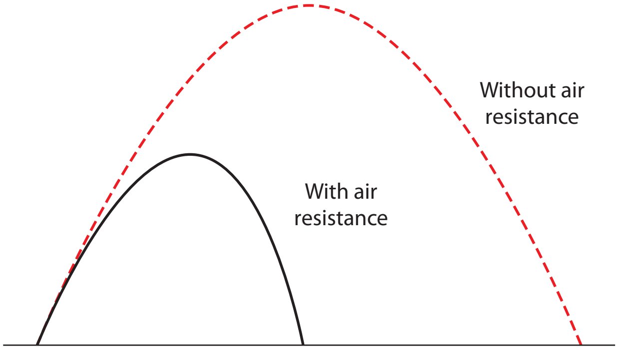


Figure 2.24

Notice both the height and range are less. It is also no longer a parabola – the way down is steeper than the way up.

2.5 Forces and dynamics

Assessment statements

2.2.1 Calculate the weight of a body using the expression $W = mg$.

2.2.2 Identify the forces acting on an object and draw free body diagrams representing the forces acting.

2.2.3 Determine the resultant force in different situations.

Forces

From experience, we know that things don't seem to move unless we push them, so movement is related to pushing. In this next section we will investigate this relationship.

The size of one newton
If you hold an object of mass 100 g in your hand then you will be exerting an upward force of about one newton (1 N).

What is a force?

A force is simply a push or a pull.

The unit of force is the newton (N).

Force is a vector quantity.

You might believe that there are hundreds of different ways to push or pull an object but there are actually surprisingly few.

1. Tension

If you attach a rope to a body and pull it, the rope is in tension. This is also the name of the force exerted on the body.

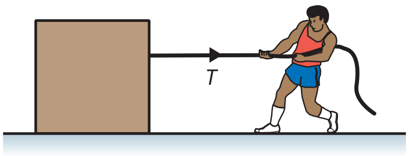


Figure 2.25 The force experienced by the block is tension, *T*.

2. Normal force

Whenever two surfaces are in contact, there will be a force between them (if not then they are not in contact). This force acts at right angles to the surface so is called the *normal force*.

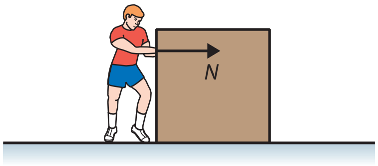


Figure 2.26 The man pushes the block with his hands. The force is called the normal force, *N*.

3. Gravitational force

We know that all objects experience a force that pulls them downwards; we call this force the *weight*. The direction of this force is always towards the centre of the Earth. The weight of a body is directly proportional to the mass of the body.
$W = mg$ where $g =$ the acceleration of free fall.

You will discover why this is the case later in the chapter.

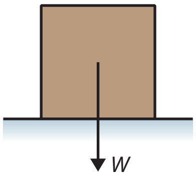


Figure 2.27 This box is pulled downwards by gravity. We call this force the weight, *W*.

Where to draw forces
It is important that you draw the point of application of the forces in the correct place. Notice where the forces are applied in these diagrams.

4. Friction force

Whenever two touching surfaces move, or attempt to move, relative to each other, there is a force that opposes the motion. This is called *frictional force*. The size of this force is dependent on the material of the surfaces and how much force is used to push them together.

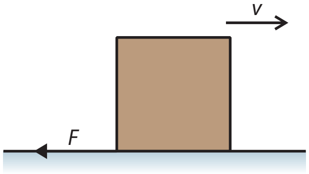


Figure 2.28 This box sliding along the floor will slow down due to the friction, *F*, between it and the floor.

5. Upthrust

Upthrust is the name of the force experienced by a body immersed in a fluid (gas or liquid). This is the force that pushes up on a boat enabling it to float in water. The size of this force is equal to the weight of fluid displaced by the boat.

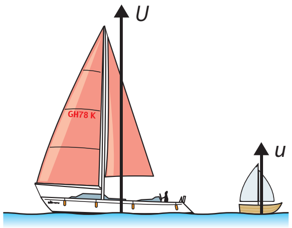
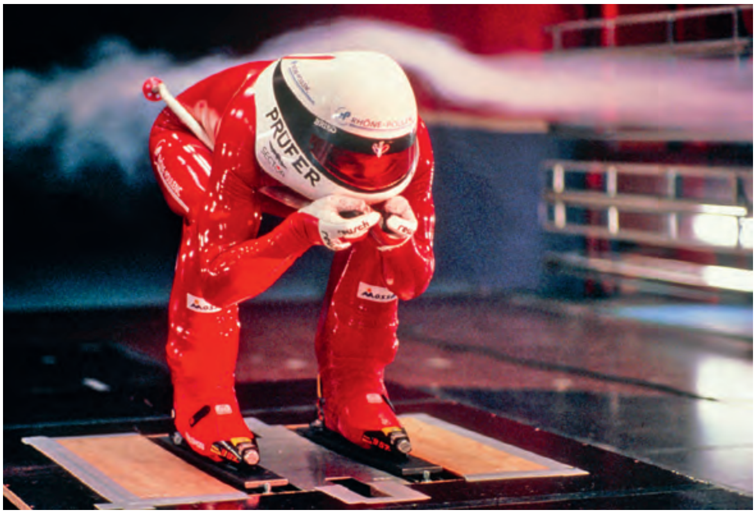


Figure 2.29 Upthrust *U* depends on how much water is displaced.

6. Air resistance

Air resistance is the force that opposes the motion of bodies through the air. This force is dependent on the speed, size and shape of the body.

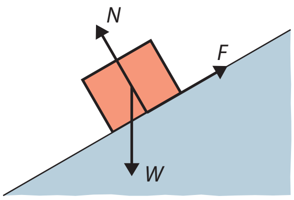


Speed skiers wear special clothes and squat down like this to reduce air resistance.

Free body diagrams

Problems often involve more than one body and more than one force. To keep things simple we always draw each body separately and only the forces acting on that body, not the forces that body exerts on something else. This is called a *free body diagram*.

Figure 2.30



A good example of this is a block resting on a ramp as shown in Figure 2.30. The block will also exert a force on the slope but this is not shown, since it is a free body diagram of the block not the ramp. Another common example that we will come across many times is a mass swinging on the end of a rope, as shown in Figure 2.31.

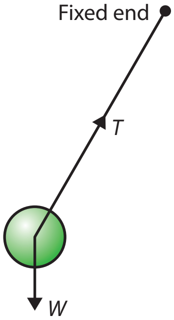


Figure 2.31

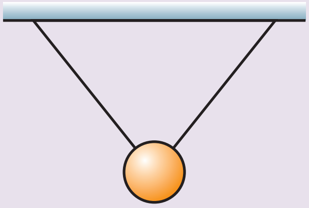
Exercise

17 Draw free body diagrams for

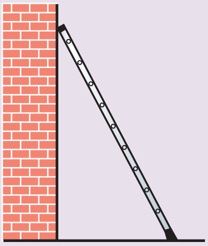
(a) a box resting on the floor

(b) the examples shown below

i



ii



(c) a free fall parachutist falling through the air

(d) a boat floating in water.

Adding forces

Force is a vector quantity, so if two forces act on the same body you must add them vectorially as with displacements and velocities.

Examples

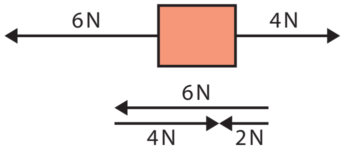


Figure 2.32

1 A body is pulled in two opposing directions by two ropes as shown in Figure 2.32. The resultant force acting is the vector sum of the forces.

The sum is found by arranging the vectors point to tail. This gives a resultant of 2 N to the left.

2 If a body is pulled by two perpendicular ropes as in Figure 2.33, then the vector addition gives a triangle that can be solved by Pythagoras.

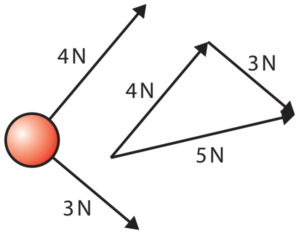
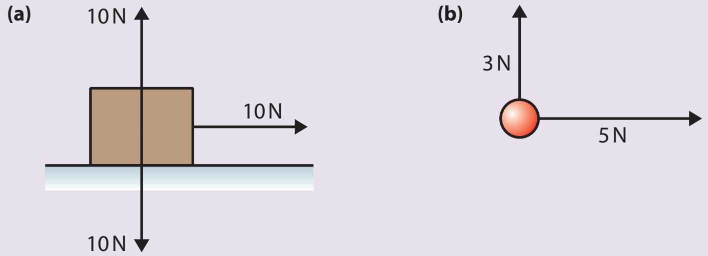


Figure 2.33

Exercise

18 Find the resultant force in the following examples:



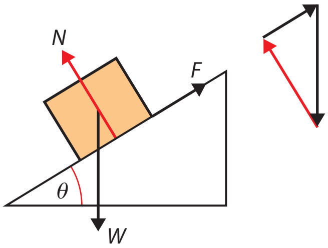


Figure 2.34

Balanced forces

If the resultant force on a body is zero, the forces are said to be balanced. For example, if we add together the vectors representing the forces on the box in Figure 2.34 then we can see that they add up to zero. The forces are therefore balanced.

This can lead to some complicated triangles so it is easier to take components of the forces; if the components in any two perpendicular directions are balanced, then the forces are balanced. Figure 2.35 shows how this would be applied to the same example. To make things clear, the vectors have been drawn away from the box.

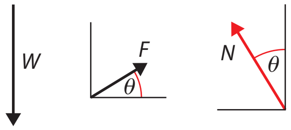


Figure 2.35

Vertical components add up to zero.

$F\sin\theta + N\cos\theta - W = 0$

$W = F\sin\theta + N\cos\theta$

Horizontal components add up to zero.

$0 + F\cos\theta - N\sin\theta = 0$

$F\cos\theta = N\sin\theta$

So we can see that

forces up = forces down

forces left = forces right

Exercises

19 A ball of weight 10 N is suspended on a string and pulled to one side by another horizontal string as shown in Figure 2.36. If the forces are balanced:

(a) write an equation for the horizontal components of the forces acting on the ball

(b) write an equation for the vertical components of the forces acting on the ball

(c) use the second equation to calculate the tension in the upper string, *T*

(d) use your answer to **(c)** plus the first equation to find the horizontal force *F*.

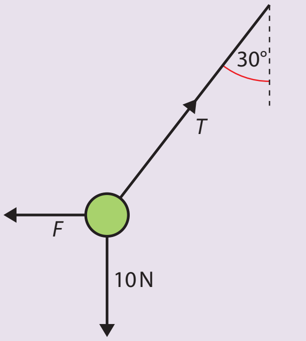


Figure 2.36

20 The condition for the forces to be balanced is that the sum of components of the forces in any two perpendicular components is zero. In the 'box on a ramp' example the vertical and horizontal components were taken. However, it is sometimes more convenient to consider components parallel and perpendicular to the ramp.

Consider the situation in Figure 2.37. If the forces on this box are balanced:

(a) write an equation for the components of the forces parallel to the ramp

(b) write an equation for the forces perpendicular to the ramp

(c) use your answers to find the friction (*F*) and normal force (*N*).

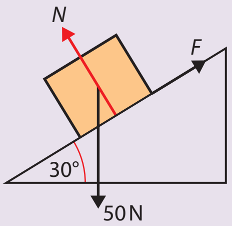


Figure 2.37

21 A rock climber is hanging from a rope attached to the cliff by two bolts as shown in Figure 2.38. If the forces are balanced

(a) write an equation for the vertical component of the forces on the knot

(b) write an equation for the horizontal forces exerted on the knot

(c) calculate the tension *T* in the ropes joined to the bolts.

The result of this calculation shows why ropes should not be connected in this way.

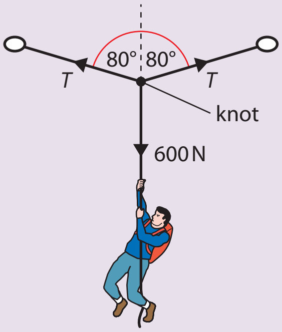


Figure 2.38

2.6 Newton's laws of motion

Assessment statements

2.2.4 State Newton's first law of motion.

2.2.5 Describe examples of Newton's first law.

2.2.6 State the condition for translational equilibrium.

2.2.7 Solve problems involving translational equilibrium.

We now have the quantities to enable us to model motion and we have observed that to make something start moving we have to exert a force – but we haven't connected the two. Newton's laws of motion connect motion with its cause. In this course there are certain fundamental concepts that everything else rests upon, Newton's laws of motion are among the most important of these.

Using laws in physics

A law in physics is a very useful tool. If applied properly, it enables us to make a very strong argument that what we say is true. If asked 'will a box move?' you can say that you think it will and someone else could say it won't. You both have your opinions and you would then argue as to who is right. However, if you say that Newton's law says it will move, then you have a much stronger argument (assuming you have applied the law correctly).

Newton's first law

A body will remain at rest or moving with constant velocity unless acted upon by an unbalanced force.

To put this the other way round, if the forces on a body are unbalanced, then it will not be at rest or moving with constant velocity. If the velocity is not constant then it is accelerating.

We know from experience that things don't start moving unless we push them, but it's not obvious from observation that things will continue moving with constant velocity unless acted upon by an unbalanced force. Usually, if you give an object a push it moves for a bit and then stops. This is because of friction. It would be a very different world without friction, as everyone would be gliding around with constant velocity, only able to stop themselves when they grabbed hold of something. Friction not only stops things moving but enables them to get going. If you stood in the middle of a friction-free room, you wouldn't be able to move. It is the friction between your feet and the floor that pushes you forward when you try to move your feet backwards.

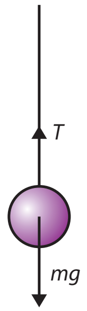


Figure 2.39

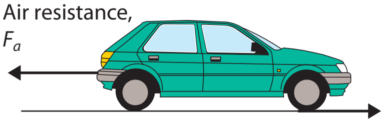


Figure 2.40 Notice that the friction acts forwards, this is because the wheels are trying to turn backwards and friction resists this motion by acting forwards.

Examples

1 Mass on a string

If a mass is hanging at rest on the end of a string as in Figure 2.39 then Newton's first law says the forces must be balanced. This means the Force up = Force down.

$T = mg$

2 Car travelling at constant velocity

If the car in Figure 2.40 is travelling at constant velocity, then Newton's first law says the forces must be balanced.

Force up = Force down

$N = mg$ (not drawn on diagram)

Force left = Force right

$F = F_a$

3 The parachutist

If the free fall parachutist in Figure 2.41 descends at a constant velocity then Newton's first law says that the forces must be balanced.

Force up = Force down

$F_a = mg$

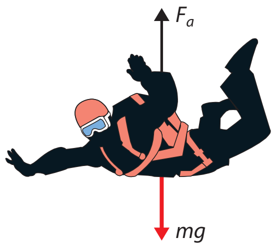


Figure 2.41 A skydiver at terminal velocity.

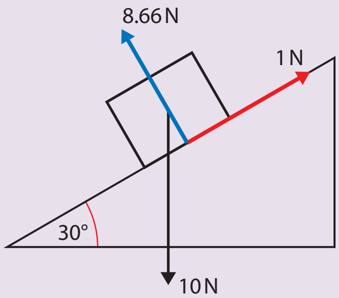
Translational equilibrium

If all the forces on a body are balanced, the body is said to be in translational equilibrium. The bodies in the previous three examples were all therefore in translational equilibrium.

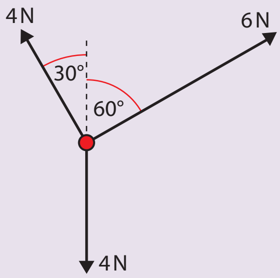
Exercises

22 By resolving the vectors into components, calculate if the following bodies are in translational equilibrium or not. If not, calculate the resultant force.

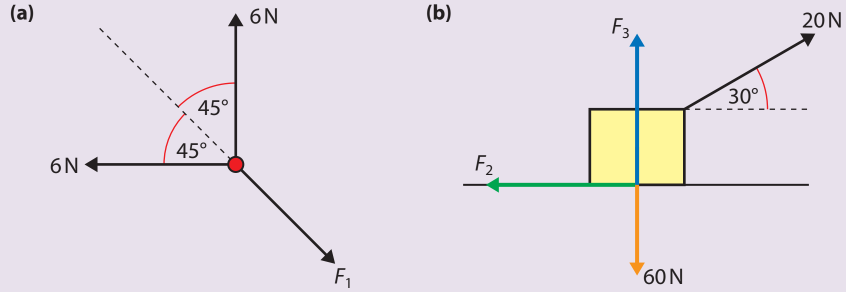
(a)



(b)



23 If the following two examples are in equilibrium, calculate the unknown forces F_1, F_2 and F_3.



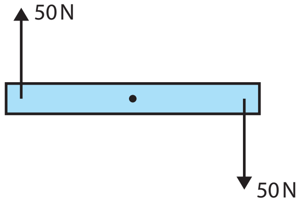


Figure 2.42 The forces are balanced but the body will rotate.

Rotation

When a body is in translational equilibrium it means that its centre will not move. However, it can rotate about the centre, as in the example shown in Figure 2.42.

2.7 The relationship between force and acceleration

Assessment statements

2.2.10 Define linear momentum and impulse.
2.2.8 State Newton's second law of motion.
2.2.9 Solve problems involving Newton's second law.

Newton's first law says that a body will accelerate if an unbalanced force is applied to it. Newton's second law tells us how big the acceleration will be and in which direction. Before we look in detail at Newton's second law we should look at the factors that affect the acceleration of a body when an unbalanced force is applied. Let us consider the example of catching a ball. When we catch the ball we change its velocity, Newton's first law tells us that we must therefore apply an unbalanced force to the ball. The size of that force depends upon two things, the mass and the velocity. A heavy ball is more difficult to stop than a light one travelling at the same speed, and a fast one is harder to stop than a slow one. Rather than having to concern ourselves with two quantities we will introduce a new quantity that incorporates both mass and velocity, *momentum*.

Momentum (p)

Momentum is defined as the product of mass and velocity.

$$p = mv$$

The unit of momentum is kg m s^{-1}.
Momentum is a vector quantity.

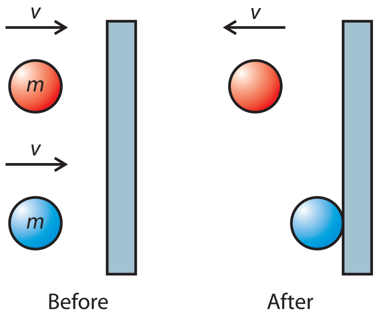


Figure 2.43 The change of momentum of the red ball is greater.

Impulse

When you get hit by a ball the effect it has on you is greater if the ball bounces off you than if you catch it. This is because the change of momentum is greater when the ball bounces, as shown in Figure 2.43.

The unit of impulse is $kg\,m\,s^{-1}$.

Impulse is a vector.

Red ball

Momentum before $= mv$

Momentum after $= -mv$ (remember momentum is a vector)

Change in momentum $= -mv - mv = -2mv$

Blue ball

Momentum before $= mv$

Momentum after $= 0$

Change in momentum $= 0 - mv = -mv$

The impulse is defined as the change of momentum.

Exercises

24 A ball of mass 200 g travelling at $10\,m\,s^{-1}$ bounces off a wall. If after hitting the wall it travels at $5\,m\,s^{-1}$, what is the impulse?

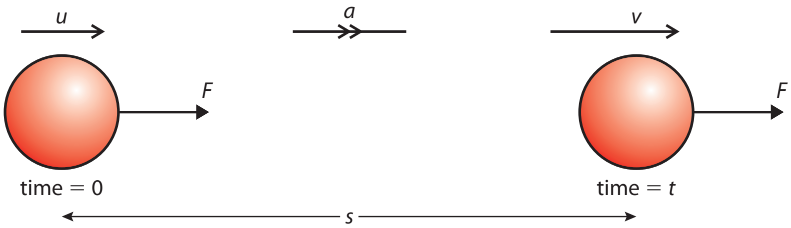
25 Calculate the impulse on a tennis racket that hits a ball of mass 67 g travelling at $10\,m\,s^{-1}$ so that is comes off the racket at a velocity of $50\,m\,s^{-1}$.

Newton's second law

The rate of change of momentum of a body is directly proportional to the unbalanced force acting on that body and takes place in same direction.

Let us once again consider the ball with a constant force acting on it as in Figure 2.44.

Figure 2.44



Unit of momentum

If F = change in momentum /time then momentum = force × time So the unit of momentum is N s. This is the same as $kg\,m\,s^{-1}$.

Firstly Newton's first law tells us that there must be an unbalanced force acting on the ball since it is accelerating.

Newton's second law tells us that the size of the unbalanced force is directly proportional to the rate of change of momentum. We know that the acceleration is constant, which means the rate of change of velocity is constant; this implies that the rate of change of momentum is also constant, so the force, F must be constant too.

If the ball has mass m we can calculate the change of momentum of the ball.

Initial momentum $= mu$

Final momentum $= mv$

Change in momentum $= mv - mu$

The time taken is t so the rate of change of momentum $= \frac{mv - mu}{t}$

This is the same as $\frac{m(v - u)}{t} = ma$

Newton's second law says that the rate of change of momentum is proportional to the force, so $F \alpha\ ma$.

To make things simple the newton is defined so that the constant of proportionality is equal to 1 so:

$$F = ma$$

So when a force is applied to a body in this way, Newton's second law can be simplified to:

The acceleration of a body is proportional to the force applied and inversely proportional to its mass.

Not all examples are so simple. Consider a jet of water hitting a wall as in Figure 2.45. The water hits the wall and loses its momentum, ending up in a puddle on the floor.

Newton's first law tells us that since the velocity of the water is changing, there must be a force on the water,

Newton's second law tells us that the size of the force is equal to the rate of change of momentum. The rate of change of momentum in this case is equal to the amount of water hitting the wall per second multiplied by the change in velocity; this is not the same as ma. For this reason it is best to use the first, more general statement of Newton's second law, since this can always be applied.

However, in this course most of the examples will be of the $F = ma$ type.

The newton
The fact that a 100 g mass has a weight of approximately 1 N is coincidental; 1 N is actually defined as the force that would cause a 1 kg body to accelerate at 1 ms^{-2}.

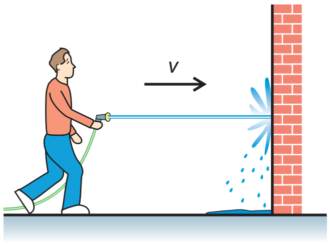


Figure 2.45

Examples

1. Elevator accelerating upwards

An elevator has an upward acceleration of 1 $m\,s^{-2}$. If the mass of the elevator is 500 kg, what is the tension in the cables pulling it up?

First draw a free body diagram as in Figure 2.46. Now we can see what forces are acting. Newton's first law tells us that the forces must be unbalanced. Newton's second law tells us that the unbalanced force must be in the direction of the acceleration (upwards). This means that T is bigger than mg.

Newton's second law also tells us that the size of the unbalanced force equals ma so we get the equation

$$T - mg = ma$$

Rearranging gives

$$T = mg + ma$$
$$= 500 \times 10 + 500 \times 1$$
$$= 5500\ \text{N}$$

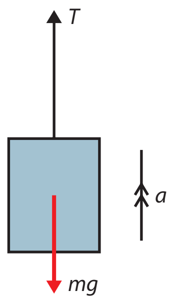


Figure 2.46 An elevator accelerating up. This could either be going up getting faster or going down getting slower.

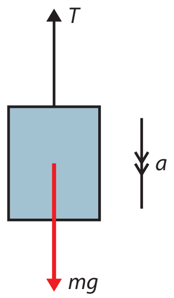


Figure 2.47 The elevator with downward acceleration.

2. Elevator accelerating down

The same elevator as in example 1 now has a downward acceleration of 1 m s^{-2} as in Figure 2.47.

This time Newton's laws tell us that the weight is bigger than the tension so $mg - T = ma$

Rearranging gives

$$\begin{aligned} T &= mg - ma \\ &= 500 \times 10 - 500 \times 1 \\ &= 4500\,\text{N} \end{aligned}$$

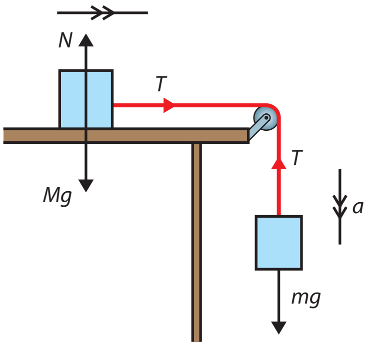


Figure 2.48

3. Joined masses

Two masses are joined by a rope. One of the masses sits on a frictionless table, the other hangs off the edge as in Figure 2.48.

M is being dragged to the edge of the table by m.

Both are connected to the same rope so T is the same for both masses, this also means that the acceleration a is the same.

We do not need to consider N and Mg for the mass on the table because these forces are balanced. However the horizontally unbalanced force is T.

Applying Newton's laws to the mass on the table gives

$$T = Ma$$

The hanging mass is accelerating down so mg is bigger than T. Newton's second law implies that $mg - T = ma$

Substituting for T gives $mg - Ma = ma$ so $a = \dfrac{mg}{M + m}$

4. The free fall parachutist

After falling freely for some time, a free fall parachutist whose weight is 60 kg opens her parachute. Suddenly the force due to air resistance increases to 1200 N. What happens?

Looking at the free body diagram in Figure 2.49 we can see that the forces are unbalanced and that according to Newton's second law the acceleration, a, will be upwards.

The size of the acceleration is given by

$$ma = 1200 - 600 = 60 \times a$$

so $$a = 10\,\text{m s}^{-2}$$

The acceleration is in the opposite direction to the motion. This will cause the parachutist to slow down. As she slows down, the air resistance gets less until the forces are balanced. She will then continue down with a constant velocity.

Figure 2.49 The parachutist just after opening the parachute.

Exercise

26 The helium in a balloon causes an upthrust of 0.1 N. If the mass of the balloon and helium is 6 g, calculate the acceleration of the balloon.

27 A rope is used to pull a felled tree (mass 50 kg) along the ground. A tension of 1000 N causes the tree to move from rest to a velocity of 0.1 m s^{-1} in 2 s. Calculate the force due to friction acting on the tree.

28 Two masses are arranged on a frictionless table as shown in Figure 2.50. Calculate:

(a) the acceleration of the masses

(b) the tension in the string.

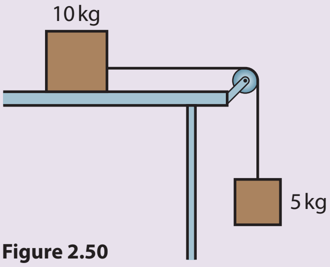


Figure 2.50

29 A helicopter is lifting a load of mass 1000 kg with a rope. The rope is strong enough to hold a force of 12 kN. What is the maximum upward acceleration of the helicopter?

30 A person of mass 65 kg is standing in an elevator that is accelerating upwards at 0.5 m s^{-2}. What is the normal force between the floor and the person?

31 A plastic ball is held under the water by a child in a swimming pool. The volume of the ball is 4000 cm^3.

(a) If the density of water is 1000 kg m^{-3}, calculate the upthrust on the ball (remember upthrust = **weight** of fluid displaced).

(b) If the mass of the ball is 250 g, calculate the theoretical acceleration of the ball when it is released. Why won't the ball accelerate this quickly in a real situation?

2.8 Newton's third law

Assessment statements

2.2.14 State Newton's third law of motion.
2.2.15 Discuss examples of Newton's third law.
2.2.12 State the law of conservation of linear momentum.
2.2.13 Solve problems involving momentum and impulse.
2.2.11 Determine the impulse due to a time–varying force by interpreting a force–time graph.

When dealing with Newton's first and second laws, we are careful to consider only the body that is *experiencing* the forces, not the body that is *exerting* the forces. Newton's third law relates these forces.

If body A exerts a force on body B then body B will exert an equal and opposite force on body A.

Incorrect statements

It is very important to realise that Newton's third law is about two bodies. Avoid statements of this law that do not mention anything about there being two bodies.

So if someone is pushing a car with a force F as shown in Figure 2.51 the car will push back on the person with a force $-F$. In this case both of these forces are the normal force.

Figure 2.51 The man pushes the car and the car pushes the man.

You might think that since these forces are equal and opposite, they will be balanced, and in that case how does the person get the car moving? This is wrong; the forces act on different bodies so can't balance each other.

Examples

1. A falling body

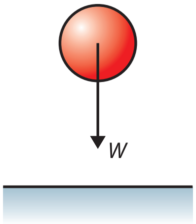


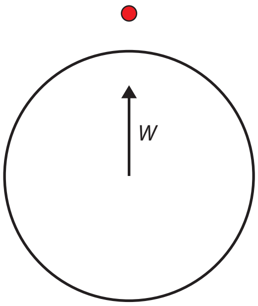
Figure 2.52 A falling body pulled down by gravity.

A body falls freely towards the ground as in Figure 2.52. If we ignore air resistance, there is only one force acting on the body – the force due to the gravitational attraction of the Earth, that we call weight.

Applying Newton's third law:

If the Earth pulls the body down, then the body must pull the Earth up with an equal and opposite force. We have seen that the gravitational force always acts on the centre of the body, so Newton's third law implies that there must be a force equal to W acting upwards on the centre of the Earth as in Figure 2.53.

Figure 2.53 The Earth pulled up by gravity.



2. A box rests on the floor

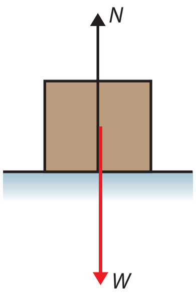


Figure 2.54 Forces acting on a box resting on the floor.

A box sits on the floor as shown in Figure 2.54. Let us apply Newton's third law to this situation.

There are two forces acting on the box.

Normal force: The floor is pushing up on the box with a force N. According to Newton's third law the box must therefore push down on the floor with a force of magnitude N.

Weight: The Earth is pulling the box down with a force *W*. According to Newton's third law, the box must be pulling the Earth up with a force of magnitude *W* as shown in Figure 2.55.

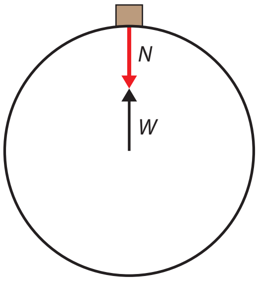


Figure 2.55 Forces acting on the Earth according to Newton's third law.

3. Recoil of a gun

When a gun is fired the velocity of the bullet changes. Newton's first law implies that there must be an unbalanced force on the bullet; this force must come from the gun. Newton's third law says that if the gun exerts a force on the bullet the bullet must exert an equal and opposite force on the gun. This is the force that makes the gun recoil or 'kick back'.

4. The water cannon

When water is sprayed at a wall from a hosepipe it hits the wall and stops. Newton's first law says that if the velocity of the water changes, there must be an unbalanced force on the water. This force comes from the wall. Newton's third law says that if the wall exerts a force on the water then the water will exert a force on the wall. This is the force that makes a water cannon so effective at dispersing demonstrators.

A boat tests its water cannons.

Exercise

32 Use Newton's first and third laws to explain the following:

(a) When burning gas is forced downwards out of a rocket motor, the rocket accelerates up.

(b) When the water cannons on the boat in the photo are operating, the boat accelerates forwards.

(c) When you step forwards off a skateboard, the skateboard accelerates backwards.

(d) A table tennis ball is immersed in a fluid and held down by a string as shown in Figure 2.56. The container is placed on a balance. What will happen to the reading of the balance if the string breaks?

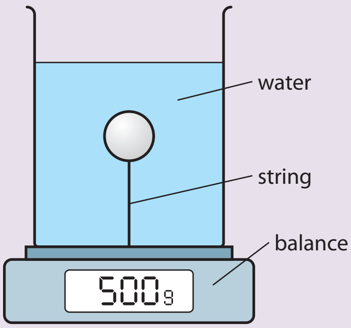


Figure 2.56

Collisions

In this section we have been dealing with the interaction between two bodies (gun-bullet, skater-skateboard, hose-water). To develop our understanding of the interaction between bodies, let us consider a simple collision between two balls as illustrated in Figure 2.57.

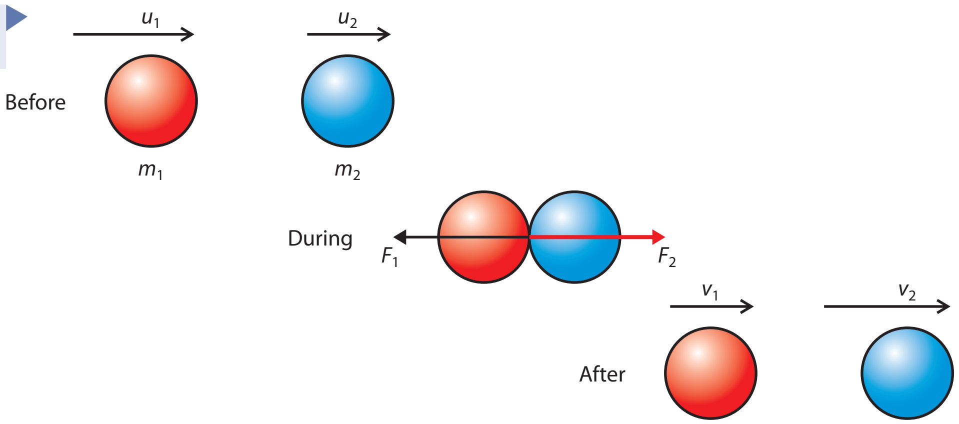


Figure 2.57 Collision between two balls.

Let us apply Newton's three laws to this problem.

Newton's first law

In the collision the red ball slows down and the blue ball speeds up. Newton's first law tells us that that this means there is a force acting to the left on the red ball (F_1) and to the right on the blue ball (F_2).

Newton's second law

This law tells us that the force will be equal to the rate of change of momentum of the balls so if the balls are touching each other for a time Δt

$$F_1 = \frac{m_1v_1 - m_1u_1}{\Delta t}$$

$$F_2 = \frac{m_2v_2 - m_2u_2}{\Delta t}$$

Isolated system
An isolated system is one in which no external forces are acting. When a ball hits a wall the momentum of the ball is not conserved because the ball and wall is not an isolated system, since the wall is attached to the ground. If the ball and wall were floating in space then momentum would be conserved.

Newton's third law

According to the third law, if the red ball exerts a force on the blue ball, then the blue ball will exert an equal and opposite force on the red ball.

$$F_1 = -F_2$$

$$\frac{m_1v_1 - m_1u_1}{\Delta t} = \frac{-m_2v_2 - m_2u_2}{\Delta t}$$

Rearranging gives $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

In other words the momentum at the start equals the momentum at the end. We find that this applies not only to this example but to all interactions.

The law of the conservation of momentum

For a system of isolated bodies the total momentum is always the same.

This is not a new law since it is really just a combination of Newton's laws. However it provides a useful short cut when solving problems.

Examples

In these examples we will have to pretend everything is in space isolated from the rest of the universe, otherwise they are not isolated and the law of conservation of momentum won't apply.

1. A collision where the bodies join together

If two balls of modelling clay collide with each other they stick together as shown in Figure 2.58. We want to find the velocity, v, of the combined lump after the collision.

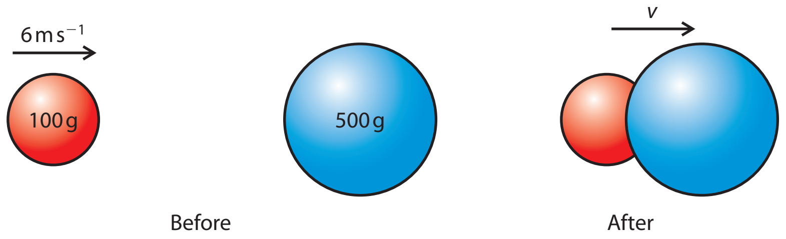


Figure 2.58 Two bodies stick together after colliding.

Simplified models
Pieces of clay floating in space are not exactly everyday examples, but most everyday examples (like balls on a pool table) are not isolated systems, so we can't solve them in this simple way.

If bodies are isolated then momentum is conserved so:

$$\text{momentum before} = \text{momentum after}$$
$$0.1 \times 6 + 0.5 \times 0.0 = 0.6 \times v$$
$$v = \frac{0.6}{0.6} = 1\ \text{m s}^{-1}$$

2. An explosion

A ball of clay floating around in space suddenly explodes into a big piece and a small piece, as shown in Figure 2.59. If the big bit has a velocity of 5 m s^{-1}, what is the velocity of the small bit?

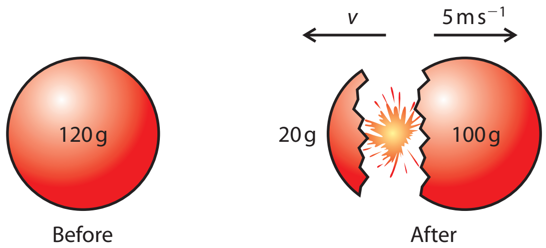


Figure 2.59 A piece of modelling clay suddenly explodes.

Since this is an isolated system, momentum is conserved so:

$$\text{momentum before} = \text{momentum after}$$
$$0 \times 0.12 = 0.02 \times (-v) + 0.1 \times 5$$
$$0.02 \times v = 0.5$$
$$v = 25\ \text{m s}^{-1}$$

Exercise

33 Draw diagrams to represent the following collisions then use the law of conservation of momentum to find the unknown velocity. Assume all collisions are head-on, in other words they take place in one dimension.

(a) Two identical isolated balls collide with each other. Before the collision, one ball was travelling at $10\,m\,s^{-1}$ and the other was at rest. After the collision the first ball continues in the same direction with a velocity of $1\,m\,s^{-1}$. Find the velocity of the other ball.

(b) Two identical balls are travelling towards each other; each is travelling at a speed of $5\,m\,s^{-1}$. After they hit, one ball bounces off with a speed of $1\,m\,s^{-1}$. What is the speed of the other?

(c) A spaceman of mass 100 kg is stranded 2 m from his spaceship as shown in Figure 2.60. He happens to be holding a hammer of mass 2 kg what must he do?

If he only has enough air to survive for 2 minutes, how fast must he throw the hammer if he is to get back in time? Is it possible?

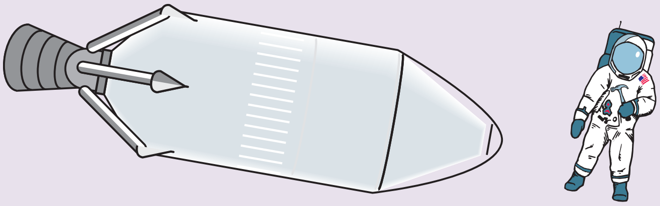


Figure 2.60 If you are ever in this position this course could save your life.

What happens during the collision?

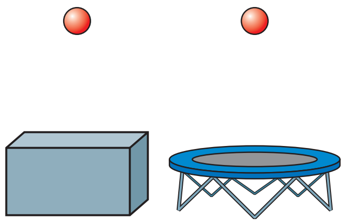


Figure 2.61 It's less painful to land on a trampoline than a concrete block.

In the previous examples, we were more interested in the difference between the conditions before and after the collision than during it. However, collisions can be very different. For example, consider a ball of mass 200 g colliding with a hard floor and a trampoline as shown in Figure 2.61. Before the collisions each ball travels downwards at $10\,m\,s^{-1}$ and each bounces up with velocity $10\,m\,s^{-1}$. So the change in momentum (impulse) is the same for each:

$$0.2 \times (-20) - 0.2 \times 20 = -8\,N\,s$$

Each has the same change of momentum but each collision was very different – the collision with the trampoline took place over a longer time. If you replace the ball with yourself, you would certainly be able to feel the difference. We can represent these two collisions by plotting a graph of force against time as shown in Figure 2.62. We see the force exerted by the concrete is much greater.

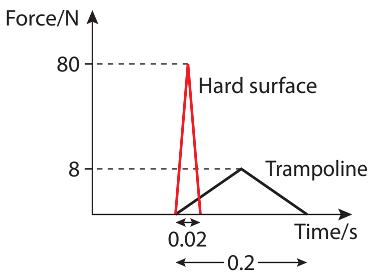


Figure 2.62

Area under the graph

We notice that the area under the graph for each interaction is the same.

$$\text{Area} = \tfrac{1}{2}\text{base} \times \text{height} = \tfrac{1}{2} \times 0.02 \times 8 = \tfrac{1}{2} \times 0.2 \times 0.8 = 8\,N\,s$$

This is equal to the impulse.

Exercise

34 (a) Calculate the impulse of the body for the motion represented in Figure 2.63.

(b) If the mass of the object is 20 g, what is the change of velocity?

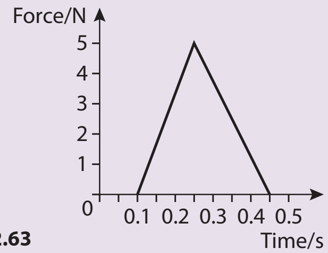


Figure 2.63

2.9 Work, energy and power

Assessment statements

2.3.1 Outline what is meant by work.
2.3.2 Determine the work done by a non-constant force by interpreting a force–displacement graph.
2.3.3 Solve problems involving the work done by a force.
2.3.4 Outline what is meant by kinetic energy.
2.3.5 Outline what is meant by change in gravitational potential energy.
2.3.6 State the principle of conservation of energy.
2.3.7 List different forms of energy and describe examples of the transformation of energy from one form to another.
2.3.8 Distinguish between elastic and inelastic collisions.
2.3.9 Define power.
2.3.10 Define and apply the concept of efficiency.
2.3.11 Solve problems involving momentum, work, energy and power.

We have so far dealt with the motion of a small red ball and understand what causes it to accelerate. We have also investigated the interaction between a red ball and a blue one and have seen that the red one can cause the blue one to move when they collide. But what enables the red one to push the blue one? To answer this question we need to define some more quantities.

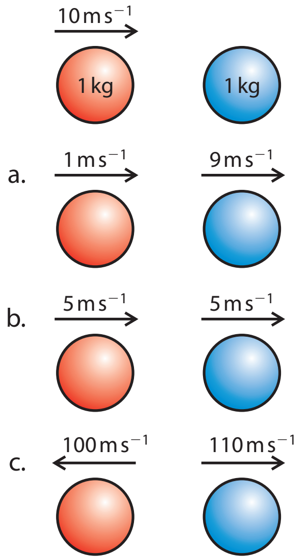


Figure 2.64 The red ball hits the blue ball but what happens?

Work

In the introduction to this book it was stated that by developing models, our aim is to understand the physical world so that we can make predictions. At this point you should understand certain concepts related to the collision between two balls, but we still can't predict the outcome. To illustrate this point let us again consider the red and blue balls. Figure 2.64 shows three possible outcomes of the collision.

If we apply the law of conservation of momentum, we realise that all three outcomes are possible. The original momentum is 10 newtonmetres (10 Nm) and the final momentum is 10 Nm in all three cases. But which one actually happens? This we cannot say (yet). All we know is that from experience the last option is not possible – but why?

When body A hits body B, body A exerts a force on body B. This force causes B to have an increase in velocity. The amount that the velocity increases depends upon how big the force is and over what distance the collision takes place. To make this simpler, consider a constant force acting on a body as in Figure 2.65.

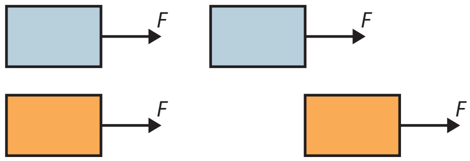


Figure 2.65 The force acts on the orange block for a greater distance.

Both blocks start at rest and are pulled by the same force, but the second block will gain more velocity because the force acts over a longer distance. To quantify this difference, we say that in the second case the force has done more work. Work is done when the point of application of a force moves in the direction of the force.

Work is defined in the following way:

Work done = force × distance moved in the direction of the force.

The unit of work is the newtonmetre (Nm) which is the same as the joule (J).

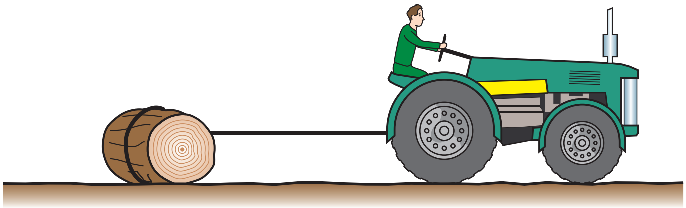
Work is a scalar quantity.

Sign of work

Since work is a scalar the sign has nothing to do with direction. Saying that you have done negative work on A is the same as saying A has done work on you.

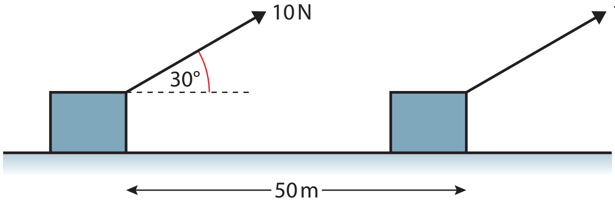
Worked examples

1 A tractor pulls a felled tree along the ground for a distance of 200 m. If the tractor exerts a force of 5000 N, how much work will be done?



2 A force of 10 N is applied to a block, pulling it 50 m along the ground as shown in Figure 2.66. How much work is done by the force?

Figure 2.66



3 When a car brakes it slows down due to the friction force between the tyres and the road. This force opposes the motion as shown in Figure 2.67. If the friction force is a constant 500 N and the car comes to rest in 25 m, how much work is done by the friction force?

Figure 2.67 Work done against friction.

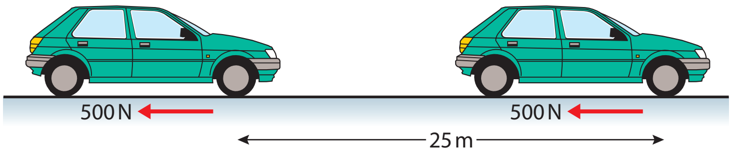
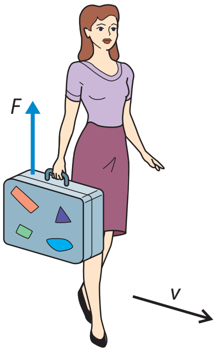


Figure 2.68



4 The woman in Figure 2.68 walks along with a constant velocity holding a suitcase.
How much work is done by the force holding the case?

Solutions

1 Work done = force × distance moved in direction of force

Work done = 5000 × 200 = 1 MJ

2 In this example the force is not in the same direction as the movement. However, the horizontal component of the force is.

Work done = 10 × cos 30° × 50 = 433 N

3 This time the force is in the opposite direction to the motion.

Work done = −500 × 25 = −12 500 J

The negative sign tells us that the friction isn't doing the work but the work is being done against the friction.

4 In this example the force is acting perpendicular to the direction of motion, so there is no movement in the direction of the force.

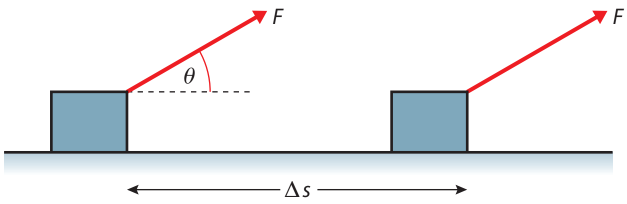
Work done = zero

General formula

In general

$$\text{Work} = F\cos\theta \times \Delta s$$

where θ is the angle between the displacement, Δs, and force, F.



All the previous examples can be solved using this formula.

If $\theta < 90°$, cos θ is positive so the work is positive.

$\theta = 90°$, cos $\theta = 0$ so the work is zero.

$\theta > 90°$, cos θ is negative so the work is negative.

Working or not?
It may seem strange that when you carry a heavy bag you are not doing any work – that's not what it feels like. In reality, lots of work is being done, since to hold the bag you use your muscles. Muscles are made of microscopic fibres, which are continuously contracting and relaxing, so are doing work.

Exercises

35 Figure 2.69 shows a boy taking a dog for a walk.
(a) Calculate the work done by the force shown when the dog moves 10 m forward.
(b) Who is doing the work?

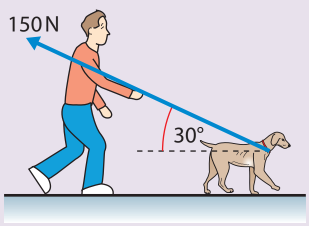


Figure 2.69

36 A bird weighing 200 g sits on a tree branch.
How much work does the bird do on the tree?

37 As a box slides along the floor it is slowed down by a constant force due to friction. If this force is 150 N and the box slides for 2 m, how much work is done against the frictional force?

Work done by a varying force

In the examples so far, the forces have been constant. If the force isn't constant, we can't simply say work = force × distance unless we use the 'average force'. In these cases, we can use a graphical method.

Let us first consider a constant force, F, acting over a distance Δs. The graph of force against distance for this motion is shown in Figure 2.70.

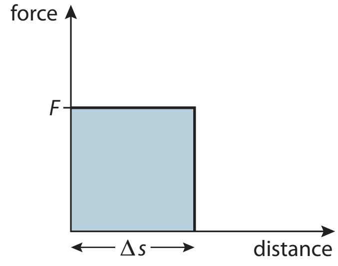


Figure 2.70 Force *vs* distance for a constant force.

From the definition of work, we know that in this case work = $F\Delta s$

This is the same as the area under the graph. From this we deduce that:

work done = area under the force–distance graph

Example

Stretching a spring

Stretching a spring is a common example of a varying force. When you stretch a spring it gets more and more difficult the longer it gets. Within certain limits the force needed to stretch the spring is directly proportional to the extension of the spring. This was first recognised by Robert Hooke in 1676, so is named 'Hooke's Law'. Figure 2.71 shows what happens if we add different weights to a spring; the more weight we add the longer it gets. If we draw a graph of force against distance as we stretch a spring, it will look like the graph in Figure 2.72. The gradient of this line, $\frac{F}{\Delta s}$ is called the spring constant, k.

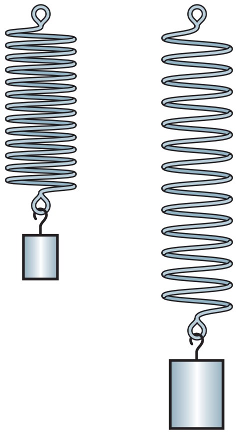


Figure 2.71 Stretching a spring.

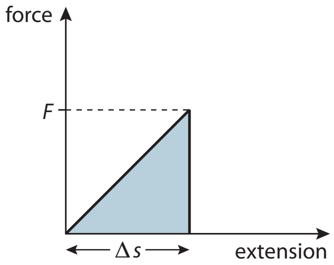


Figure 2.72 Force *vs* extension for a spring.

The work done as the spring is stretched is found by calculating the area under the graph.

$$\text{area} = \tfrac{1}{2}\text{base} \times \text{height} = \tfrac{1}{2}F\Delta s$$

So $\quad \text{work done} = \tfrac{1}{2}F\Delta s$

But if $\quad \frac{F}{\Delta s} = k$ then $F = k\Delta s$

Substituting for F gives

$$\text{work done} = \tfrac{1}{2}k\Delta s^2$$

Exercises

38 A spring of spring constant $2\,\text{N cm}^{-1}$ and length 6 cm is stretched to a new length of 8 cm.

(a) How far has the spring been stretched?

(b) What force will be needed to hold the spring at this length?

(c) Sketch a graph of force against extension for this spring.

(d) Calculate the work done in stretching the spring.

(e) The spring is now stretched a further 2 cm. Draw a line on your graph to represent this and calculate how much additional work has been done.

39 Calculate the work done by the force represented by Figure 2.73.

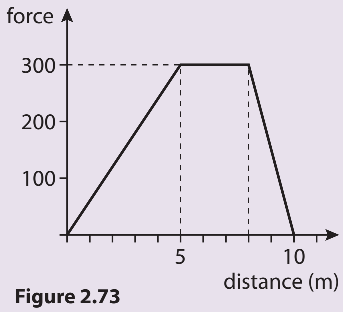


Figure 2.73

Energy

We have seen that it is sometimes possible for body A to do work on body B but what does A have that enables it to do work on B? To answer this question we must define a new quantity, energy.

Energy is the quantity that enables body A to do work on body B.

If body A collides with body B as shown in Figure 2.74, body A has done work on body B. This means that body B can now do work on body C. Energy has been transferred from A to B.

When body A does work on body B, energy is transferred from body A to body B.

The unit of energy is the joule (J).

Energy is a scalar quantity.

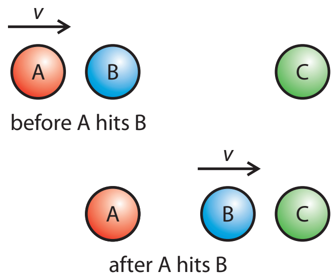


Figure 2.74 The red ball gives energy to the blue ball.

Different types of energy

If a body can do work then it has energy. There are two ways that a simple body such as a red ball can do work. In the example above, body A could do work because it was moving – this is called *kinetic energy*. Figure 2.75 shows an example where A can do work even though it isn't moving. In this example, body A is able to do work on body B because of its position above the Earth. If the hand is removed, body A will be pulled downwards by the force of gravity, and the string attached to it will then drag B along the table. If a body is able to do work because of its position, we say it has *potential energy*.

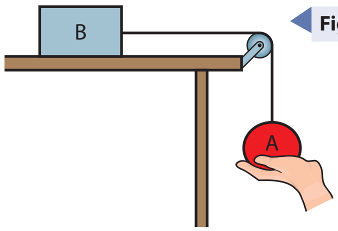


Figure 2.75

Kinetic energy (KE)

This is the energy a body has due to its movement. To give a body KE, work must be done on the body. The amount of work done will be equal to the increase in KE. If a constant force acts on a red ball of mass m as shown in Figure 2.76, then the work done is Fs.

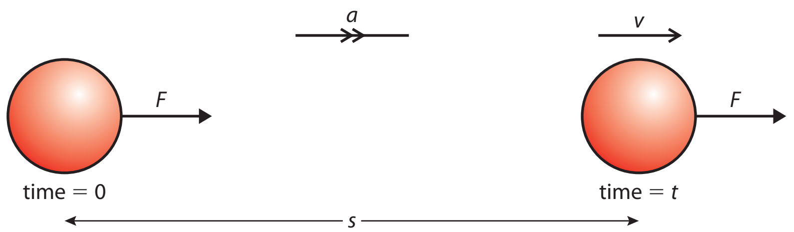


Figure 2.76

From Newton's second law we know that $F = ma$ which we can substitute in work $= Fs$ to give work $= mas$

We also know that since acceleration is constant we can use the *suvat* equation $v^2 = u^2 + 2as$ which since $u = 0$ simplifies to $v^2 = 2as$.

Rearranging this gives $as = \frac{v^2}{2}$ so work $= \frac{1}{2}mv^2$

This work has increased the KE of the body so we can deduce that:

$$KE = \frac{1}{2}mv^2$$

Use of words

If we say a body has potential energy it sounds as though it has the potential to do work. This is true, but a body that is moving has the potential to do work too. This can lead to misunderstanding. It would have been better to call it *positional energy*.

Other types of PE
In this section we only deal with examples of PE due to a body's position close to the Earth. However there are other positions that will enable a body to do work (for example, in an electric field). These will be introduced after the concept of fields has been introduced in Chapter 6.

Gravitational potential energy (PE)

This is the energy a body has due to its position above the Earth.

For a body to have PE it must have at some time been lifted to that position. The amount of work done in lifting it equals the PE. Taking the example shown in Figure 2.75, the work done in lifting the mass, m, to a height h is mgh (this assumes that the body is moving at a constant velocity so the lifting force and weight are balanced).

If work is done on the body then energy is transferred so:

gain in PE $= mgh$

The law of conservation of energy

We could not have derived the equations for KE or PE without assuming that the work done is the same as the gain in energy. The law of conservation of energy is a formal statement of this fact.

Energy can neither be created nor destroyed – it can only be changed from one form to another.

This law is one of the most important laws that we use in physics. If it were not true you could suddenly find yourself at the top of the stairs without having done any work in climbing them, or a car suddenly has a speed of 200 km h^{-1} without anyone touching the accelerator pedal. These things just don't happen, so the laws we use to describe the physical world should reflect that.

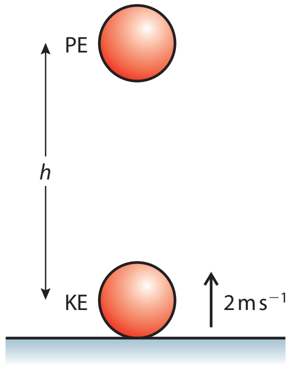


Figure 2.77 Work is done lifting the ball so it gains PE.

Worked examples

1 A ball of mass 200 g is thrown vertically upwards with a velocity of 2 m s^{-1} as shown in Figure 2.77. Use the law of conservation of energy to calculate its maximum height.

2 A block slides down the frictionless ramp shown in Figure 2.78. Use the law of conservation of energy to find its speed when it gets to the bottom.

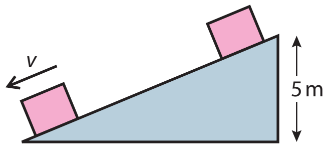


Figure 2.78 As the block slides down the slope it gains KE.

Solutions

1 At the start of its motion the body has KE. This enables the body to do work against gravity as the ball travels upwards. When the ball reaches the top, all the KE has been converted into PE. So applying the law of conservation of energy:

$$\text{loss of KE} = \text{gain in PE}$$

$$\tfrac{1}{2}mv^2 = mgh$$

so $$h = \frac{v^2}{2g} = \frac{2^2}{2 \times 10} = 0.2\text{ m}$$

This is exactly the same answer you would get by calculating the acceleration from $F = ma$ and using the *suvat* equations.

2 This time the body loses PE and gains KE so applying the law of conservation of energy:

$$\text{loss of PE} = \text{gain of KE}$$

$$mgh = \tfrac{1}{2}mv^2$$

So $$v = \sqrt{2gh} = \sqrt{(2 \times 10 \times 5)} = 10\,\text{m s}^{-1}$$

Again, this is a much simpler way of getting the answer than using components of the forces.

Exercises

Use the law of conservation of energy to solve the following:

40 A stone of mass 500 g is thrown off the top of a cliff with a speed of $5\,\text{m s}^{-1}$. If the cliff is 50 m high, what is its speed just before it hits the ground?

41 A ball of mass 250 g is dropped 5 m onto a spring as shown in Figure 2.79.

(a) How much KE will the ball have when it hits the spring?

(b) How much work will be done as the spring is compressed?

(c) If the spring constant is $250\,\text{kN m}^{-1}$, calculate how far the spring will be compressed.

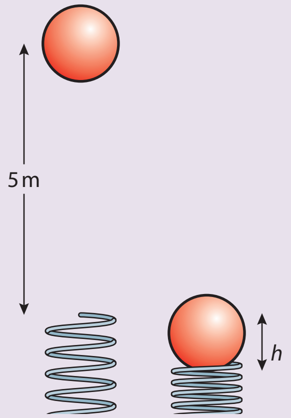


Figure 2.79 In this example the spring is compressed, not stretched, but Hooke's law still applies.

42 A ball of mass 100 g is hit vertically upwards with a bat. The bat exerts a constant force of 15 N on the ball and is in contact with it for a distance of 5 cm.

(a) How much work does the bat do on the ball?

(b) How high will the ball go?

43 A child pushes a toy car of mass 200 g up a slope. The car has a speed of $2\,\text{m s}^{-1}$ at the bottom of the slope.

(a) How high up the slope will the car go?

(b) If the speed of the car were doubled how high would it go now?

Forms of energy

When we are describing the motion of simple red balls there are only two forms of energy, KE and PE. However, when we start to look at more complicated systems, we discover that we can do work using a variety of different machines, such as petrol engine, electric engine etc. To do work, these machines must be given energy and this can come in many forms, for example:

- Petrol
- Gas
- Electricity
- Solar
- Nuclear

As you learn more about the nature of matter in Chapter 3, you will discover that all of these (except solar) are related to either KE or PE of particles.

Energy and collisions

One of the reasons that we brought up the concept of energy was related to the collision between two balls as shown in Figure 2.80. We now know that if no energy is lost when the balls collide, then the KE before the collision = KE after. This enables us to calculate the velocity afterwards and the only solution in this example is quite a simple one. The red ball gives all its KE to the blue one, so the red one stops and the blue one continues, with velocity = 10 m s^{-1}. If the balls become squashed, then some work needs to be done to squash them. In this case not all the KE is transferred, and we can only calculate the outcome if we know how much energy is used in squashing the balls.

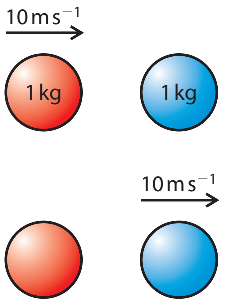


Figure 2.80

Elastic collisions

An elastic collision is a collision in which both momentum and KE are conserved.

Example

Two balls of equal mass collide with each other elastically with the velocities shown in Figure 2.81. What is the velocity of the balls after the collision?

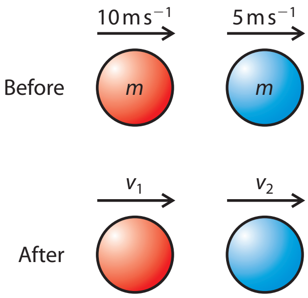


Figure 2.81

Examiner's hint: If the masses are the same in an elastic collision the velocities of the two bodies swap

Conservation of momentum:

Momentum before = momentum after

$m \times 10 + m \times 5 = mv_1 + mv_2$

$10 + 5 = v_1 + v_2$

Conservation of KE:

$\frac{1}{2}m \times 10^2 + \frac{1}{2}m \times 5^2$

$= \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2$

$10^2 + 5^2 = v_1^2 + v_2^2$

There are only two possible solutions to these two equations; either the velocities don't change, which means there isn't a collision, or the velocities swap, so $v_1 = 5$ m s^{-1} and $v_2 = 10$ m s^{-1}.

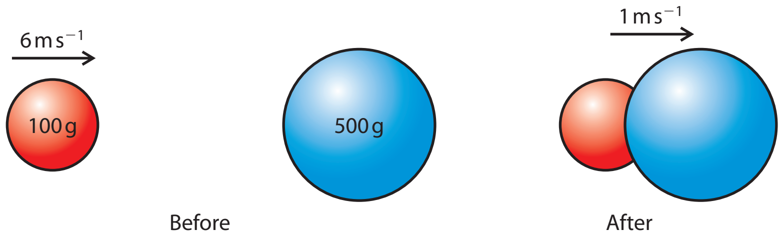
Inelastic collisions

There are many outcomes of an inelastic collision but here we will only consider the case when the two bodies stick together. We call this *totally inelastic collision.*

Example

When considering the conservation of momentum in collisions, we used the example shown in Figure 2.82. How much work was done to squash the balls in this example?

Figure 2.82



According to the law of conservation of energy, the work done squashing the balls is equal to the loss in KE.

KE loss = KE before − KE after = $\frac{1}{2} \times 0.1 \times 6^2 - \frac{1}{2} \times 0.6 \times 1^2$

KE loss = 1.8 − 0.3 = 1.5 J

So work done = 1.5 J

Explosions

Explosions can never be elastic since, without doing work and therefore increasing the KE, the parts that fly off after the explosion would not have any KE and would therefore not be moving. The energy to initiate an explosion often comes from the chemical energy contained in the explosive.

Example

Again consider a previous example where a ball exploded (shown again in Figure 2.83). How much energy was supplied to the balls by the explosive?

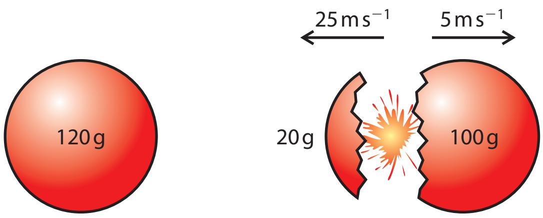


Figure 2.83

Sharing of energy
The result of this example is very important; we will use it when dealing with nuclear decay later on. So remember, when a body explodes into two unequal bits, the small bit gets most energy.

According to the law of conservation of energy, the energy from the explosive equals the gain in KE of the balls.

KE gain = KE after − KE before

KE gain = $(\frac{1}{2} \times 0.02 \times 25^2 + \frac{1}{2} \times 0.1 \times 5^2) - 0 = 6.25 + 1.25 = 7.5$ J

Notice how the small bit gets most of the energy. We will come back to this when studying nuclear decay.

Exercise

44 Two balls are held together by a spring as shown in Figure 2.84. The spring has a spring constant of 10 N cm^{-1} and has been compressed a distance 5 cm.

(a) How much work was done to compress the spring?

(b) How much KE will each gain?

(c) If each ball has a mass of 10 g calculate the velocity of each ball.

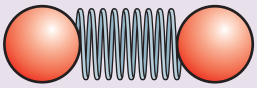


Figure 2.84

45 Two pieces of modelling clay as shown in Figure 2.85 collide and stick together.

(a) Calculate the velocity of the lump after the collision.

(b) How much KE is lost during the collision?

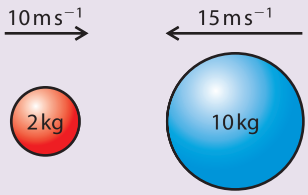


Figure 2.85

46 A red ball travelling at 10 m s^{-1} to the right collides with a blue ball with the same mass travelling at 15 m s^{-1} to the left. If the collision is elastic, what are the velocities of the balls after the collision?

Power

We know that to do work requires energy, but work can be done quickly or it can be done slowly. This does not alter the energy exchanged but the situations are certainly different. For example we know that to lift one thousand, 1 kg bags of sugar from the floor to the table is not an impossible task – we can simply lift them one by one. It will take a long time but we would manage it in the end. However, if we were asked to do the same task in 5 seconds, we would either have

Power and velocity

If power $= \frac{\text{work done}}{\text{time}}$ then we can also write $P = \frac{F\Delta s}{t}$ so $P = F\left(\frac{\Delta s}{t}\right)$ which is the same as $P = Fv$ where v is the velocity.

to lift all 1000 kg at the same time or move each bag in 0.005 s, both of which are impossible. Power is the quantity that distinguishes between these two tasks.

Power is defined as:

Power = work done per unit time.

Unit of power is the J/s which is the same as the watt (W).

Power is a scalar quantity.

Examples

1. The powerful car

We often use the term power to describe cars. A powerful car is one that can accelerate from 0 to 100 km h^{-1} in a very short time. When a car accelerates, energy is being converted from the chemical energy in the fuel to KE. To have a big acceleration the car must gain KE in a short time, hence be powerful.

Horse power

Horse power is often used as the unit for power when talking about cars and boats.

746 W = 1 hp

So in Example 1, the power of the car is 105 hp.

2. Power lifter

A power lifter is someone who can lift heavy weights, so shouldn't we say they are strong people rather than powerful? A power lifter certainly is a strong person (if they are good at it) but they are also powerful. This is because they can lift a big weight in a short time.

Worked example

A car of mass 1000 kg accelerates from rest to 100 km h^{-1} in 5 seconds. What is the average power of the car?

Solution

100 km h^{-1} = 28 m s^{-1}.

The gain in KE of the car $= \frac{1}{2}mv^2 = \frac{1}{2} \times 1000 \times 28^2 = 392$ kJ

If the car does this in 5 s then

$$\text{power} = \frac{\text{work done}}{\text{time}} = \frac{392}{5} = 78.4\text{ kW}$$

Exercises

47 A weightlifter lifts 200 kg 2 m above the ground in 5 s. Calculate the power of the weightlifter in watts.

48 In 25 s a trolley of mass 50 kg runs down a hill. If the difference in height between the top and the bottom of the hill is 50 m, how much power will have been dissipated?

49 A car moves along a road at a constant velocity of 20 m s^{-1}. If the resistance force acting against the car is 1000 N, what is the power developed by the engine?

Efficiency

When a box is pulled along the floor, the person pulling has to do work. This work is converted into the KE of the box and some of it is done against friction. Since the result they are trying to achieve is to move the box, the energy transferred to KE could be termed 'useful energy' and the rest is 'wasted'. (The waste energy in this example turns into heat, but we will deal with that in the next chapter). The efficiency tells us how good a system is at turning the input energy into useful work.

Efficiency and power

Since the work is going in and out at the same rate, efficiency can also be expressed as

$$\frac{\text{power out}}{\text{power in}}$$

Efficiency is defined as:

$$\frac{\text{useful work out}}{\text{energy put in}}$$

Efficiency has no units.

Efficiency is a scalar quantity.

It is worth noting here that due to the law of conservation of energy, the efficiency can never be greater than 1.

Worked example

A box of mass 10 kg is pulled along the floor for 2 m by a horizontal force of 50 N. If the frictional force is 20 N, what is the efficiency of the system?

Solution

The work done by the pulling force = force × distance = 50 × 2 = 100 J

The unbalanced force on the box = 50 − 20 = 30 N

So the work done on the box = 30 × 2 = 60 J

This work is exchanged to the box and increases its KE.

Work done against friction = 20 × 2 = 40 J

So work in = 100 J and total work out = 60 + 40 = 100 J

Efficiency $= \frac{\text{useful work out}}{\text{work in}} = \frac{60}{100} = 0.6$

Expressed as a percentage, this is 60%.

Exercise

50 A motor is used to lift a 10 kg mass 2 m above the ground in 4 s. If the power input to the motor is 100 W, what is the efficiency of the motor?

51 A motor is 70% efficient. If 60 kJ of energy is put into the engine, how much work is got out?

52 The drag force that resists the motion of a car travelling at 80 km h^{-1} is 300 N.
- **(a)** What power is required to keep the car travelling at that speed?
- **(b)** If the efficiency of the engine is 60%, what is the power of the engine?

2.10 Uniform circular motion

Assessment statements

2.4.1 Draw a vector diagram to illustrate that the acceleration of a particle moving with constant speed in a circle is directed towards the centre of the circle.

2.4.2 Apply the expression for centripetal acceleration.

2.4.3 Identify the force producing circular motion in various situations.

2.4.4 Solve problems involving circular motion.

Circular motion quantities
When describing motion in a circle we often use quantities referring to the angular rather than linear quantities.

Time period (*T*)
The time taken for one complete circle.

Angular displacement (*θ*)
The angle swept out by a line joining the body to the centre.

Angular velocity (*ω*)
The angle swept out per unit time.
$\omega = \left(\frac{2\pi}{T}\right)$

Frequency (*f*)
The number of complete circles per unit time $\left(f = \frac{1}{T}\right)$.

If a car travels around a bend at 30 km h^{-1}, it is obviously travelling at a constant speed, since the speedometer would register 30 km/hr all the way round. However it is not travelling at constant velocity. This is because velocity is a vector quantity and for a vector quantity to be constant, both magnitude and direction must remain the same. Bends in a road can be many different shapes, but to simplify things, we will only consider circular bends.

Centripetal acceleration

From the definition of acceleration, we know that if the velocity of a body changes, it must be accelerating, and that the direction of acceleration is in the direction of the change in velocity. Let us consider a body moving in a circle with a constant speed *v*. Figure 2.86 shows two positions of the body separated by a short time.

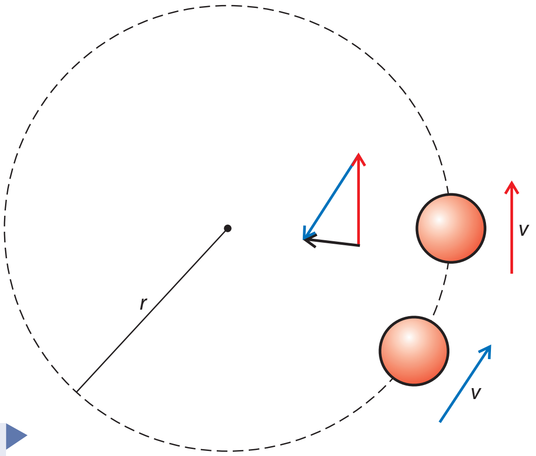


Figure 2.86 A body travels at constant speed around a circle of radius *r*.

Size of centripetal acceleration
The centripetal acceleration is given by the following formula

$$a = \frac{v^2}{r}$$

Or in circular terms

$$a = \omega^2 r$$

Since this is a two-dimensional problem, we will need to draw the vectors to find out the change in velocity. This has been done on the diagram. It can be seen that the vector representing the change in velocity points to the centre of the circle. This means that the acceleration is also directed towards the centre, and this acceleration is called *centripetal acceleration*. All bodies moving in a circle accelerate towards the centre.

Centripetal force

From Newton's first law, we know that if a body accelerates, there must be an unbalanced force acting on it. The second law tells us that that force is in the direction of the acceleration. This implies that there must be a force acting towards the centre. This force is called the *centripetal force*.

Centripetal force, $F = \frac{mv^2}{r} = m\omega^2 r$

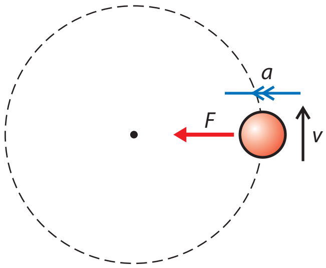


Figure 2.87 Showing the direction of the force and acceleration.

Examples

All bodies moving in a circle must be acted upon by a force towards the centre of the circle. However, this can be provided by many different forces.

1. Ball on a string

You can make a ball move in a circle by attaching it to a string and swinging it round your head. In this case the centripetal force is provided by the tension in the string. If the string suddenly broke, the ball would fly off in a straight line at a tangent to the circle.

2. Car going round a bend

When a car goes round a bend, the force causing it to move in a circle is the friction between the road and the tyres. Without this friction the car would move in a straight line.

3. Wall of Death

In the Wall of Death the rider travels around the inside walls of a cylinder. Here the centripetal force is provided by the normal force between the wall and the tyres.

Wall of Death

Exercise

53 Calculate the centripetal force for a 1000 kg car travelling around a circular track of radius 50 m at 30 km h^{-1}.

54 A 200 g ball is made to travel in a circle of radius 1 m on the end of a string. If the maximum force that the string can withstand before breaking is 50 N, what is the maximum speed of the ball?

55 A body of mass 250 g moves in a circle of radius 50 cm with a speed of 2 m s^{-1}. Calculate
- **(a)** the distance travelled per revolution
- **(b)** the time taken for one revolution
- **(c)** the angle swept out per revolution
- **(d)** the angular velocity
- **(e)** the centripetal acceleration
- **(f)** the centripetal force.

Centripetal is not an extra force
Remember when solving circular motion problems, centripetal force is not an extra force – it is one of the existing forces. Your task is to find which force (or a component of it) points towards the centre.

Practice questions

1 This question is about linear motion.
A police car P is stationary by the side of a road. A car S, exceeding the speed limit, passes the police car P at a constant speed of 18 m s^{-1}. The police car P sets off to catch car S just as car S passes the police car P. Car P accelerates at 4.5 m s^{-2} for a time of 6.0 s and then continues at constant speed. Car P takes a time *t* seconds to draw level with car S.

(a) **(i)** State an expression, in terms of *t*, for the distance car S travels in *t* seconds. (1)

(ii) Calculate the distance travelled by the police car P during the first 6.0 seconds of its motion. (1)

(iii) Calculate the speed of the police car P after it has completed its acceleration. (1)

(iv) State an expression, in terms of t, for the distance travelled by the police car P during the time that it is travelling at constant speed. (1)

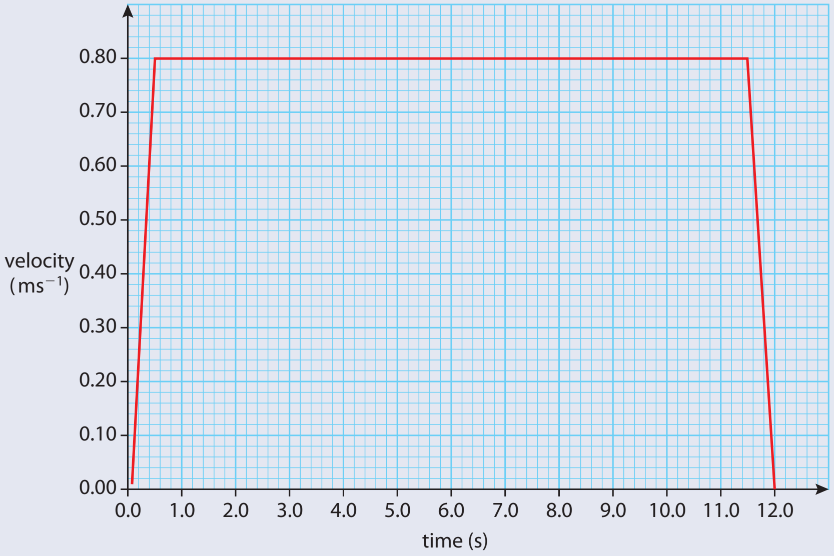
(b) Using your answers to **(a)**, determine the total time *t* taken for the police car P to draw level with car S. (2)

(Total 6 marks)

2 This question is about the kinematics of an elevator (lift).

(a) Explain the difference between the gravitational mass and the inertial mass of an object. (3)

An elevator (lift) starts from rest on the ground floor and comes to rest at a higher floor. Its motion is controlled by an electric motor. A simplified graph of the variation of the elevator's velocity with time is shown below.



(b) The mass of the elevator is 250 kg. Use this information to calculate

(i) the acceleration of the elevator during the first 0.50 s. (2)

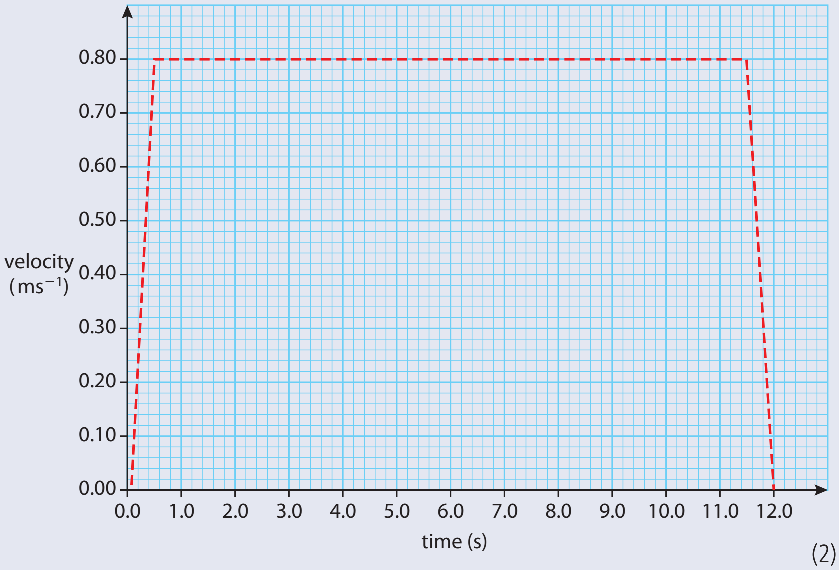
(ii) the total distance travelled by the elevator. (2)

(iii) the minimum work required to raise the elevator to the higher floor. (2)

(iv) the minimum average power required to raise the elevator to the higher floor. (2)

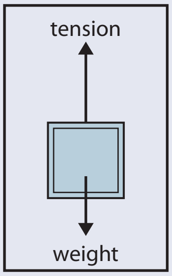
(v) the efficiency of the electric motor that lifts the elevator, given that the input power to the motor is 5.0 kW. (2)

(c) On the graph axes below, sketch a realistic variation of velocity for the elevator. Explain your reasoning. *(The simplified version is shown as a dotted line)*



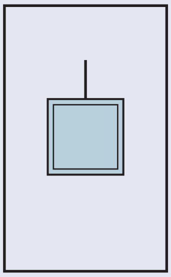
(2)

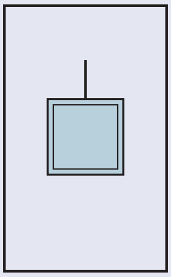
The elevator is supported by a cable. The diagram opposite is a free-body force diagram for when the elevator is moving upwards during the first 0.50 s.



(d) In the space below, draw free-body force diagrams for the elevator during the following time intervals.

(i) 0.5 to 11.50 s **(ii)** 11.50 to 12.00 s

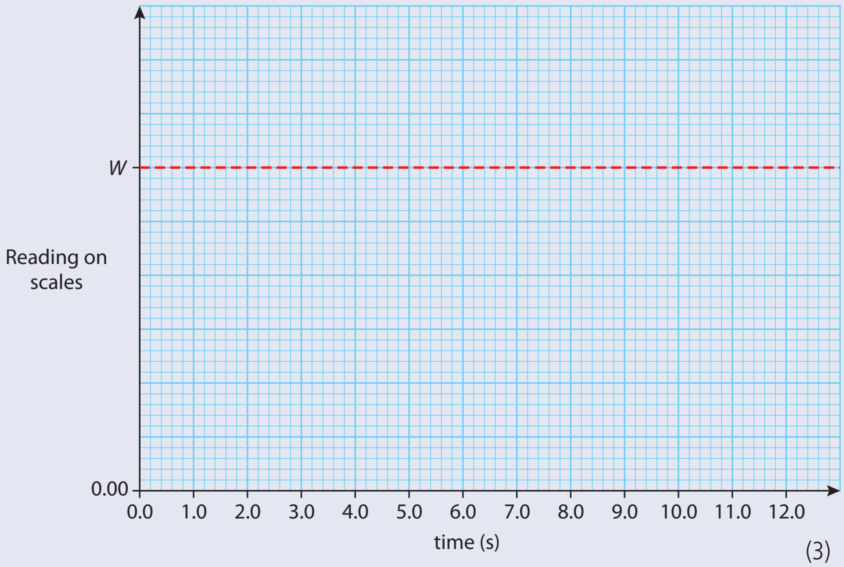




(3)

A person is standing on weighing scales in the elevator. Before the elevator rises, the reading on the scales is *W*.

(e) On the axes below, sketch a graph to show how the reading on the scales varies during the whole 12.00 s upward journey of the elevator. *(Note that this is a sketch graph – you do not need to add any values.)*



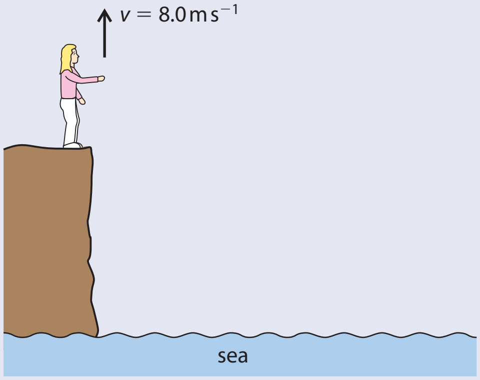
(3)

(f) The elevator now returns to the ground floor where it comes to rest. Describe and explain the energy changes that take place during the whole up and down journey.

(4)

(Total 25 marks)

3 This question is about throwing a stone from a cliff.
Antonia stands at the edge of a vertical cliff and throws a stone vertically upwards.



The stone leaves Antonia's hand with a speed $v = 8.0\ \text{m s}^{-1}$.
The acceleration of free fall g is $10\ \text{m s}^{-2}$ and all distance measurements are taken from the point where the stone leaves Antonia's hand.

(a) Ignoring air resistance calculate

(i) the maximum height reached by the stone. (2)

(ii) the time taken by the stone to reach its maximum height. (1)

The time between the stone leaving Antonia's hand and hitting the sea is 3.0 s.

(b) Determine the height of the cliff. (3)

(Total 6 marks)

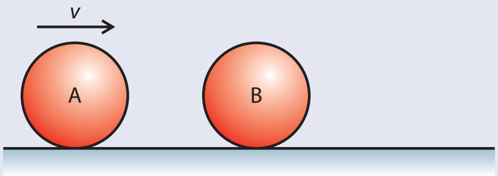
4 This question is about conservation of momentum and conservation of energy.

(a) State Newton's third law. (1)

(b) State the law of conservation of momentum. (2)

The diagram below shows two identical balls A and B on a horizontal surface. Ball B is at rest and ball A is moving with speed v along a line joining the centres of the balls. The mass of each ball is M.

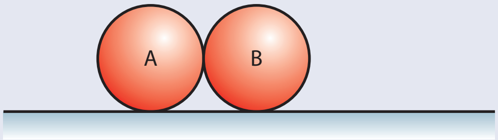
Before collision



During the collision of the balls, the magnitude of the force that ball A exerts on ball B is F_{AB} and the magnitude of the force that ball B exerts on ball A is F_{BA}.

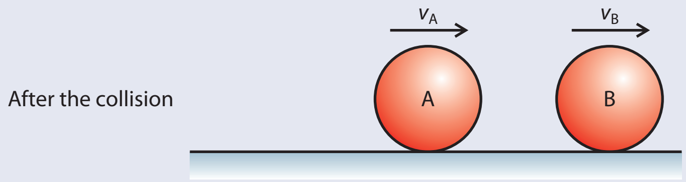
(c) On the diagram below, add labelled arrows to represent the magnitude and direction of the forces F_{AB} and F_{BA}.

During the collision



(3)

The balls are in contact for a time t. After the collision, the speed of ball A is $+v_A$ and the speed of ball B is $+v_B$ in the directions shown.



As a result of the collision, there is a change in momentum of ball A and of ball B.

(d) Use Newton's second law of motion to deduce an expression relating the forces acting during the collision to the change in momentum of

  (i) ball B. (2)

  (ii) ball A. (2)

(e) Apply Newton's third law and your answers to **(d)** to deduce that the change in momentum of the system (ball A and ball B) as a result of this collision, is zero. (4)

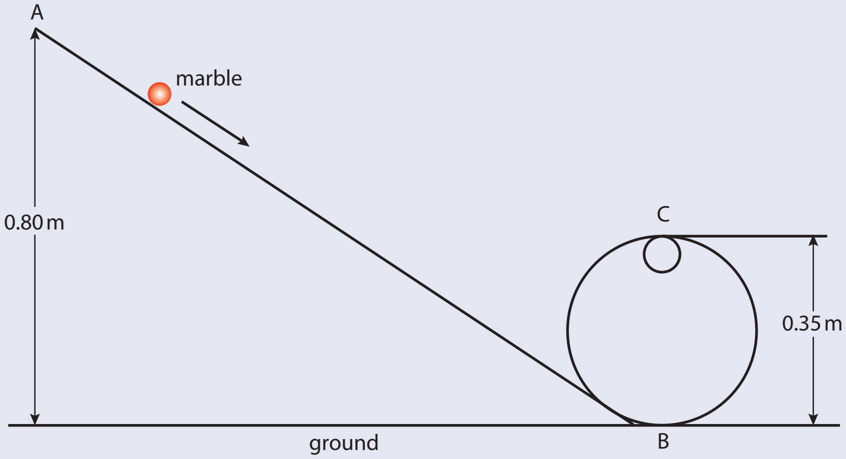
(f) Deduce that if kinetic energy is conserved in the collision, then after the collision, ball A will come to rest and ball B will move with speed v. (3)

(Total 17 marks)

5 This question is about the kinematics and dynamics of circular motion.

(a) A car goes round a curve in a road at constant speed. Explain why, although its speed is constant, it is accelerating. (2)

In the diagram below, a marble (small glass sphere) rolls down a track, the bottom part of which has been bent into a loop. The end A of the track, from which the marble is released, is at a height of 0.80 m above the ground. Point B is the lowest point and point C the highest point of the loop. The diameter of the loop is 0.35 m.



The mass of the marble is 0.050 kg. Friction forces and any gain in kinetic energy due to the rotating of the marble can be ignored. The acceleration due to gravity, $g = 10\ \text{m s}^{-2}$.

Consider the marble when it is at point C.

(b) **(i)** On the diagram, draw an arrow to show the direction of the resultant force acting on the marble. (1)

  (ii) State the names of the two forces acting on the marble. (2)

  (iii) Deduce that the speed of the marble is $3.0\ \text{m s}^{-1}$. (3)

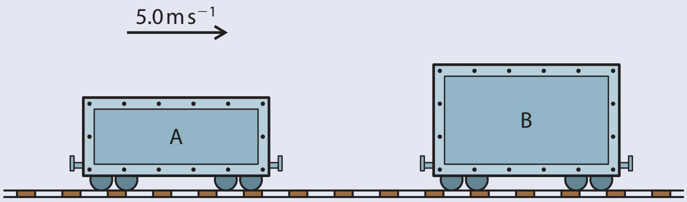
(iv) Determine the resultant force acting on the marble and hence determine the reaction force of the track on the marble. (4)

(Total 12 marks)

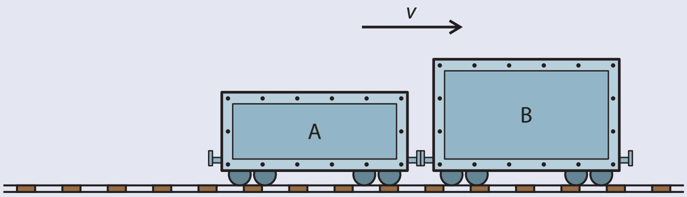
6 This question is about the collision between two railway trucks (carts).

(a) Define *linear momentum*. (1)

In the diagram below, railway truck A is moving along a horizontal track. It collides with a stationary truck B and on collision, the two join together. Immediately before the collision, truck A is moving with speed 5.0 m s^{-1}. Immediately after collision, the speed of the trucks is *v*.



Immediately before collision



Immediately after collision

The mass of truck A is 800 kg and the mass of truck B is 1200 kg.

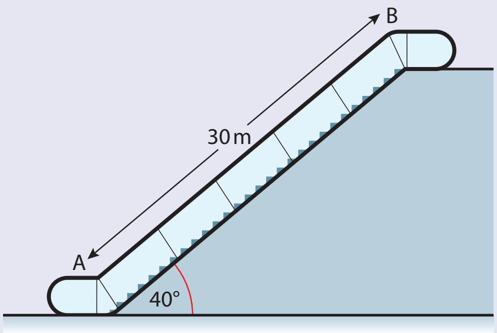
(b) **(i)** Calculate the speed v immediately after the collision. (3)

(ii) Calculate the total kinetic energy lost during the collision. (2)

(c) Suggest what has happened to the lost kinetic energy. (2)

(Total 8 marks)

7 This question is about estimating the energy changes for an escalator (moving staircase). The diagram below represents an escalator. People step on to it at point A and step off at point B.



(a) The escalator is 30 m long and makes an angle of 40° with the horizontal. At full capacity, 48 people step on at point A and step off at point B every minute.

(i) Calculate the potential energy gained by a person of weight 700 N in moving from A to B. (2)

(ii) Estimate the energy supplied by the escalator motor to the people every minute when the escalator is working at full capacity. (1)

(iii) State **one** assumption that you have made to obtain your answer to **(ii)**. (1)

The escalator is driven by an electric motor that has an efficiency of 70 %.

(b) **(i)** Using your answer to **(a)(ii),** calculate the minimum input power required by the motor to drive the escalator. (3)

(ii) Explain why it is not necessary to take into account the weight of the escalator when calculating the input power. (1)

(c) Explain why in practice, the power of the motor will need to be greater than that calculated in **(b)(i)**. (1)

(Total 9 marks)