

5

Electrical currents

5.1

Electric potential difference, current and resistance

Assessment statements

- 5.1.1 Define electric potential difference.
- 5.1.2 Determine the change in potential energy when a charge moves between two points at different potentials.
- 5.1.3 Define the electronvolt.
- 5.1.4 Solve problems involving electric potential difference.
- 5.1.5 Define electric current.
- 5.1.6 Define resistance.
- 5.1.7 Apply the equation for resistance in the form $R = \frac{\rho l}{A}$ where ρ is the resistivity of the material of the resistor.
- 5.1.8 State Ohm's law.
- 5.1.9 Compare ohmic and non-ohmic behaviour.
- 5.1.10 Derive and apply expressions for electrical power dissipation in resistors.
- 5.1.11 Solve problems involving potential difference, current and resistance.



All electrical appliances convert electrical energy into other forms.

In this chapter we will develop the theory behind electric circuits, but first we need to understand how the individual components work.

The battery

All electrical devices convert electrical energy into other forms; an MP3 player produces sound, a torch produces light and an electric motor produces mechanical energy. In cordless devices (ones that don't plug into the mains) the source of this energy is the battery.

Inside the battery, chemicals react with each other to convert chemical energy into electrical energy.

When the light bulb is connected across the ends of the battery, it gets hot and starts to glow, giving out energy in the form of light and heat. This energy comes from the battery. To make the bulb light up, we must make a complete circuit. If this circuit is broken, the bulb goes out. It seems like something is flowing through the wire; we call the flow *current* and the thing that is flowing we call *charge*.

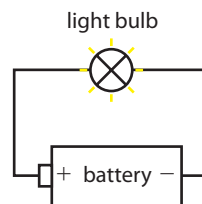


Figure 5.1 This simple circuit consists of a battery and a light bulb. The battery has two ends, one is positive (+) and the other is negative (-). To make a circuit, the bulb must be connected as shown.



To build your own circuits and test the theories in this chapter, visit www.heinemann.co.uk/hotlinks, enter the express code 4426P and click on the Weblink 5.1.

Water analogy

To explain the observations, we can use an analogy.

Figure 5.2 Water flow is similar to the flow of electricity.

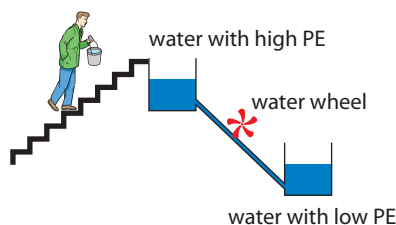
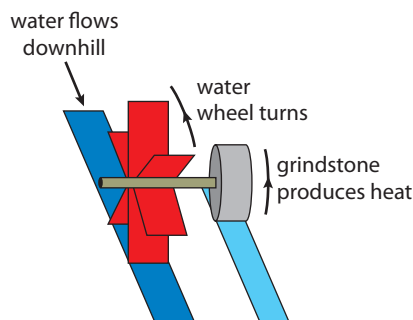


Figure 5.2 shows a man carrying water up the stairs and pouring it into a tank. The water flows from this tank through a pipe into a lower one. As it flows, the water turns a water wheel. The water wheel turns a grindstone that rubs against a hard surface, producing heat.

Energy explanation

We can explain what is happening using the principle of conservation of energy. As the man carries the water up the stairs he does work on the water, increasing its potential energy (PE). When the water flows downhill into the lower tank, it loses PE. This energy is given to the water wheel, which turns the grindstone, producing heat. So we can say that the work done by the man is the same as the heat produced at the grindstone.

Figure 5.3 Energy changes.



Charge

Unlike with water, we cannot see charge flowing through the wire. We will find out later (Chapter 6) that there are two types of charge (+ and -) but to make things simple we will just consider + charge. The unit of charge is the coulomb (C).

Electrical potential energy

To make the water flow through the pipe, the man did work on the water to raise its potential energy. In the electrical circuit, the battery does work to increase the electrical potential energy of the charges. The charges on the + end of the battery have a higher potential energy than the ones on the - end, so charge flows from + to - outside the cell.

Current

Current is the flow of charge, and since charges at the + end of the battery have higher PE than those at the - end, the current flows from + to -.

The unit of current is the amp (A).

The amp is a scalar quantity.

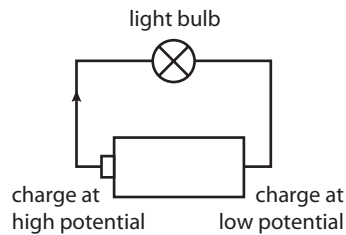
$$\text{Current} = \frac{\text{charge}}{\text{time}}$$

1 amp = 1 coulomb per second.



The complete picture

The battery takes charges and puts them in a position of high potential at the + end. If a bulb is connected between the ends of the battery, this charge flows from the position of high potential to a position of low potential. The potential energy is converted to heat and light.



Microscopic model

Metals allow charge to flow because they contain small negatively charged particles called *electrons*. Since the electrons are negative, they flow from low potential to high potential. Trying to imagine something flowing uphill is not very easy, so we will stick with positive charges flowing downhill. This is not a problem but we must always remember that electrons flow in the opposite direction to conventional current.

Energy transformation in a battery

Since energy is conserved, the energy given to the charges must come from somewhere – in fact, the battery converts chemical energy to electrical PE. The chemicals in the battery contain charges that are all mixed up. When these chemicals react, the charges are rearranged so that there are more + charges at the + end of the battery. After the reaction, the chemicals have less energy, as the chemical energy has been converted to electrical PE.

Potential difference

If we move a + charge from the negative end of the battery to the positive end, we need to do work. This is because the charge has a higher potential energy at the positive end than the negative end. The amount of work done per unit charge is defined as the potential difference (p.d.) between the plates or terminals.

The unit of p.d. is the volt. The volt is a scalar quantity. 1 volt = 1 joule per coulomb.

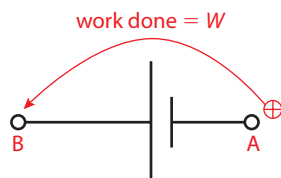
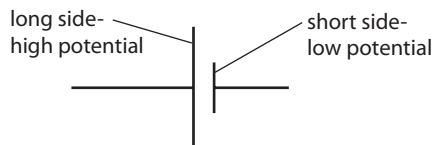


Figure 5.4 As the charge flows from + to –, its potential energy is converted into heat and light in the bulb.



The electron is a small particle responsible for carrying charge in conductors.
Charge = -1.6×10^{-19} C
Electrons are negative so flow in the opposite direction to conventional current.



We consider current to flow in the opposite direction to the way that electrons actually flow. Does it matter that our model is actually completely backwards?



The PhET simulation 'Battery voltage' shows how the charges are rearranged. The little people represent chemical energy. To view this, visit www.heinemann.co.uk/hotlinks, enter the express code 4426P and click on the Weblink 5.2.

Figure 5.5 The symbol for a cell makes a lot of sense; the side that is at the highest potential has the longest line.



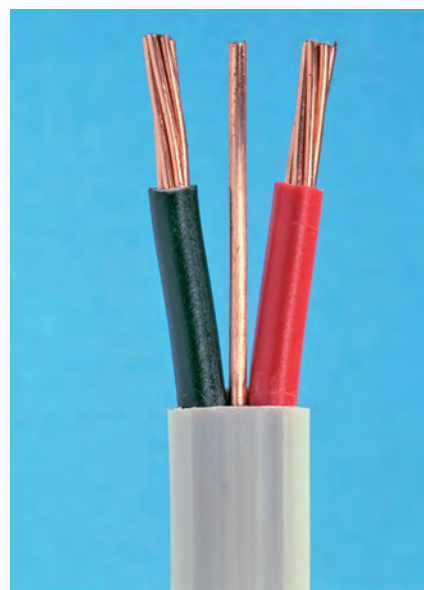
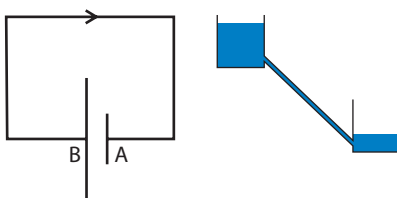
The p.d. between A and B is defined as the amount of work done per unit charge in taking a small +ve test charge from A to B.

Figure 5.6 If an amount of work W is done in taking charge q from A to B then the p.d. (V) between A and B is given by the equation $V = W/q$.

Conductors and insulators

A *conductor* is a material that allows charge to flow through it. An *insulator* doesn't. All metals are good conductors of electricity, but plastics are insulators.

Figure 5.7 Current flows from B to A like water flowing downhill.



The metal core of the cable conducts electricity; the plastic cover is an insulator.

Worked examples

- 1 If a current of 10 A flows for 15 s, how much charge flows?
- 2 How much energy is lost when +5 C of charge flows from the +ve terminal to the -ve terminal of a 9 V battery?

Solutions

- 1 Current $I = \text{charge per unit time} = \frac{Q}{t}$
 $Q = It$
 $= 10 \times 15 \text{ C}$
 $= 150 \text{ C}$
- 2 Energy to move 1 C = 9 J
 Energy to move 5 C = $5 \times 9 \text{ J} = 45 \text{ J}$

● **Examiner's hint:** From definition of p.d., the energy lost per unit charge moving from one terminal to the other is 9 J C^{-1} .

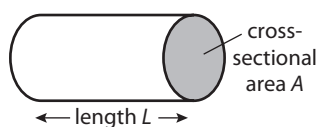


Figure 5.8 Dimensions of a conductor.

Resistance

The rate at which charge can flow through a conductor depends on the size and material of the conductor. A conductor that does not let much current flow for a given p.d. is said to have a high resistance.

Resistance (R) is related to: cross-sectional area (A), length (L) and the material.

$$R \propto \frac{L}{A}$$

The constant of proportionality is called the *resistivity* (ρ).

So
$$R = \rho \frac{L}{A}$$

The unit of resistance is the ohm (Ω).

This means that more current flows through a short fat conductor than a long thin one.

Figure 5.9 More current flows through the bottom branch than the top one.

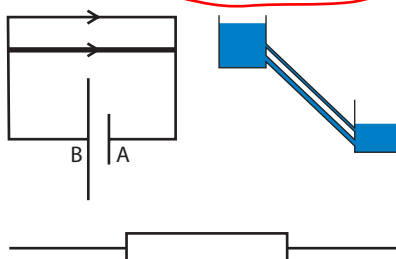


Figure 5.10 The symbol for a resistor.

Resistivity

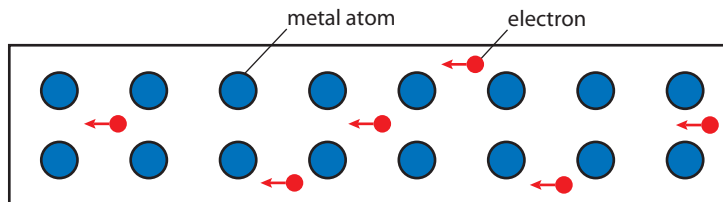


By rearranging $R = \rho \frac{L}{A}$ we get $\rho = \frac{RA}{L}$ (units Ωm).

From this we can deduce that if the length of a sample of material is 1m and the cross-sectional area is 1m^2 , then $\rho = R$. You can probably imagine that the resistance of such a large piece of metal will be very small – that’s why the values of resistivity are so low (e.g. for copper $\rho = 1.72 \times 10^{-8} \Omega\text{m}$).

Microscopic model of resistance

As mentioned earlier, it is actually the negative charges called *free electrons* that move. As they move through the metal, they give energy to the metal atoms, causing the temperature of the metal to increase.



Worked example

The resistivity of copper is $1.72 \times 10^{-8} \Omega\text{m}$. What is the resistance of a 1 m length of 2 mm diameter copper wire?

Solution

$$\begin{aligned} \text{Cross-sectional area} &= \pi (0.001)^2 \\ &= 3.14 \times 10^{-6} \text{m}^2 \\ R &= \rho \frac{L}{A} \\ &= 1.72 \times 10^{-8} \times \frac{1}{3.14 \times 10^{-6}} \Omega \\ &= 0.0055 \Omega \end{aligned}$$

A resistor is a component with a known resistance. You can work out the resistance from the colours.



W To try the battery-resistance circuit, visit www.heinemann.co.uk/hotlinks, enter the express code 4426P and click on Weblink 5.3.

Figure 5.11 Electrons move through metal, giving energy to atoms.

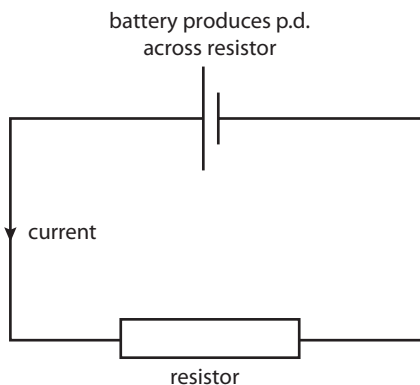
Examiner’s hint:

$$\begin{aligned} \text{Area} &= \pi r^2 \text{ where radius} \\ &= \frac{1}{2} \times 2 \text{ mm} = 0.001 \text{ m.} \end{aligned}$$

Ohm's law

Ohm's law relates the current flowing through a conductor with the potential difference across it.

Figure 5.12 We can now make a simple circuit with a battery and a resistor.



The current flowing through an ohmic conductor is directly proportional to the potential difference across it, provided temperature and other physical conditions remain constant.

Figure 5.13 Doubling p.d. doubles the current.

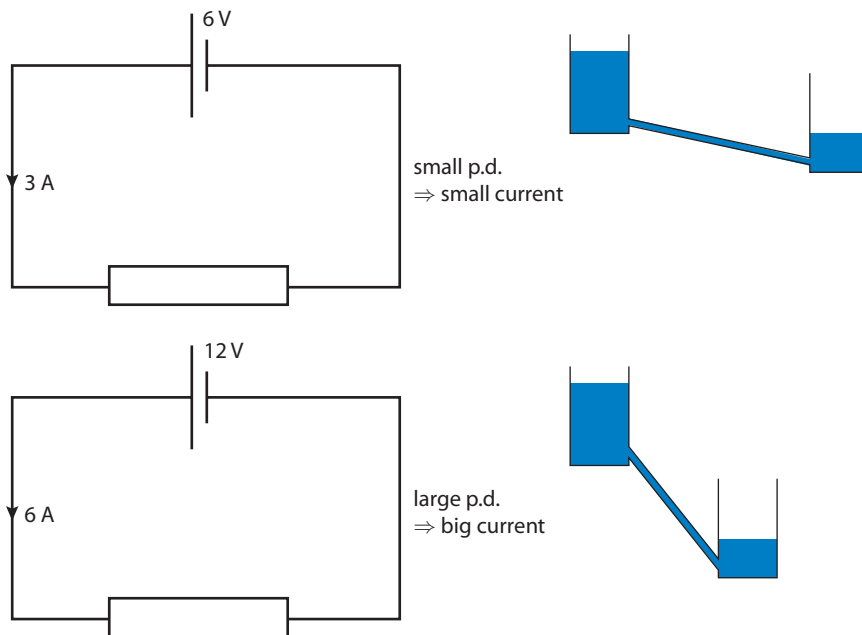
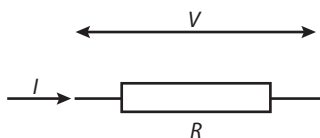


Figure 5.14 V , I and R



If the p.d. across a conductor is V and the current flowing through it is I , then according to Ohm's law:

$$V \propto I$$

The constant of proportionality is the resistance, R .

So:

$$V = IR$$

Worked example

If the p.d. across a $3\ \Omega$ resistance is $9\ \text{V}$ what current will flow?

Solution

$$V = IR \quad \text{From Ohm's law.}$$

$$I = \frac{V}{R} \quad \text{Rearranging.}$$

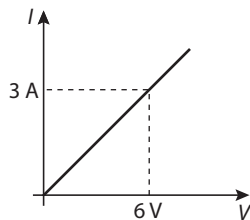
$$I = \frac{9}{3}\ \text{A} \\ = 3\ \text{A}$$

Graphical treatment

Ohmic conductor

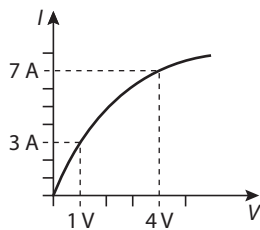
Since $V \propto I$ for an ohmic conductor, a graph of I against V will be a straight line.

In this example, resistance = $\frac{V}{I} = 2\ \Omega$



Non-ohmic conductors

Not all conductors obey Ohm's law. I - V graphs for these conductors will not be straight. A light bulb filament is an example of a non-ohmic conductor.



In this example, the resistance at the start is $\frac{1}{3}\ \Omega$ ($0.33\ \Omega$) and at the end it is $\frac{4}{7}\ \Omega$ ($0.57\ \Omega$).

The reason for this is that when the light bulb gets hot, the metal atoms vibrate more. This means that there are more collisions between the electrons and metal atoms, leading to an increase in resistance.

The water behind this dam has a high potential energy but because the hole is small only a small current of water can flow out. This is equivalent to a high resistance leading to a small current.



Figure 5.15 I - V for an ohmic conductor



Note: The resistance is found by taking the ratio $\frac{V}{I}$ – this is the same as $\frac{1}{\text{gradient}}$ for an ohmic conductor.

Figure 5.16 This I - V graph for a light bulb shows that the resistance has increased.



Why do we plot V on the x -axis? To produce an I - V graph, the current through the resistor is measured as the p.d. is changed. The variable that is changed is called the independent variable, the variable that changes is the dependent variable. The independent variable is generally plotted on the x -axis; that's why we plot I against V . So if an I - V graph gets steeper it means that the resistance is getting lower.

Exercises

- 1 If a p.d. of 9 V causes a current of 3 mA to flow through a wire, what is the resistance of the wire?
- 2 A current of 1 μA flows through a 300 k Ω resistor. What is the p.d. across the resistor?
- 3 If the p.d. across a 600 Ω resistor is 12 V, how much current flows?
- 4 Below is a table of the p.d. and current through a device called a thermistor.

p.d./V	Current/A
1.0	0.01
10	0.1
25	1.0

Calculate the resistance at different potential differences.

5.2 Electric circuits

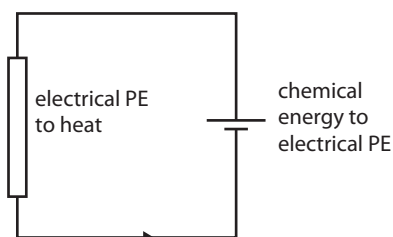
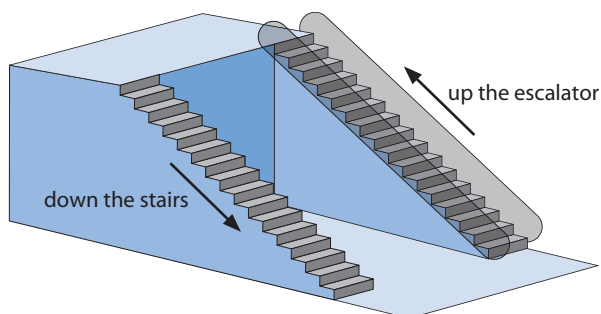
Assessment statements

- 5.2.1 Define electromotive force (emf).
- 5.2.2 Describe the concept of internal resistance.
- 5.2.3 Apply the equations for resistors in series and in parallel.
- 5.2.4 Draw circuit diagrams.
- 5.2.5 Describe the use of ideal ammeters and ideal voltmeters.
- 5.2.6 Describe a potential divider.
- 5.2.7 Explain the use of sensors in potential divider circuits.
- 5.2.8 Solve problems involving electric circuits.

The simple circuit – an energy view

We can now consider the energy changes that take place as current flows around the simple circuit.

Figure 5.17 The shopping centre analogy. The energy changes of the charges that flow round an electric circuit are similar to the energy changes as you walk around a multi-storey shopping centre. The escalator gives you potential energy as it takes you up. When you come down the stairs your potential energy gets less. This energy is given to the stairs; as your feet hit each step, the step gets hot.



The cell uses chemical energy to place the charges in a position of high potential. As the charges flow through the resistor, this potential energy is converted to heat, so the resistor gets hot. These changes in energy are defined by the following terms:

- Emf (\mathcal{E})

The emf of a cell is the amount of chemical energy converted to electrical energy, per unit charge. The unit is the volt (V).

- Potential difference (p.d. or V)

The p.d. across a resistance is the amount of electrical energy converted to heat, per unit charge. The unit is the volt (V).

Both of these quantities are energy per unit charge, but emf specifically applies to cells, batteries, generators and any other device that gives the charges potential energy. (One volt is the same as one joule per coulomb.)

Applying the law of conservation of energy

Since energy cannot be created or destroyed, the energy converted from chemical to electrical in the cell must be equal to the amount converted from electrical to heat in the resistor.

$$\text{From Ohm's law, } V = IR$$

Worked examples

- 1 If the emf of a battery is 9 V, how much energy is converted from chemical to electrical when 2 C of charge flow?
- 2 What is the p.d. across a resistor if 24 J of heat are produced when a current of 2 A flows through it for 10 s?

Solutions

- 1 emf = energy converted from chemical to electrical per unit charge.

$$\begin{aligned} \text{So energy converted} &= 2 \times 9 \text{ J} \\ &= 18 \text{ J} \end{aligned}$$

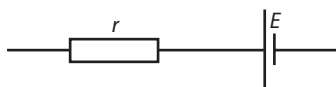
- 2 2 A for 10 s = $2 \times 10 \text{ C} = 20 \text{ C}$ of charge.

$$\text{If } 20 \text{ C of charge flows, then the energy per unit charge} = \frac{24}{20} \text{ V} = 1.2 \text{ V}$$

Internal resistance of cells

All cells are made of materials that have resistance. The resistance of the cell is called the *internal resistance*.

If a cell with internal resistance is connected to a resistor, current will flow from the cell. As current flows through the internal resistance, some energy is converted from electrical to heat inside the cell (so the cell gets hot). This means that there is less energy to be converted to heat in the resistor. The p.d. across the resistor is therefore less than the emf of the cell.



To look at the simulations on the PhET website, visit www.heinemann.co.uk/hotlinks, enter the express code 4426P and click on Weblink 5.4. 'Battery voltage' shows how the battery changes chemical energy to electrical potential energy by moving the charges. 'Battery-resistor current' shows how the electrical energy is converted to heat in the resistor. Note: these simulations show the movement of electrons, not conventional current.

• **Examiner's hint:** If the amount of charge = 2 C then energy converted = $2 \times \text{emf}$

• **Examiner's hint:** Current is charge flowing per unit time.

• **Examiner's hint:** From the definition, p.d. = energy converted from electrical to heat per unit charge.



The shopping centre analogy is an example of how we use models to visualize things that we can't see. It is much easier to understand the changes in height as you walk around the shops than the changes of energy as electrons flow around a circuit.

Figure 5.18 The symbol for a cell includes a resistor r next to the cell to show internal resistance.

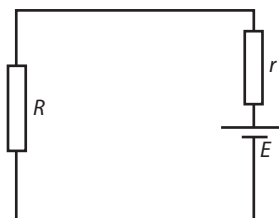
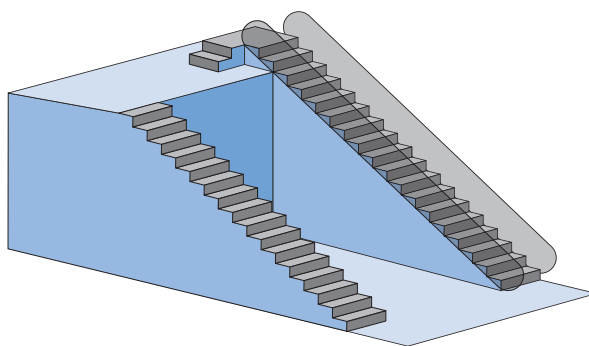
Figure 5.19 Internal resistance acts like a small step down.

Shopping centre analogy

In the shopping centre analogy you can see that the elevator still lifts you up to the same level but you must come down a few steps as soon as you step off it.

In the electrical circuit the emf is still the same but the charges lose some energy before they leave the battery.

If the resistance connected to the battery is very small, then the current will be large. This means that most of the electrical energy is converted to heat inside the battery so the battery gets very hot. This is why you shouldn't connect a wire between the ends of a battery. You can try this with the PhET circuit construction kit, but don't try it with a real battery.



Applying Ohm's law to the internal resistance, the p.d. across it will be Ir .

From the law of conservation of energy, when a certain charge flows, the amount of energy converted from chemical to electrical equals the amount converted from electrical to heat.

$$\mathcal{E} = IR + Ir$$

Rearranging this formula, we can get an equation for the current from the battery.

$$I = \frac{\mathcal{E}}{R + r}$$

Worked example

A battery of emf 9 V with an internal resistance 1 Ω is connected to a 2 Ω resistor, as shown in Figure 5.20.

How much current will flow?

Solution

$$I = \frac{\mathcal{E}}{R + r}$$

$$\begin{aligned} I &= \frac{9}{2 + 1} \text{ A} \\ &= 3 \text{ A} \end{aligned}$$

What is the p.d. across the 2 Ω resistor?

$$\begin{aligned} V &= IR \\ V &= 3 \times 2 \text{ V} \\ &= 6 \text{ V} \end{aligned}$$

● **Examiner's hint:** The current through the resistor is 3 A.

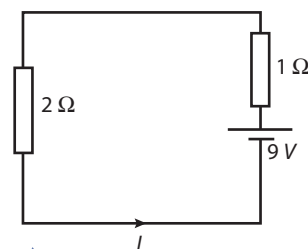


Figure 5.20 Always start by drawing a circuit showing the quantities you know and labelling the ones you want to find.

Exercises

- A current of 0.5 A flows when a battery of emf 6 V is connected to an 11 Ω resistor. What is the internal resistance of the battery?
- A 12 V battery with internal resistance 1 Ω is connected to a 23 Ω resistor. What is the p.d. across the 23 Ω resistor?

Electrical power

Electrical power is the rate at which energy is changed from one form to another.

Power delivered

In a perfect battery, the power is the amount of chemical energy converted to electrical energy per unit time.

If the emf of a battery is ε , then if a charge Q flows, the amount of energy converted from chemical to electrical is εQ .

If this charge flows in a time t then the power delivered $= \frac{\varepsilon Q}{t}$

But $\frac{Q}{t} =$ the current, I

So the power delivered $= \varepsilon I$

In a real battery, the actual power delivered will be a bit less, since there will be some power dissipated in the internal resistance.

Power dissipated

The power dissipated in the resistor is the amount of electrical energy converted to heat per unit time.

Consider a resistance R with a p.d. V across it. If a charge q flows in time t then the current, $I = \frac{q}{t}$

The p.d., V , is defined as the energy converted to heat per unit charge, so the energy converted to heat in this case $= Vq$

Power is the energy used per unit time, so $P = \frac{Vq}{t}$

but $\frac{q}{t} = I$, so $P = VI$

Worked examples

- 1 If a current of 2 A flows through a resistor that has a p.d. of 4 V across it, how much power is dissipated?
- 2 What power will be dissipated when a current of 4 A flows through a resistance of 55 Ω ?

Solutions

1 $P = VI$ where $V = 4 \text{ V}$ and $I = 2 \text{ A}$

$$\begin{aligned} P &= 4 \times 2 \text{ W} \\ &= 8 \text{ W} \end{aligned}$$

2 Using Ohm's law $V = IR$

$$\begin{aligned} &= 4 \times 55 \text{ V} \\ &= 220 \text{ V} \\ P &= VI \\ &= 220 \times 4 \text{ W} \\ &= 880 \text{ W} \end{aligned}$$

First we need to find out the p.d. across the resistor



Kilowatt-hours

The electrical energy used at home is measured in kilowatt-hours. This is the amount of energy used by a 1 kilowatt heater switched on for 1 hour. When the electricity bill comes, you have to pay for each kilowatt-hour of energy that you have used.

Since $1 \text{ W} = 1 \text{ J s}^{-1}$

1 kilowatt-hour $= 1000 \times 60 \times 60 \text{ J}$

1 kWh $= 3.6 \times 10^6 \text{ J}$

Power

$P = VI$

$P = I^2R$

$P = V^2/R$

**Alternative ways of writing $P = VI$**

In Example 2, we had to calculate the p.d. before finding the power. It would be convenient if we could solve this problem in one step. We can write alternative forms of the equation by substituting for I and V from Ohm's law.

We have shown that power $P = VI$

But from Ohm's law $V = IR$

If we substitute for V we get $P = IR \times I = I^2R$

We can also substitute for $I = \frac{V}{R}$

$$\text{Power} = V \times I = \frac{V \times V}{R} = \frac{V^2}{R}$$

Exercises

- 7 5 A flows through a $20\ \Omega$ resistor.
 - (a) How much electrical energy is converted to heat per second?
 - (b) If the current flows for one minute, how much energy is released?
- 8 If a battery has an internal resistance of $0.5\ \Omega$, how much power will be dissipated in the battery when 0.25 A flows?
- 9 A current of 0.5 A flows from a battery of emf 9V. If the power delivered is 4W, how much power is dissipated in the internal resistance?

Electric kettle (water boiler)

An electric kettle transfers the heat produced when current flows through a wire element to the water inside the kettle.

Worked example

A current of 3 A flows through an electric kettle connected to the 220 V mains. What is the power of the kettle and how long will it take to boil 1 litre of water?

Solution

The power of the kettle = $VI = 220 \times 3 = 660\ \text{W}$

To calculate energy needed to boil the water, we use the formula

$$\text{heat required} = \text{mass} \times \text{specific heat capacity} \times \text{temperature change.}$$

The specific heat capacity of water is $4180\ \text{J kg}^{-1}\text{ }^\circ\text{C}^{-1}$

The mass of 1 litre of water is 1 kg, so if we assume that the water was at room temperature, 20°C , then to raise it to 100°C the energy required is:

$$1 \times 4180 \times 80 = 334\ 400\ \text{J}$$

$$\begin{aligned} \text{power} = \text{energy/time, so the time taken} &= \frac{\text{energy}}{\text{power}} \\ &= \frac{334\ 400}{660} \end{aligned}$$

$$\text{Time} = 506.67\ \text{s}$$

$$= 8\ \text{minutes } 27\ \text{seconds}$$

The light bulb

If the power dissipated in a wire is large, then a lot of heat is produced per second. When heat is added quickly, the wire doesn't have time to lose this heat to the surroundings. The result is that the temperature of the wire increases, and if the

temperature is high enough, the wire will begin to glow, giving out light. Only about 10% of the energy dissipated in a light bulb is converted to light - the rest is heat.

The electric motor

A motor converts electrical energy to mechanical energy; this could be in the form of potential energy, if something is lifted by the motor, or kinetic energy, if the motor is accelerating something like a car.

Worked example

An electric motor is used to lift 10 kg through 3 m in 5 seconds. If the p.d. across the motor is 12 V, how much current flows (assuming no energy is lost)?

Solution

$$\begin{aligned}\text{Work done by the motor} &= mgh \\ &= 10 \times 10 \times 3 \text{ J} \\ &= 300 \text{ J}\end{aligned}$$

$$\begin{aligned}\text{Power} &= \frac{\text{work done}}{\text{time}} \\ &= \frac{300}{5} \text{ W} \\ &= 60 \text{ W}\end{aligned}$$

$$\begin{aligned}\text{Electrical power, } P &= IV \text{ so } I = P/V \\ &= \frac{60}{12} \text{ A} \\ &= 5 \text{ A}\end{aligned}$$

Exercises

- 10** An electric car of mass 1000 kg uses twenty-five 12 V batteries connected together to create a p.d. of 300 V. The car accelerates from rest to a speed of 30 m s^{-1} in 12 seconds.
- What is the final kinetic energy of the car?
 - What is the power of the car?
 - How much electrical current flows from the battery? What assumptions have you made in calculating **a**) to **c**)?
- 11** A light bulb for use with the 220 V mains is rated at 100 W.
- What current will flow through the bulb?
 - If the bulb converts 20% of the energy to light, how much light energy is produced per second?
- 12** A 1 kW electric heater is connected to the 220 V mains and left on for 5 hours.
- How much current will flow through the heater?
 - How much energy will the heater release?

Combinations of components

In practical situations, resistors and cells are often joined together in combinations e.g. fairy lights, flashlight batteries.

There are many ways of connecting a number of components – we will consider two simple arrangements, series and parallel.

i Fluorescent tubes are much more efficient than incandescent light bulbs, converting most of the electrical energy to light.

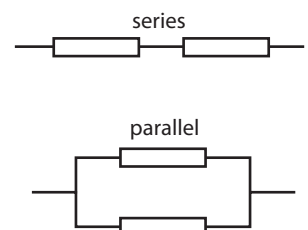
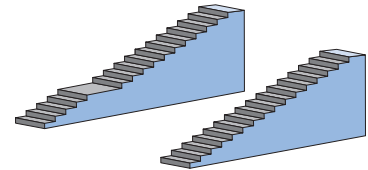
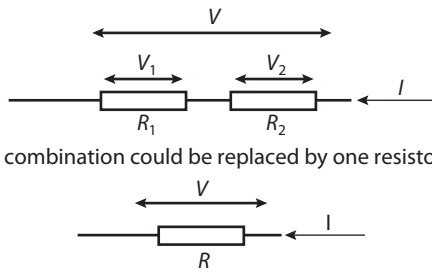


Figure 5.21 Two simple combinations of resistors.

Figure 5.22 Two resistors in series are similar to two flights of stairs.



The combination could be replaced by one resistor.

Applying the law of conservation of energy, the p.d. across R_1 plus the p.d. across R_2 must be equal to the p.d. across the combination.

$$V_1 + V_2 = V$$

Applying Ohm's law to each resistor: $IR_1 + IR_2 = IR$

Dividing by I : $R_1 + R_2 = R$



These coloured lights are connected in series – if you take one out they all go out.

Worked example

What is the total resistance of a 4Ω and an 8Ω resistor in series?

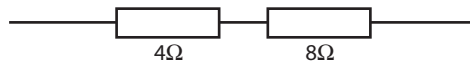


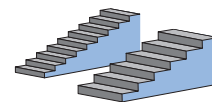
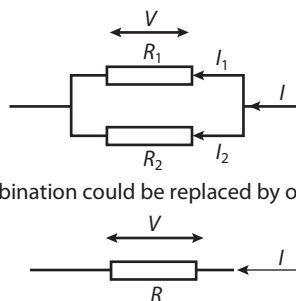
Figure 5.23

Solution

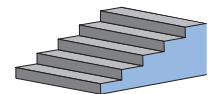
$$\begin{aligned} \text{Total resistance} &= R_1 + R_2 \\ &= 4 + 8 \Omega \\ &= 12 \Omega \end{aligned}$$

Resistors in parallel

In a parallel circuit the current splits in two.



The combination could be replaced by one resistor.



Applying the law of conservation of charge, we know that the current going into a junction must equal the current coming out.

$$I = I_1 + I_2$$

Applying Ohm's law to each resistor gives: $\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2}$

Dividing by V : $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$

Worked examples

- 1 What is the total resistance of a $4\ \Omega$ and an $8\ \Omega$ resistor in parallel?
- 2 What is the total resistance of two $8\ \Omega$ resistors in parallel?

Solutions

1 Using $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$

$$\frac{1}{R} = \frac{1}{4} + \frac{1}{8}$$

$$= \frac{2+1}{8} = \frac{3}{8}$$

$$\text{so } R = \frac{8}{3}\ \Omega$$

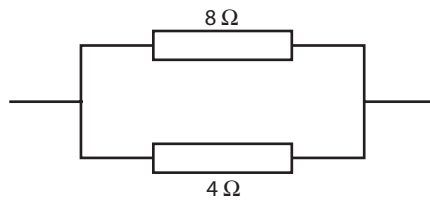


Figure 5.25

2 Using $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$

$$\frac{1}{R} = \frac{1}{8} + \frac{1}{8} = \frac{2}{8}$$

$$R = \frac{8}{2}\ \Omega$$

$$= 4\ \Omega$$

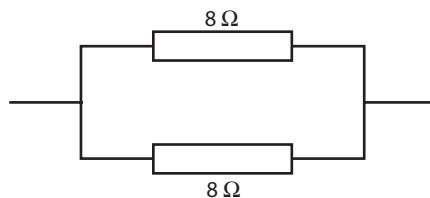


Figure 5.26 The total resistance of two equal resistors in parallel is half the resistance of one of them.

Multiple combinations

When there are many resistors, the total is found by splitting the circuit into small units of parallel and series resistors; for example in the circuit shown in Figure 5.27.

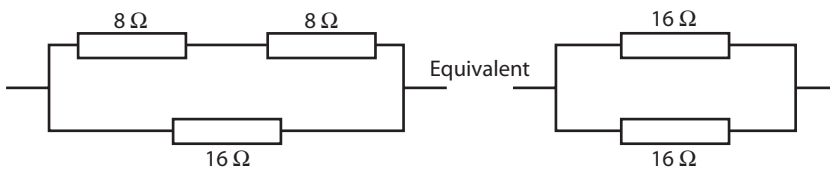


Figure 5.27 Equivalent circuits.

The two $8\ \Omega$ resistors at the top are equivalent to one $16\ \Omega$ resistor.

This gives two $16\ \Omega$ resistors in parallel, a total of $8\ \Omega$.

Exercises

Calculate the total resistance for the circuits in Figure 5.28.

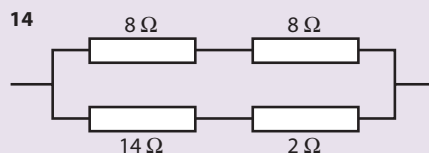
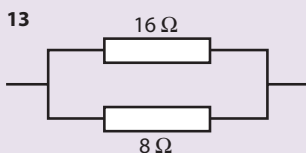
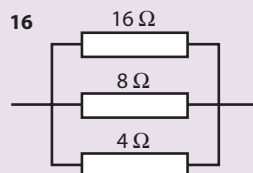
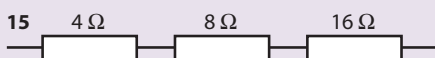


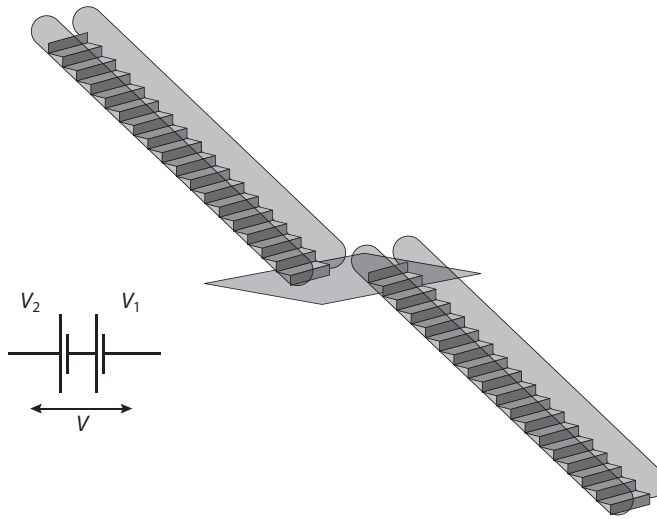
Figure 5.28



Cells in series

Cells are very often added in series to produce a larger p.d.

Figure 5.29 Cells in series are similar to two flights of escalators.



The p.d. of cells in series simply add

$$V = V_1 + V_2$$

Worked example

Two 12 V batteries are connected in series to a 10 Ω resistor. If each battery has an internal resistance of 1 Ω , how much current will flow?

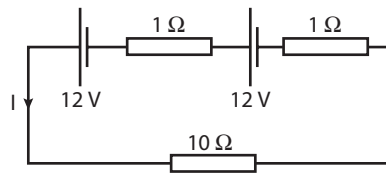
Solution

The total p.d. for two batteries in series = 12 + 12 = 24 V

The three resistors are in series (it looks like the 10 Ω is in parallel with the 1 Ω resistors, but they are in series because the same current flows through them all).



Figure 5.30 The circuit diagram.



Total resistance = 1 + 1 + 10 = 12 Ω .

Applying Ohm's law,

$$I = \frac{V}{R}$$

$$= \frac{24}{12} \text{ A}$$

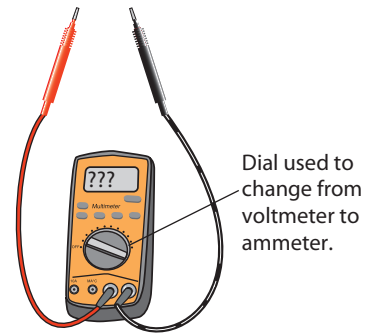
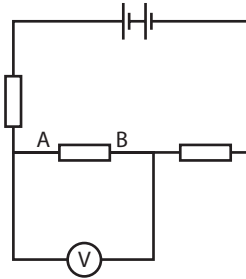
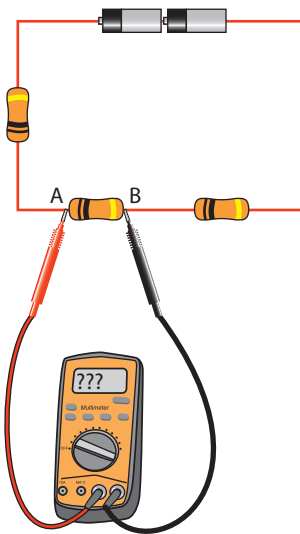
$$= 2 \text{ A}$$

Electrical measurement

Measurement of potential difference

The p.d. can be measured using a voltmeter. There are two main types of voltmeter, digital and analogue.

The p.d. is the difference in potential between two points. To measure the p.d. between A and B, one lead of the voltmeter must be connected to A, the other to B.



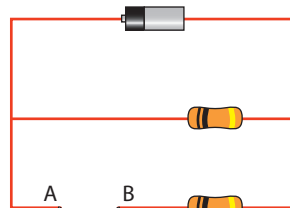
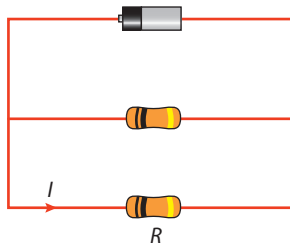
Dial used to change from voltmeter to ammeter.

Figure 5.31 A multimeter is a common instrument that can measure both p.d. and current. It can also measure resistance.

An ideal voltmeter has infinitely high resistance so that it does not take any current from the circuit.

Measurement of current

To measure the current flowing through a resistor, the ammeter must be connected so that the same current will flow through the ammeter as flows through the resistor. This means disconnecting one of the wires and connecting the ammeter.



circuit broken here so that ammeter can be connected

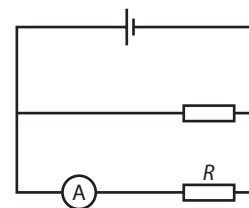


Figure 5.33 The ammeter is connected to measure the current through R .

An ideal ammeter has zero resistance so that it doesn't change the current in the circuit.

Worked example

Calculate the current and potential difference measured by the meters in the circuit in Figure 5.34. Assume the battery has no internal resistance and that the meters are ideal.

Solution

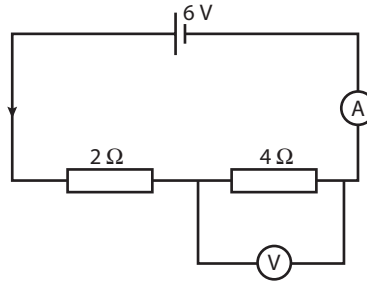


Figure 5.34

$$\begin{aligned} \text{Total resistance} &= 2 + 4 \Omega \\ &= 6 \Omega \end{aligned}$$

The two resistors are in series.

$$\begin{aligned} I &= \frac{V}{R} && \text{Applying Ohm's law, to find the current.} \\ &= \frac{6}{6} \text{ A} \\ &= 1 \text{ A} \end{aligned}$$

So ammeter reading = 1A

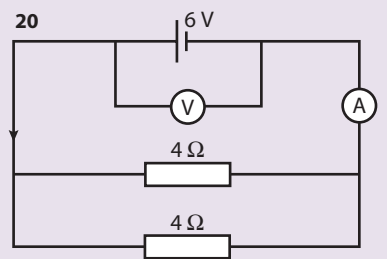
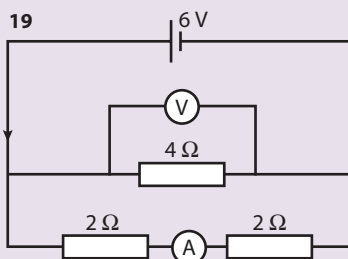
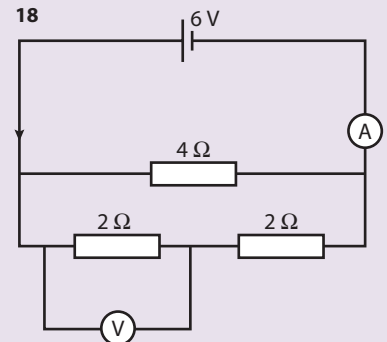
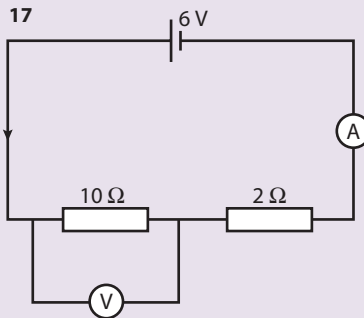
$$\begin{aligned} V &= IR && \text{Applying Ohm's law to the } 4 \Omega \text{ resistor.} \\ &= 1 \times 4 \text{ V} \\ &= 4 \text{ V} \end{aligned}$$

So the voltmeter reads 4V.

Exercises

Find the ammeter and voltmeter readings in the circuits in Figure 5.35. All meters are ideal and the batteries have no internal resistance. (You can build them in the PhET 'circuit construction kit' to see whether your answers agree.)

Figure 5.35



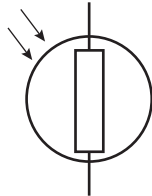
Electrical sensors

An electrical sensor is a device whose electrical properties change with changing physical conditions.

Thermistor

A thermistor is made from a semiconducting material whose resistance decreases as temperature increases. As the thermistor gets hotter, more charge carriers are released, so the current can flow more easily.

Light sensor (LDR)



A light sensor or light-dependent resistor (LDR) is also a semiconducting device, but, unlike the thermistor, it is light that releases more charge carriers, resulting in a lower resistance.

Strain gauge

A strain gauge is a thin metal wire. If it is stretched, its length increases and its cross-sectional area gets smaller. This results in an increase in resistance.

Use of sensors

The resistance of all three sensors varies with some physical property. However, it would be much more useful if the devices gave a changing p.d. rather than a changing resistance. To convert the changing resistance to a changing p.d., we use a potential divider.

The potential divider

The battery creates a p.d. across the resistors equal to V_{in}

From Ohm's law we know that the current in the circuit $I = \frac{V}{R}$

Since the total resistance is $R_1 + R_2$

The current $I = \frac{V_{in}}{R_1 + R_2}$ Equation (1)

The p.d. across $R_2 = V_{out}$

Applying Ohm's law to R_2 gives

$$V_{out} = IR_2$$

Substituting from equation (1) gives

$$V_{out} = V_{in} \frac{R_2}{R_1 + R_2}$$

This is the potential divider formula.

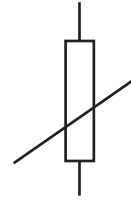
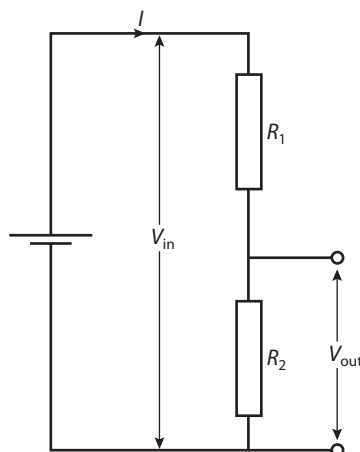


Figure 5.36 A thermistor.

Figure 5.37 An LDR.

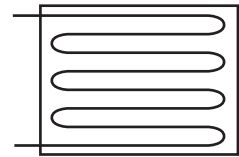


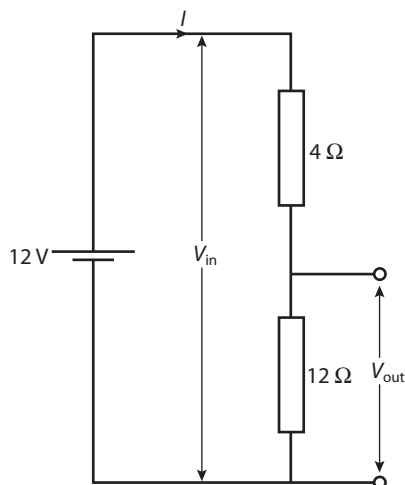
Figure 5.38 A strain gauge.

Figure 5.39 A potential divider circuit consists of two series resistors.

Worked example

Calculate the output voltage for the potential divider in Figure 5.40.

Figure 5.40

**Solution**

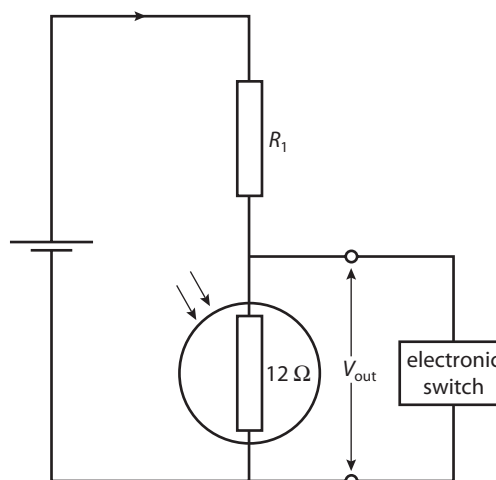
Using the potential divider formula

$$V_{\text{out}} = V_{\text{in}} \frac{R_2}{R_1 + R_2}$$

$$V_{\text{out}} = 12 \frac{12}{4 + 12} = 9\text{V}$$

Using the potential divider with sensors**Automatic light switch**

Figure 5.41 An LDR and a potential divider can be used to operate an automatic light switch.



When light stops shining on the LDR, its resistance increases, resulting in an increase in V_{out} . The increase in V_{out} in turn activates the electronic switch that puts on the lights. The electronic switch needs a minimum p.d. to activate it, so it doesn't switch on the lights until V_{out} is big enough.

The important point here is that the electronic switch needs a p.d. to activate it, hence the need for a potential divider.

Worked example

The battery in Figure 5.41 has an emf of 12 V and no internal resistance. The p.d. required to activate the switch is 5 V. Find the value of R_1 that will cause the lights to turn on when the resistance of the LDR rises to 200 k Ω .

Solution

$$V_{\text{in}} = 12 \text{ V}$$

$$R_2 = 200 \text{ k}\Omega$$

$$V_{\text{out}} = 5 \text{ V}$$

Rearranging the potential divider equation $V_{\text{out}} = V_{\text{in}} \frac{R_2}{R_1 + R_2}$

$$\begin{aligned} \text{gives } R_1 &= R_2 \frac{V_{\text{in}} - V_{\text{out}}}{V_{\text{out}}} \\ &= 200 \text{ k}\Omega \times \frac{(12 - 5)}{5} \\ &= 280 \text{ k}\Omega \end{aligned}$$

The fire alarm

When the thermistor gets hot, its resistance decreases, resulting in an increased current through R_2 , which in turn leads to an increase in V_{out} . The increase in V_{out} activates an electronic switch that rings a bell.

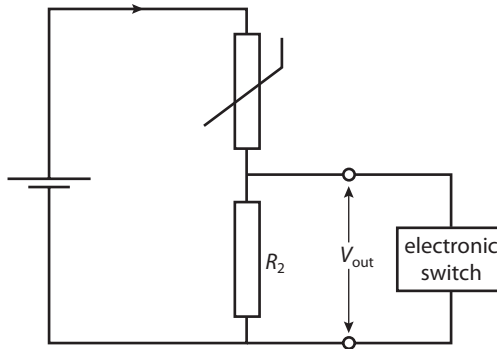


Figure 5.42 The fire alarm bell must ring if the thermistor gets hot.

Exercises

- 21** Assume that the circuit in Figure 5.42 has a 12 V battery and a switch that activates when the p.d. is 5 V. Calculate the value of R_2 , if the bell rings when the resistance of the thermistor drops to 1 k Ω ?

Using a strain gauge

A strain gauge can be used to detect whether parts of a building are stretching. For example, a strain gauge stuck to the underside of a bridge will be stretched if the bridge bends when a heavy truck crosses it. If the strain gauge is connected in a potential divider circuit, the V_{out} can be used to measure how much the bridge stretches.

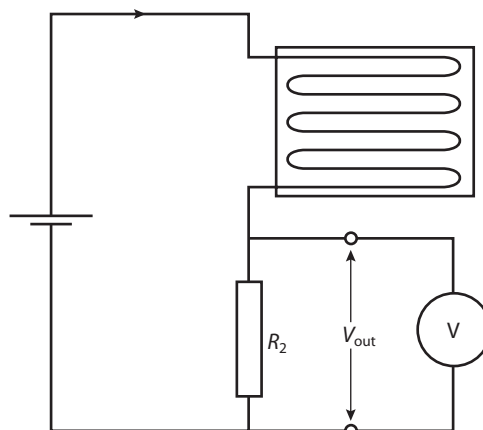


Figure 5.43 Strain gauge circuit.

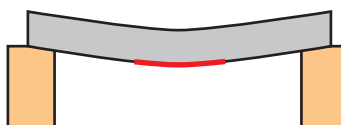
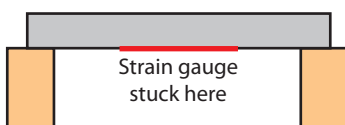
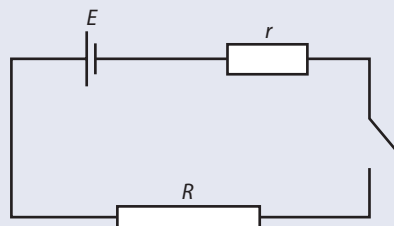


Figure 5.44 As the bridge bends, the strain gauge gets longer.

Practice questions

1 This question is about electrical energy and associated phenomena.

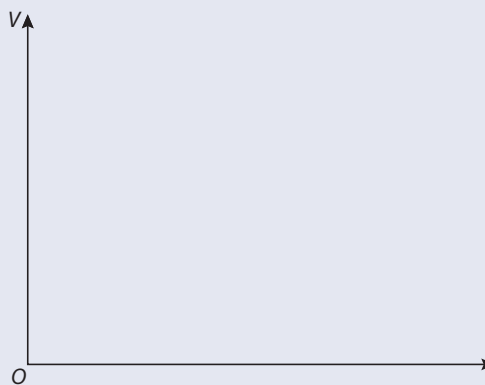
A cell of emf E and internal resistance r is connected in series with a resistor R , as shown below. The cell supplies $8.1 \times 10^3 \text{ J}$ of energy when $5.8 \times 10^3 \text{ C}$ of charge moves completely round the circuit. The current in the circuit is constant.



- (i) Calculate the emf E of the cell. (2)
- (ii) The resistor R has resistance 6.0Ω . The potential difference between its terminals is 1.2 V . Determine the internal resistance r of the cell. (3)
- (iii) Calculate the total energy transfer in the resistor R . (2)
- (iv) Describe, in terms of a simple model of electrical conduction, the mechanism by which the energy transfer in the resistor R takes place. (5)

2 This question is about a filament lamp.

- (a) On the axes below, draw a sketch graph to show the variation with potential difference V of the current I in a typical filament lamp (the I - V characteristic). (Note: this is a sketch graph; you do not need to add any values to the axes). (1)



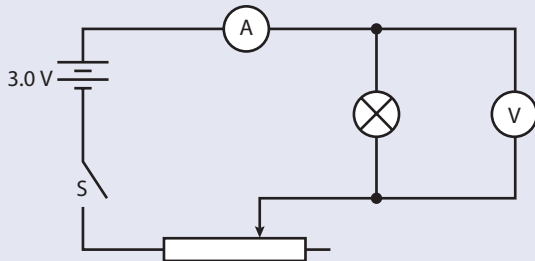
- (b) (i) Explain how the resistance of the filament is determined from the graph. (1)
- (ii) Explain whether the graph you have sketched indicates ohmic behaviour or non-ohmic behaviour. (1)

A filament lamp operates at maximum brightness when connected to a 6.0 V supply. At maximum brightness, the current in the filament is 120 mA .

- (c) (i) Calculate the resistance of the filament when it is operating at maximum brightness. (1)
- (ii) You have available a 24 V supply and a collection of resistors of a suitable power rating and with different values of resistance. Calculate the resistance of the resistor that is required to be connected in series with the supply such that the voltage across the filament lamp will be 6.0 V . (2)

3 This question is about electric circuits.

Susan sets up the circuit below in order to measure the current–voltage (I – V) characteristic of a small filament lamp.



The supply is a battery that has an emf of 3.0 V and the ammeter and voltmeter are considered to be ideal. The lamp is labelled by the manufacturer as '3 volts, 0.6 watts'.

- (a) (i) Explain what information this labelling provides about the normal operation of the lamp. (2)
- (ii) Calculate the current in the filament of the lamp when it is operating at normal brightness. (2)

Susan sets the variable resistor to its maximum value of resistance. She then closes the switch S and records the following readings.

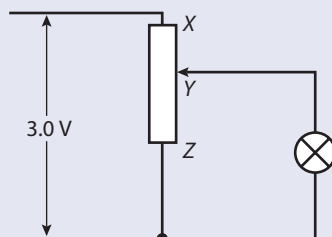
Ammeter reading = 0.18 A Voltmeter reading = 0.60 V

She then sets the variable resistor to its zero value of resistance and records the following readings.

Ammeter reading = 0.20 A Voltmeter reading = 2.6 V

- (b) (i) Explain why, by changing the value of the resistance of the variable resistance, the potential difference across the lamp cannot be reduced to zero or be increased to 3.0 V. (2)
- (ii) Determine the internal resistance of the battery. (3)
- (c) Calculate the resistance of the filament when the reading on the voltmeter is
- (i) 0.60 V. (1)
- (ii) 2.6 V. (1)
- (d) Explain why there is a difference between your answers to (c) (i) and (c) (ii). (2)
- (e) Using the axes as in question 2, draw a sketch-graph of the I – V characteristic of the filament of the lamp. (Note: this is a sketch-graph; you do not need to add any values to the axis.) (1)

The diagram below shows an alternative circuit for varying the potential difference across the lamp.

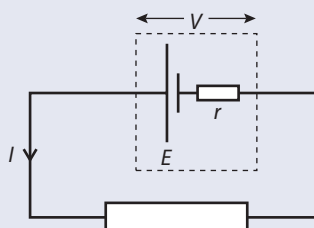


The potential divider XZ has a potential of 3.0 V across it. When the contact is at the position Y, the resistance of XY equals the resistance of YZ which equals 12 Ω . The resistance of the lamp is 4 Ω .

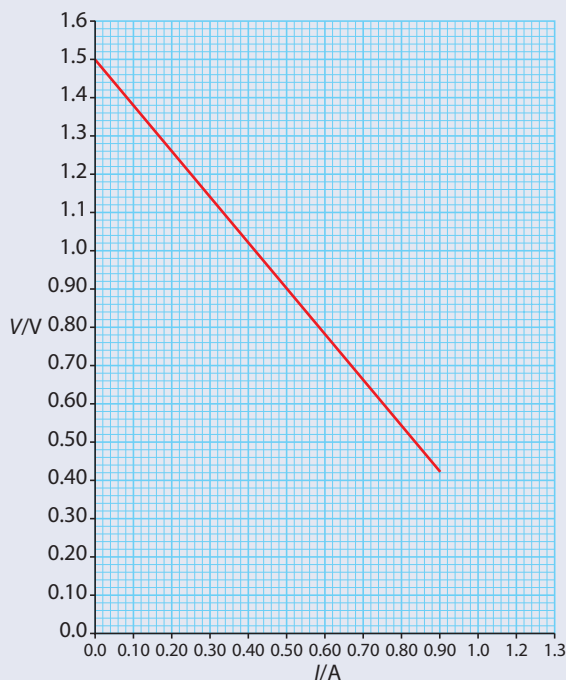
- (f) Calculate the potential difference across the lamp. (4)

4 This question is about emf and internal resistance.

A dry cell has an emf E and internal resistance r and is connected to an external circuit. There is a current I in the circuit when the potential difference across the terminals of the cell is V .



- (a) State expressions, in terms of E , V , r and I where appropriate, for
 - (i) the total power supplied by the cell. (1)
 - (ii) the power dissipated in the cell. (1)
 - (iii) the power dissipated in the external circuit. (1)
 - (b) Use your answers to (a) to derive a relationship between V , E , I and r . (2)
- The graph below shows the variation of V with I for the dry cell.



- (c) Draw the circuit that could be used to obtain the data from which the graph was plotted. (3)
- (d) Use the graph, explaining your answers, to
 - (i) determine the emf E of the cell. (2)
 - (ii) determine the current in the external circuit when the resistance R of the external circuit is very small. (2)
 - (iii) deduce that the internal resistance r of the cell is about 1.2Ω . (1)
- (e) The maximum power dissipated in the external circuit occurs when the resistance of the external circuit has the same value as the internal resistance of the cell. Calculate the maximum power dissipation in the external circuit. (3)