6 Fields and forces

6.1 Gravitational force and field

Assessment statements

6.1.1 State Newton's universal law of gravitation.

6.1.2 Define *gravitational field strength*.

6.1.3 Determine the gravitational field due to one or more point masses.

6.1.4 Derive an expression for gravitational field strength at the surface of a planet, assuming that all its mass is concentrated at its centre.

6.1.5 Solve problems involving gravitational forces and fields.

Gravitational force and field

We have all seen how an object falls to the ground when released. Newton was certainly not the first person to realize that an apple falls to the ground when dropped from a tree. However, he did recognize that the force that pulls the apple to the ground is the same as the force that holds the Earth in its orbit around the Sun; this was not obvious - after all, the apple moves in a straight line and the Earth moves in a circle. In this chapter we will see how these forces are connected.

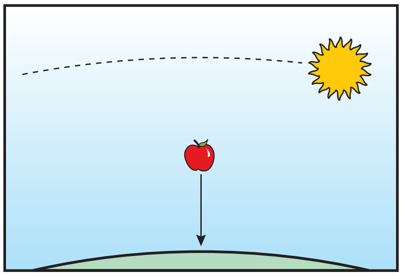


Figure 6.1 The apple drops and the Sun seems to move in a circle, but it is gravity that makes both things happen.

Newton's universal law of gravitation

Newton extended his ideas further to say that every single particle of mass in the universe exerts a force on every other particle of mass. In other words, everything in the universe is attracted to everything else. So there is a force between the end of your nose and a lump of rock on the Moon.

Was it reasonable for Newton to think that his law applied to the whole universe?

The modern equivalent of the apparatus used by Cavendish to measure G in 1798.

Newton's universal law of gravitation states that:

> every single point mass attracts every other point mass with a force that is directly proportional to the product of their masses and inversely proportional to the square of their separation.

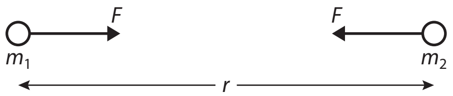


Figure 6.2 The gravitational force between two point masses.

If two point masses with mass m_1 and m_2 are separated by a distance r then the force, F, experienced by each will be given by:

$$F \propto \frac{m_1 m_2}{r^2}$$

The constant of proportionality is the universal gravitational constant G.

$$G = 6.6742 \times 10^{-11}\,\text{m}^3\,\text{kg}^{-1}\,\text{s}^{-2}$$

Therefore the equation is $$F = G\frac{m_1 m_2}{r^2}$$

Spheres of mass

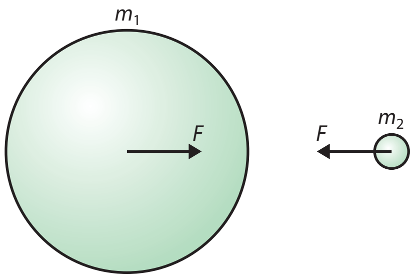


Figure 6.3 Forces between two spheres. Even though these bodies don't have the same mass, the force on them is the same size. This is due to Newton's third law – if mass m_1 exerts a force on mass m_2 then m_2 will exert an equal and opposite force on m_1.

By working out the total force between every particle of one sphere and every particle of another, Newton deduced that spheres of mass follow the same law, where the separation is the separation between their centres. Every object has a centre of mass where the gravity can be taken to act. In regularly-shaped bodies, this is the centre of the object.

How fast does the apple drop?

If we apply Newton's universal law to the apple on the surface of the Earth, we find that it will experience a force given by

$$F = G\frac{m_1 m_2}{r^2}$$

where:

m_1 = mass of the Earth = 5.97×10^{24} kg

m_2 = mass of the apple = 250 g

r = radius of the Earth = 6378 km (at the equator)

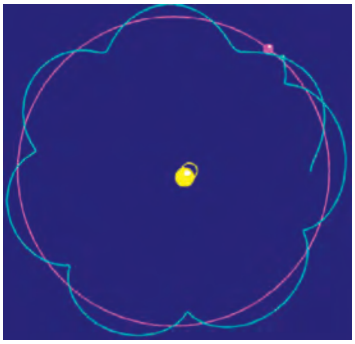
So $F = 2.43$ N

From Newton's 2nd law we know that $F = ma$.

So the acceleration (a) of the apple $= \frac{2.43}{0.25}\,\text{m}\,\text{s}^{-2}$

$a = 9.79\,\text{m}\,\text{s}^{-2}$

This is very close to the average value for the acceleration of free fall on the Earth's surface. It is not exactly the same, since 9.82 m s^{-2} is an average for the whole Earth, the radius of the Earth being maximum at the equator.



Exercise

1 The mass of the Moon is 7.35×10^{22} kg and the radius 1.74×10^{3} km.
What is the acceleration due to gravity on the Moon's surface?

How often does the Earth go around the Sun?

To build your own solar system with the 'solar system' simulation from PhET, visit www.heinemann.co.uk/hotlinks, enter the express code 4426P and click on Weblink 6.1.

Applying Newton's universal law, we find that the force experienced by the Earth is given by:

$$F = G\frac{m_1 m_2}{r^2}$$

where

m_1 = mass of the Sun = 1.99×10^{30} kg

m_2 = mass of the Earth = 5.97×10^{24} kg

r = distance between the Sun and Earth = 1.49×10^{11} m

So $F = 3.56 \times 10^{22}$ N

The planets orbit the Sun.

We know that the Earth travels in an elliptical orbit around the Sun, but we can take this to be a circular orbit for the purposes of this calculation. From our knowledge of circular motion we know that the force acting on the Earth towards the centre of the circle is the centripetal force given by the equation $F = \frac{mv^2}{r}$

Here the law is used to make predictions that can be tested by experiment.

So the velocity $v = \sqrt{\frac{Fr}{m}}$

$= 29\,846$ m s^{-1}

The circumference of the orbit $= 2\pi r = 9.38 \times 10^{11}$ m

Time taken for 1 orbit $= \frac{9.38 \times 10^{11}}{29\,846}$

$= 3.14 \times 10^{7}$ s

This is equal to 1 year.

This agrees with observation. Newton's law has therefore predicted two correct results.

Gravitational field

The fact that both the apple and the Earth experience a force without being in contact makes gravity a bit different from the other forces we have come across. To model this situation, we introduce the idea of a *field*. A field is simply a region of space where something is to be found. A potato field, for example, is a region where you find potatoes. A gravitational field is a region where you find gravity. More precisely, gravitational field is defined as a region of space where a mass experiences a force because of its mass.

So there is a gravitational field in your classroom since masses experience a force in it.

Field strength on the Earth's surface:
Substituting M = mass of the Earth $= 5.97 \times 10^{24}$ kg
r = radius of the Earth = 6367 km
gives $g = Gm_1M/r^2 = 9.82\ \mathrm{N\,kg^{-1}}$
This is the same as the acceleration due to gravity, which is what you might expect, since Newton's 2nd law says $a = F/m$.

Gravitational field strength (*g*)

This gives a measure of how much force a body will experience in the field. It is defined as the force per unit mass experienced by a small test mass placed in the field.

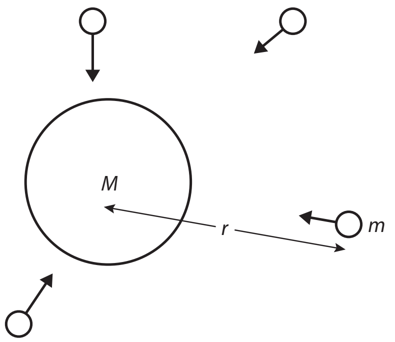
So if a test mass, m, experiences a force F at some point in space, then the field strength, g, at that point is given by $g = \dfrac{F}{m}$.

g is measured in $\mathrm{N\,kg^{-1}}$, and is a vector quantity.

Note: The reason a small test mass is used is because a big mass might change the field that you are trying to measure.

Gravitational field around a spherical object

Figure 6.4 The region surrounding M is a gravitational field since all the test masses experience a force.



The force experienced by the mass, m is given by;

$$F = G\frac{Mm}{r^2}$$

So the field strength at this point in space, $g = \dfrac{F}{m}$

So $$g = G\frac{M}{r^2}$$

Exercises

2 The mass of Jupiter is 1.89×10^{27} kg and the radius 71 492 km.
What is the gravitational field strength on the surface of Jupiter?

3 What is the gravitational field strength at a distance of 1000 km from the surface of the Earth?

Field lines

Field lines are drawn in the direction that a mass would accelerate if placed in the field – they are used to help us visualize the field.

The field lines for a spherical mass are shown in Figure 6.5.

The arrows give the direction of the field.

The field strength (g) is given by the density of the lines.

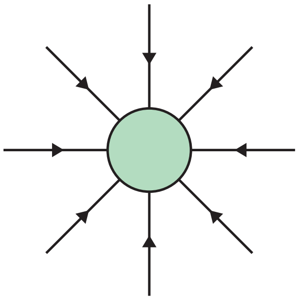


Figure 6.5 Field lines for a sphere of mass.

Gravitational field close to the Earth

When we are doing experiments close to the Earth, in the classroom for example, we assume that the gravitational field is uniform. This means that wherever you put a mass in the classroom it is always pulled downwards with the same force. We say that the field is *uniform*.

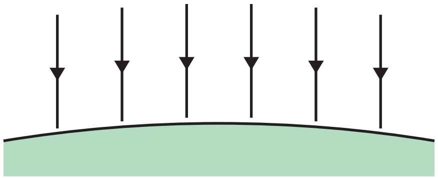


Figure 6.6 Regularly spaced parallel field lines imply that the field is uniform.

Addition of field

Since field strength is a vector, when we add field strengths caused by several bodies, we must remember to add them vectorially.

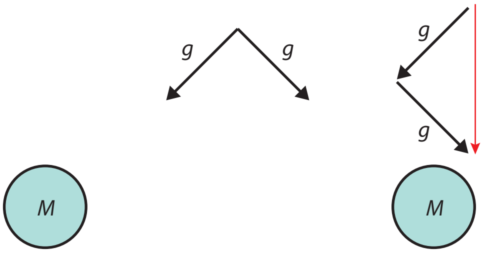


Figure 6.7 Vector addition of field strength.

In this example, the angle between the vectors is 90°. This means that we can use Pythagoras to find the resultant.

$$g = \sqrt{g_1^2 + g_2^2}$$

Worked example

Calculate the gravitational field strength at points A and B in Figure 6.8.

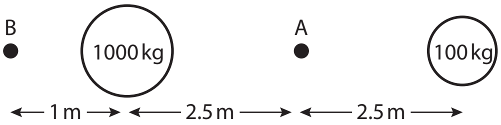


Figure 6.8

Solution

The gravitational field strength at A is equal to the sum of the field due to the two masses.

Field strength due to large mass $= G \times 1000/2.5^2 = 1.07 \times 10^{-8}\ \text{N kg}^{-1}$

Field strength due to small mass $= G \times 100/2.5^2 = 1.07 \times 10^{-9}\ \text{N kg}^{-1}$

Field strength $= 1.07 \times 10^{-8} - 1.07 \times 10^{-9}$
$= 9.63 \times 10^{-9}\ \text{N kg}^{-1}$

● **Examiner's hint:** Since field strength g is a vector, the resultant field strength equals the vector sum.

Exercises

4 Calculate the gravitational field strength at point B.

5 Calculate the gravitational field strength at A if the big mass were changed for a 100 kg mass.

6.2 Gravitational potential

Assessment statements

9.2.1 Define gravitational potential and gravitational potential energy.
9.2.2 State and apply the expression for gravitational potential due to a point mass.
9.2.3 State and apply the formula relating gravitational field strength to gravitational potential gradient.
9.2.4 Determine the potential due to one or more point masses.
9.2.5 Describe and sketch the pattern of equipotential surfaces due to one and two point masses.
9.2.6 State the relation between equipotential surfaces and gravitational field lines.

Gravitational potential in a uniform field

As you lift a mass m from the ground, you do work. This increases the PE of the object. As PE $= mgh$, we know the PE gained by the mass depends partly on the size of the mass (m) and partly on where it is (gh). The 'where it is' part is called the 'gravitational potential (V)'. This is a useful quantity because, if we know it, we can calculate how much PE a given mass would have if placed there.

Contours
Close to the Earth, lines of equipotential join points that are the same height above the ground. These are the same as contours on a map.

Rearranging the equation for PE we get $gh = \frac{PE}{m}$ so potential is the PE per unit mass and has units $J\,kg^{-1}$.

In the simple example of masses in a room, the potential is proportional to height, so a mass m placed at the same height in the room will have the same PE. By joining all positions of the same potential we get a line of equal potential, and these are useful for visualizing the changes in PE as an object moves around the room.

Worked examples

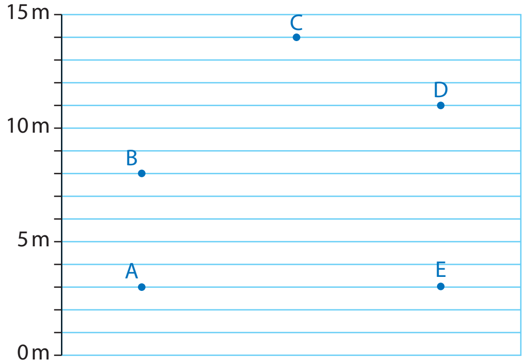


Figure 6.9

Referring to Figure 6.9.

1 What is the potential at A?

2 If a body is moved from A to B what is the change in potential?

3 How much work is done moving a 2 kg mass from A to B?

Solutions

1 $V_A = gh$ so potential at A $= 10 \times 3 = 30\,J\,kg^{-1}$

2 $V_A = 30\,J\,kg^{-1}$
$V_B = 80\,J\,kg^{-1}$
Change in potential $= 80 - 30 = 50\,J\,kg^{-1}$

3 The work done moving from A to B is equal to the change in potential $\times$ mass $= 50 \times 2 = 100\,J$

Exercises

6 What is the difference in potential between C and D?

7 How much work would be done moving a 3 kg mass from D to C?

8 What is the PE of a 3 kg mass placed at B?

9 What is the potential difference between A and E?

10 How much work would be done taking a 2 kg mass from A to E?

Equipotentials and field lines

If we draw the field lines in our 15 m room they will look like Figure 6.10. The field is uniform so they are parallel and equally spaced. If you were to move upwards along a field line (A–B), you would have to do work and therefore your PE would increase. On the other hand, if you travelled perpendicular to the field lines (A–E), no work would be done, in which case you must be travelling along a line of equipotential. For this reason, field lines and equipotentials are perpendicular.

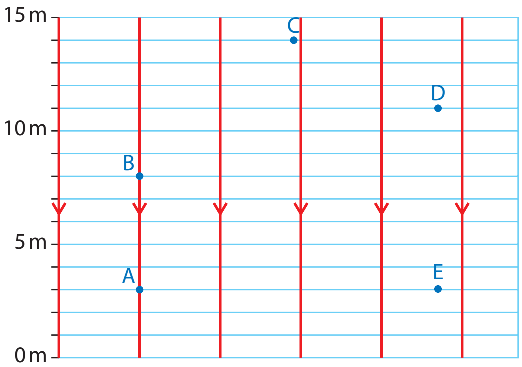


Figure 6.10 Equipotentials and field lines.

The amount of work done as you move up is equal to the change in potential × mass

Work = ΔVm

But the work done is also equal to

force × distance = $mg\Delta h$

So $\Delta Vm = mg\Delta h$

Rearranging gives

$$\frac{\Delta V}{\Delta h} = g$$

or the potential gradient = the field strength.

So lines of equipotential that are close together represent a strong field.

This is similar to the situation with contours as shown in Figure 6.11. Contours that are close together mean that the gradient is steep and where the gradient is steep, there will be a large force pulling you down the slope.

Figure 6.11 Close contours mean a steep mountain.

In this section we have been dealing with the simplified situation. Firstly we have only been dealing with bodies close to the Earth, where the field is uniform, and secondly we have been assuming that the ground is a position of zero potential. A more general situation would be to consider large distances away from a sphere of mass. This is rather more difficult but the principle is the same, as are the relationships between field lines and equipotentials.

Gravitational potential due to a massive sphere

The gravitational potential at point P is defined as:

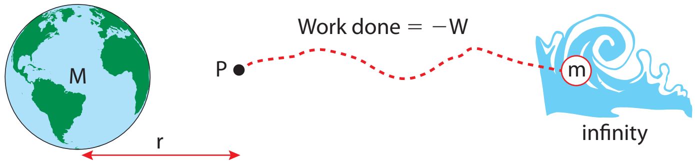
The work done per unit mass taking a small test mass from a position of zero potential to the point P.

In the previous example we took the Earth's surface to be zero but a better choice would be somewhere where the mass isn't affected by the field at all. Since $g = \frac{GM}{r^2}$ the only place completely out of the field is at an infinite distance from the mass – so let's start there.

Infinity
We can't really take a mass from infinity and bring it to the point in question, but we can calculate how much work would be required if we did. Is it OK to calculate something we can never do?

Figure 6.12 represents the journey from infinity to point P, a distance r from a large mass M. The work done making this journey $= -W$ so the potential $V = \frac{-W}{m}$

Figure 6.12 The journey from infinity to point *P*.

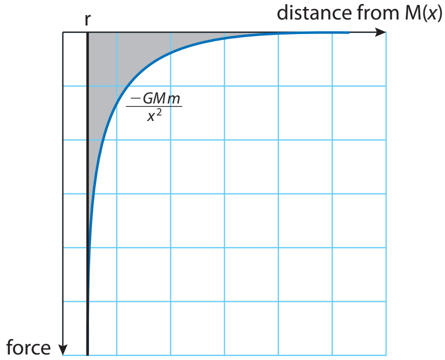


The negative sign is because if you were taking mass m from infinity to P you wouldn't have to pull it, it would pull you. The direction of the force that you would exert is in the opposite direction to the way it is moving, so work done is negative.

Calculating the work done

There are two problems when you try to calculate the work done from infinity to P; firstly the distance is infinite (obviously) and secondly the force gets bigger as you get closer. To solve this problem, we use the area under the force–distance graph (remember the work done stretching a spring?). From Newton's universal law of gravitation we know that the force varies according to the equation: $F = \frac{GMm}{r^2}$ so the graph will be as shown in Figure 6.13.

Figure 6.13 Graph of force against distance as the test mass is moved towards *M*.



Integration

The integral mentioned here is

$$V = \int_{\infty}^{r} \frac{GM}{x^2}\,dx$$

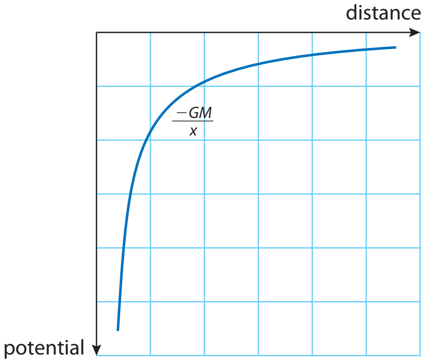
The area under this graph can be found by integrating the function $-\frac{GMm}{x^2}$ from infinity to r (you'll do this in maths). This gives the result:

$$W = -\frac{GMm}{r}$$

So the potential, $V = \frac{W}{m} = -\frac{GM}{r}$

The graph of potential against distance is drawn in Figure 6.14. The gradient of this line gives the field strength, but notice that the gradient is positive and the field strength negative so we get the formula $g = -\frac{\Delta V}{\Delta x}$

Figure 6.14 Graph of potential against distance.



Equipotentials and potential wells

If we draw the lines of equipotential for the field around a sphere, we get concentric circles, as in Figure 6.15. In 3D these would form spheres, in which case they would be called *equipotential surfaces* rather the lines of equipotential.

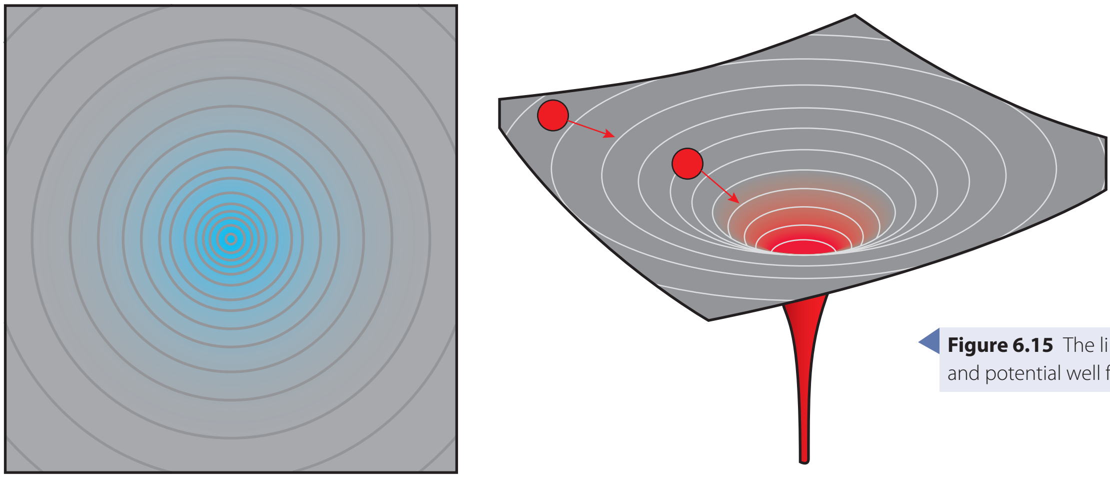


Figure 6.15 The lines of equipotential and potential well for a sphere.

An alternative way of representing this field is to draw the hole or well that these contours represent. This is a very useful visualization, since it not only represents the change in potential but by looking at the gradient, we can also see where the force is biggest. If you imagine a ball rolling into this well you can visualize the field.

Relationship between field lines and potential

If we draw the field lines and the potential as in Figure 6.16, we see that as before they are perpendicular. We can also see that the lines of equipotential are closest together where the field is strongest (where the field lines are most dense). This agrees with our earlier finding that $g = \frac{-\Delta V}{\Delta x}$

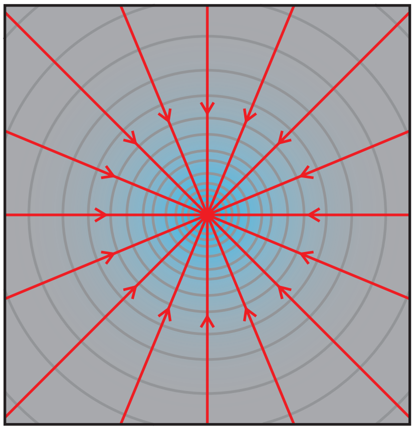


Figure 6.16 Equipotentials and field lines.

Addition of potential

Potential is a scalar quantity, so adding potentials is just a matter of adding the magnitudes. If we take the example shown in Figure 6.17, to find the potential at point P we calculate the potential due to A and B then add them together.

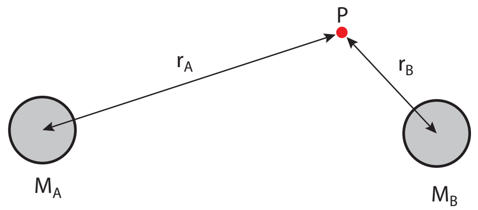


Figure 6.17 Two masses.

● **Hint:** When you add field strengths you have to add them vectorially, making triangles using Pythagoras etc. Adding potential is much simpler because it's a scalar.

The total potential at P $= -\frac{GM_A}{r_A} + -\frac{GM_B}{r_B}$

The lines of equipotential for this example are shown in Figure 6.18.

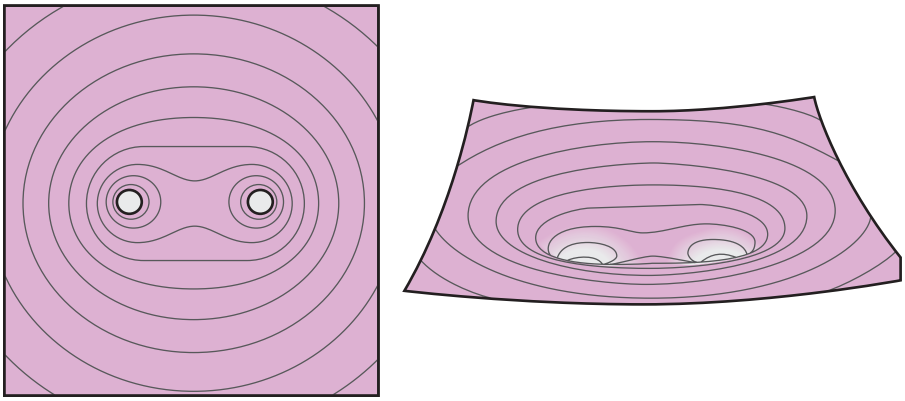


Figure 6.18 Equipotentials and potential wells for two equal masses. If you look at the potential well, you can imagine a ball could sit on the hump between the two holes. This is where the field strength is zero.

Exercise

11 The Moon has a mass of 7.4×10^{22} kg and the Earth has mass of 6.0×10^{24} kg. The average distance between the Earth and the Moon is 3.8×10^5 km. If you travel directly between the Earth and the Moon in a rocket of mass 2000 kg

(a) calculate the gravitational potential when you are 1.0×10^4 km from the Moon

(b) calculate the rocket's PE at the point in part **(a)**

(c) draw a sketch graph showing the change in potential

(d) mark the point where the gravitational field strength is zero.

6.3 Escape speed

Assessment statements

9.2.7 Explain the concept of escape speed from a planet.

9.2.8 Derive an expression for the escape speed of an object from the surface of a planet.

9.2.9 Solve problems involving gravitational potential energy and gravitational potential.

If a body is thrown straight up, its KE decreases as it rises. If we ignore air resistance, this KE is converted into PE. When it gets to the top, the final PE will equal the initial KE, so $\frac{1}{2}mv^2 = mgh$.

If we throw a body up really fast, it might get so high that the gravitational field strength would start to decrease. In this case, we would have to use the formula for the PE around a sphere.

$$\mathrm{PE} = -\frac{GMm}{r}$$

So when it gets to its furthest point as shown in Figure 6.19

$$\text{loss of KE} = \text{gain in PE}$$

$$\tfrac{1}{2}mv^2 - 0 = -\frac{GMm}{R_2} - -\frac{GMm}{R_E}$$

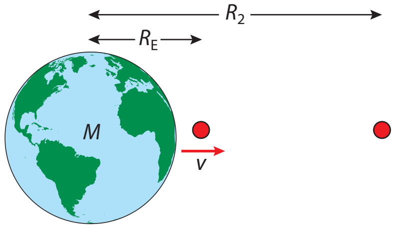


Figure 6.19 A mass *m* thrown away from the Earth.

If we throw the ball fast enough, it will never come back. This means that it has reached a place where it is no longer attracted back to the Earth, infinity. Of course it can't actually reach infinity but we can substitute $R_2 = \infty$ into our equation to find out how fast that would be.

$$\tfrac{1}{2}mv^2 = -\frac{GMm}{\infty} - -\frac{GMm}{R_E}$$

$$\tfrac{1}{2}mv^2 = \frac{GMm}{R_E}$$

Rearranging gives: $$v_{escape} = \sqrt{\frac{2GM}{R_E}}$$

If we calculate this for the Earth it is about 11 km s^{-1}.

Air resistance

If you threw something up with a velocity of 11 km s^{-1} it would be rapidly slowed by air resistance. The work done against this force would be converted to thermal energy causing the body to vaporize. Rockets leaving the Earth do not have to travel anywhere near this fast, as they are not thrown upwards, but have a rocket engine that provides a continual force.

Why the Earth has an atmosphere but the Moon doesn't

The average velocity of an air molecule at the surface of the Earth is about 500 m s^{-1}. This is much less than the velocity needed to escape from the Earth, and for that reason the atmosphere doesn't escape.

The escape velocity on the Moon is 2.4 km s^{-1} so you might expect the Moon to have an atmosphere. However, 500 m s^{-1} is the *average* speed; a lot of the molecules would be travelling faster than this leading to a significant number escaping, and over time all would escape.

Black holes

A star is a big ball of gas held together by the gravitational force. The reason this force doesn't cause the star to collapse is that the particles are continuously given KE from the nuclear reactions taking place (fusion). As time progresses, the nuclear fuel gets used up, so the star starts to collapse. As this happens, the escape velocity increases until it is bigger than the speed of light, at this point not even light can escape and the star has become a black hole.

Exercises

12 The mass of the Moon is 7.4×10^{22} kg and its radius is 1738 km. Show that its escape speed is 2.4 km s^{-1}.

13 Why doesn't the Earth's atmosphere contain hydrogen?

14 The mass of the Sun is 2.0×10^{30} kg. Calculate how small its radius would have to be for it to become a black hole.

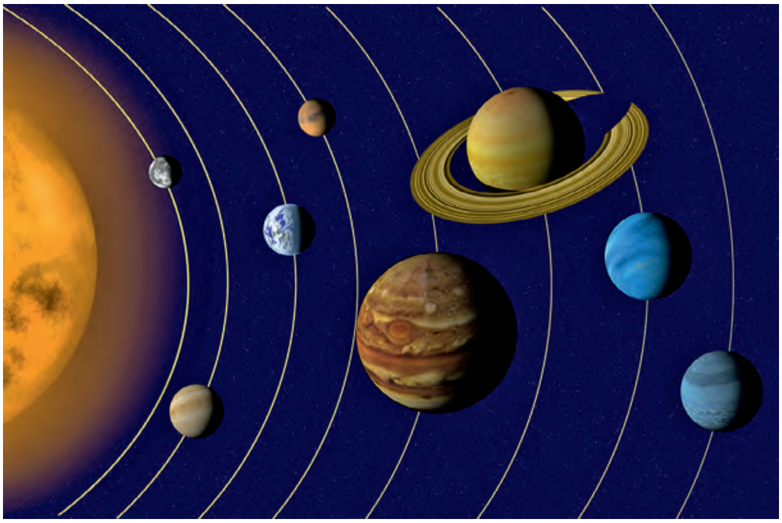
15 When travelling away from the Earth, a rocket runs out of fuel at a distance of 1.0×10^5 km. How fast would the rocket have to be travelling for it to escape from the Earth? (Mass of the Earth = 6.0×10^{24} kg, radius = 6400 km.)

How can light be slowed down by the effect of gravity, when according to Newton's law, it has no mass, therefore isn't affected by gravity? This can't be explained by Newton's theories, but Einstein solved the problem with his general theory of relativity.

6.4 Orbital motion

Assessment statements

9.4.1 State that gravitation provides the centripetal force for circular orbital motion.
9.4.2 Derive Kepler's third law.
9.4.3 Derive expressions for the kinetic energy, potential energy and total energy of an orbiting satellite.
9.4.4 Sketch graphs showing the variation with orbital radius of the kinetic energy, gravitational potential energy and total energy of a satellite.
9.4.5 Discuss the concept of weightlessness in orbital motion, in free fall and in deep space.
9.4.6 Solve problems involving orbital motion.



An artist's impression of the solar system.

The solar system

The solar system consists of the Sun at the centre surrounded by eight orbiting planets. The shape of the orbits is actually slightly elliptical but to make things simpler, we will assume them to be circular. We know that for a body to travel in a circle, there must be an unbalanced force (called the centripetal force, $m\omega^2 r$) acting towards the centre. The force that holds the planets in orbit around the Sun is the gravitational force $\frac{GMm}{r^2}$. Equating these two expressions gives us an equation for orbital motion.

$$m\omega^2 r = \frac{GMm}{r^2} \qquad (1)$$

Now ω is the angular speed of the planet, that is the angle swept out by a radius per unit time. If the time taken for one revolution (2π radians) is T then $\omega = \frac{2\pi}{T}$.

Substituting into equation (1) gives

$$m\left(\frac{2\pi}{T}\right)^2 r = \frac{GMm}{r^2}$$

Rearranging gives: $$\frac{T^2}{r^3} = \frac{4\pi^2}{GM}$$

where M is the mass of the Sun.

So for planets orbiting the Sun, $\frac{T^2}{r^3}$ is a constant, or T^2 is proportional to r^3. This is Kepler's third law.

Kepler thought of his law before Newton was born, so couldn't have derived the equation in the way we have here. He came up with the law by manipulating the data that had been gathered from many years of measurement, realizing that if you square the time period and divide by the radius cubed you always get the same number.

From this we can deduce that the planet closest to the Sun (Mercury) has a shorter time period than the planet furthest away. This is supported by measurement:

Time period of Mercury = 0.24 years.

Time period of Neptune = 165 years.

Exercise

16 Use a database to make a table of the values of time period and radius for all the planets. Plot a graph to show that T^2 is proportional to r^3.

To find an example of a database, visit www.heinemann.co.uk/hotlinks, enter the express code 4426P and click on Weblink 6.2.

Energy of an orbiting body

As planets orbit the Sun they have KE due to their movement and PE due to their position. We know that their PE is given by the equation:

$$\text{PE} = -\frac{GMm}{r}$$

and $$\text{KE} = \tfrac{1}{2}mv^2$$

We also know that if we approximate the orbits to be circular then equating the centripetal force with gravity gives:

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

Rearranging and multiplying by $\frac{1}{2}$ gives $$\tfrac{1}{2}mv^2 = \frac{GMm}{2r}$$

$$\text{KE} = \frac{GMm}{2r}$$

The total energy $= \text{PE} + \text{KE} = -\dfrac{GMm}{r} + \dfrac{GMm}{2r}$

Total energy $= -\dfrac{GMm}{2r}$

● **Hint:** There are two versions of the equation for centripetal force

Speed version:

$F = \frac{mv^2}{r}$

Angular speed version:

$F = m\omega^2 r$

Earth satellites

The equations we have derived for the orbits of the planets also apply to the satellites that man has put into orbits around the Earth. This means that the satellites closer to the Earth have a time period much shorter than the distant ones. For example, a low orbit spy satellite could orbit the Earth once every two hours and a much higher TV satellite orbits only once a day.

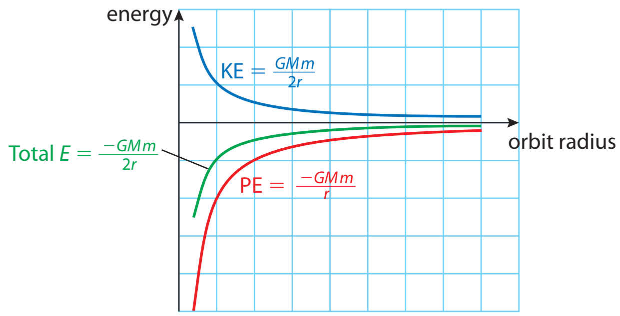
The total energy of an orbiting satellite $= -\dfrac{GMm}{2r}$ so the energy of a high satellite (big r) is less negative and hence bigger than a low orbit. To move from a low orbit into a high one therefore requires energy to be added (work done).

Imagine you are in a spaceship orbiting the Earth in a low orbit. To move into a higher orbit you would have to use your rocket motor to increase your energy. If you kept doing this you could move from orbit to orbit, getting further and further from the Earth. The energy of the spaceship in each orbit can be displayed as a graph as in Figure 6.20 (overleaf).

From the graph we can see that low satellites have greater KE but less total energy than distant satellites, so although the distant ones move with slower speed, we have to do work to increase the orbital radius. Going the other way, to move from a distant orbit to a close orbit, the spaceship needs to lose energy. Satellites in low

Earth orbit are not completely out of the atmosphere, so lose energy due to air resistance. As they lose energy they spiral in towards the Earth.

Figure 6.20 Graph of KE, PE and total energy for a satellite with different orbital radius.



The physicist Stephen Hawking experiencing weightlessness in a free falling aeroplane.

Weightlessness

The only place you can be truly without weight is a place where there is no gravitational field; this is at infinity or a place where the gravitational fields of all the bodies in the universe cancel out. If you are a long way from everything, somewhere in the middle of the universe, then you could say that you are pretty much weightless.

To understand how it feels to be weightless, we first need to think what it is that makes us feel weight. As we stand in a room we can't feel the Earth pulling our centre downwards but we can feel the ground pushing our feet up. This is the normal force that must be present to balance our weight. If it were not there we would be accelerating downwards. Another thing that makes us notice that we are in a gravitational field is what happens to things we drop; it is gravity that pulls them down. Without gravity they would float in mid-air. So if we were in a place where there was no gravitational field then the floor would not press on our feet and things we drop would not fall. It would feel exactly the same if we were in a room that was falling freely as in Figure 6.21. If we accelerate down along with the room then the only force acting on us is our weight; there is no normal force between the floor and our feet. If we drop something it falls with us. From outside the room we can see that the room is in a gravitational field falling freely but inside the room it feels like someone has turned off gravity (not for long though). An alternative and rather longer lasting way of feeling weightless is to orbit the Earth inside a space station. Since the space station and everything inside it is accelerating towards the Earth, it will feel exactly like the room in Figure 6.21, except you won't hit the ground.

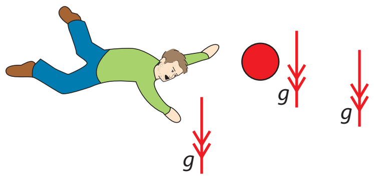


Figure 6.21 As the room, the man and the ball accelerate downwards, the man will feel weightless.

Exercises

17 So that they can stay above the same point on the Earth, TV satellites have a time period equal to one day. Calculate the radius of their orbit.

18 A spy satellite orbits 400 km above the Earth. If the radius of the Earth is 6400 km, what is the time period of the orbit?

19 If the satellite in question 18 has a mass of 2000 kg, calculate its
(a) KE
(b) PE
(c) total energy.

6.5 Electric force and field

Assessment statements

6.2.1 State that there are two types of electric charge.
6.2.2 State and apply the law of conservation of charge.
6.2.3 Describe and explain the difference in the electrical properties of conductors and insulators.
6.2.4 State Coulomb's law.
6.2.5 Define *electric field strength*.
6.2.6 Determine the electric field strength due to one or more point charges.
6.2.7 Draw the electric field patterns for different charge configurations.
6.2.8 Solve problems involving electric charges, forces and fields.

Electric force

So far we have dealt with many forces; for example, friction, tension, upthrust, normal force, air resistance and gravitational force. If we rub a balloon on a woolen pullover, we find that the balloon is attracted to the wool of the pullover – this cannot be explained in terms of any of the forces we have already considered, so we need to develop a new model to explain what is happening. First we need to investigate the effect.

This is another example of how models are used in physics.

Consider a balloon and a woollen pullover – if the balloon is rubbed on the pullover, we find that it is attracted to the pullover. However, if we rub two balloons on the pullover, the balloons repel each other.

Whatever is causing this effect must have two different types, since there are two different forces. We call this force the *electric force*.

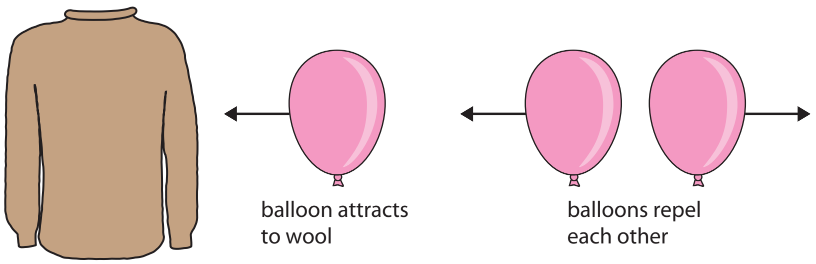
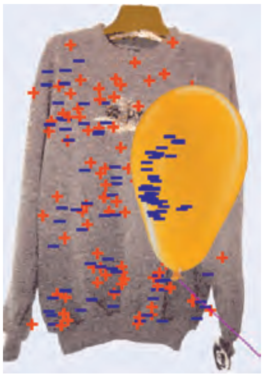


Figure 6.22 Balloons are attracted to the wool but repel each other.



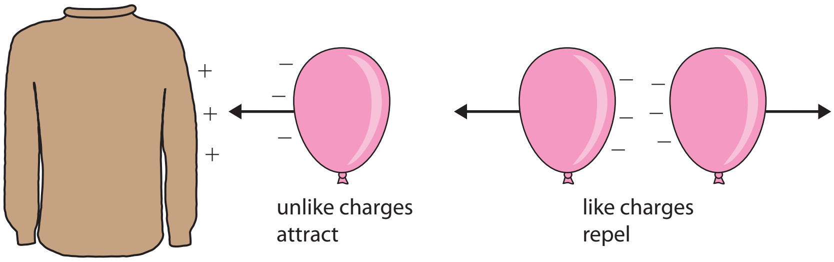
Charge

The balloon and pullover must have some property that is causing this force. We call this property *charge*. There must be two types of charge, traditionally called positive (+ve) and negative(−ve). To explain what happens, we can say that, when rubbed, the balloon gains −ve charge and the pullover gains +ve charge. If like charges repel and unlike charges attract, then we can explain why the balloons repel and the balloon and pullover attract.

You can try this with real balloons or, to use the simulation 'Balloons and static electricity', visit www.heinemann.co.uk/hotlinks, enter the express code 4426P and click on Weblink 6.3.

Figure 6.23 The force is due to charges.

Here +ve and −ve numbers are used to represent something that they were not designed to represent.

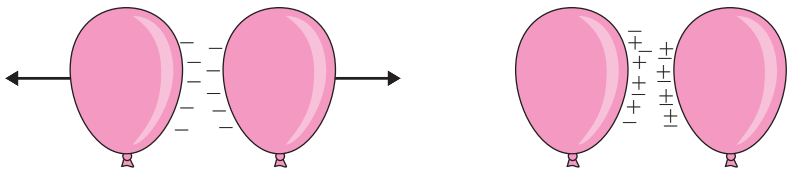


The unit of charge is the coulomb (C).

Conservation of charge

If we experiment further, we find that if we rub the balloon more, then the force between the balloons is greater. We also find that if we add +ve charge to an equal −ve charge, the charges cancel.

Figure 6.24 Charges cancel each other out.



We can add and take away charge but we cannot destroy it.

The law of conservation of charge states that charge can neither be created nor destroyed.

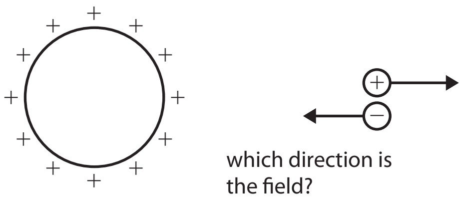
Electric field

We can see that there are certain similarities with the electric force and gravitational force; they both act without the bodies touching each other. We used the concept of a field to model gravitation and we can use the same idea here.

Electric field is defined as a region of space where a charged object experiences a force due to its charge.

Field lines

Figure 6.25 Field is in the direction in which a +ve charge would accelerate.



Field lines can be used to show the direction and strength of the field. However, because there are two types of charge, the direction of the force could be one of two possibilities.

It has been decided that we should take the direction of the field to be the direction that a small +ve charge would accelerate if placed in the field. So we will always consider what would happen if +ve charges are moved around in the field. The field lines will therefore be as shown in Figures 6.26 to 6.28.

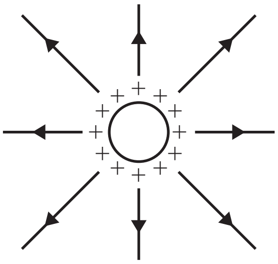


Figure 6.26 Field lines close to a sphere of charge.

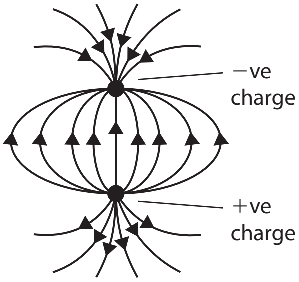


Figure 6.27 Field due to a dipole.

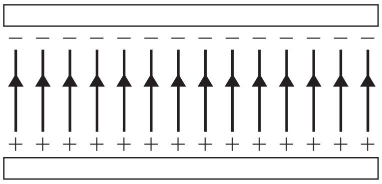


Figure 6.28 A uniform field.

Coulomb's law

In a gravitational field, the force between masses is given by Newton's law, and the equivalent for an electric field is Coulomb's law.

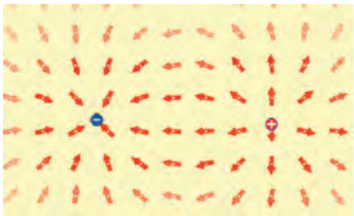
Coulomb's law states that the force experienced by two point charges is directly proportional to the product of their charge and inversely proportional to the square of their separation.

The force experienced by two point charges Q_1 and Q_2 separated by a distance r in a vacuum is given by the formula

$$F = k\frac{Q_1 Q_2}{r^2}$$

The constant of proportionality $k = 9 \times 10^9\ \mathrm{Nm^2\,C^{-2}}$

Note: Similarly to gravitational fields, Coulomb's law also applies to spheres of charge, the separation being the distance between the centres of the spheres.



In the PhET simulation 'Charges and fields' you can investigate the force experienced by a small charge as it is moved around an electric field. To try this, visit www.heinemann.co.uk/hotlinks, enter the express code 4426P and click on Weblink 6.4.

Electric field strength (E)

The electric field strength is a measure of the force that a +ve charge will experience if placed at a point in the field. It is defined as the force per unit charge experienced by a small +ve test charge placed in the field.

So if a small +ve charge q experiences a force F in the field, then the field strength at that point is given by $E = \frac{F}{q}$. The unit of field strength is $\mathrm{NC^{-1}}$, and it is a vector quantity.

Worked examples

A 5 μC point charge is placed 20 cm from a 10 μC point charge.

1 Calculate the force experienced by the 5 μC charge.

2 What is the force on the 10 μC charge?

3 What is the field strength 20 cm from the 10 μC charge?

Solutions

1 Using the equation $F = \dfrac{kQ_1Q_2}{r^2}$

$Q_1 = 5 \times 10^{-6}\,\text{C}$, $Q_2 = 10 \times 10^{-6}\,\text{C}$ and $r = 0.20\,\text{m}$

$$F = \frac{9 \times 10^9 \times 5 \times 10^{-6} \times 10 \times 10^{-6}}{0.20^2}\,\text{N}$$
$$= 11.25\,\text{N}$$

2 According to Newton's third law, the force on the 10 μC charge is the same as the 5 μC.

3 Force per unit charge $= \dfrac{11.25}{5 \times 10^{-6}}$

$E = 2.25 \times 10^6\,\text{N}\,\text{C}^{-1}$

● **Examiner's hint:** Field strength is defined as the force per unit charge so if the force on a 5 μC charge is 11.25 N, the field strength E is equal to 11.25 N divided by 5 μC.

Exercises

20 If the charge on a 10 cm radius metal sphere is 2 μC, calculate
(a) the field strength on the surface of the sphere
(b) the field strength 10 cm from the surface of the sphere
(c) the force experienced by a 0.1 μC charge placed 10 cm from the surface of the sphere.

21 A small sphere of mass 0.01 kg and charge 0.2 μC is placed at a point in an electric field where the field strength is $0.5\,\text{N}\,\text{C}^{-1}$.
(a) What force will the small sphere experience?
(b) If no other forces act, what is the acceleration of the sphere?

● **Examiner's hint:** When solving field problems you always assume one of the charges is in the field of the other. E.g in Example 1 the 5 μC charge is in the field of the 10 μC charge. Don't worry about the fact that the 5 μC charge also creates a field – that's not the field you are interested in.

Electric field strength in a uniform field

A uniform field can be created between two parallel plates of equal and opposite charge as shown in Figure 6.29. The field lines are parallel and equally spaced. If a test charge is placed in different positions between the plates, it experiences the same force.

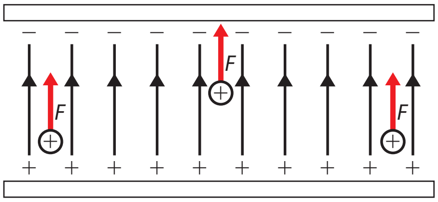


Figure 6.29 In a uniform field the force on a charge q is the same everywhere.

So if a test charge q is placed in the field above, then $E = \dfrac{F}{q}$ everywhere between the two charged plates.

Worked example

If a charge of 4 μC is placed in a uniform field of field strength $2\,\text{N}\,\text{C}^{-1}$ what force will it experience?

Solution

$F = EQ$ Rearranging the formula $E = \dfrac{F}{Q}$

$= 2 \times 4 \times 10^{-6}\,\text{N}$

$= 8\,\mu\text{N}$

Electric field strength close to a sphere of charge

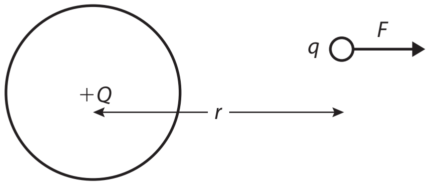


Figure 6.30

From definition: $E = \dfrac{F}{q}$

From Coulomb's law: $F = k\dfrac{Qq}{r^2}$

Substituting: $E = k\dfrac{Q}{r^2}$

Addition of field strength

Field strength is a vector, so when the field from two negatively charged bodies act at a point, the field strengths must be added vectorially. In Figure 6.31, the resultant field at two points A and B is calculated. At A the fields act in the same line but at B a triangle must be drawn to find the resultant.

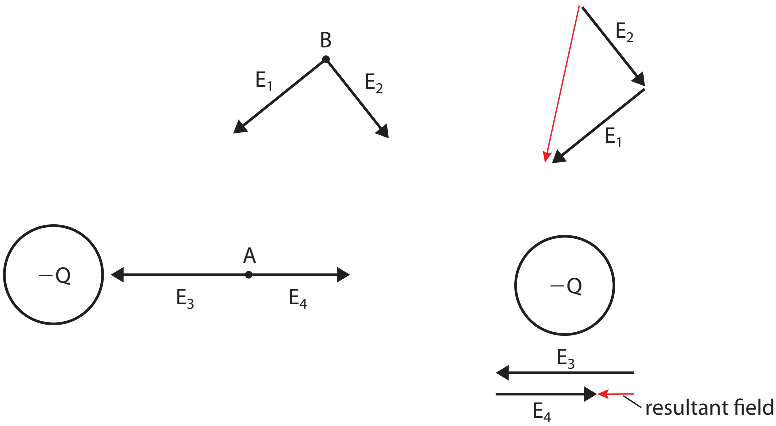
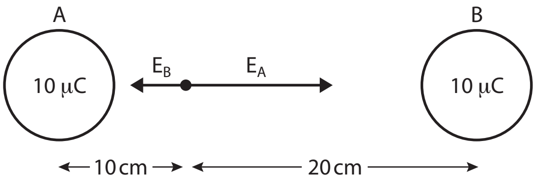


Figure 6.31 Since both charges are negative, the field strength is directed towards the charges. Since they are at right angles to each other, Pythagoras can be used to sum these vectors.

E_3 is the field due to the charge on the left, E_4 is due to the charge on the right. E_3 is bigger than E_4 because the charge on the left is closer.

Worked example



Two $+10\,\mu C$ charges are separated by 30 cm. What is the field strength between the charges 10 cm from A?

Solution

Field strength due to A, $E_A = \dfrac{9 \times 10^9 \times 10 \times 10^{-6}}{0.1^2}$

$= 9 \times 10^6\,\mathrm{N\,C^{-1}}$

$E_B = \dfrac{9 \times 10^9 \times 10 \times 10^{-6}}{0.2^2}$

$= 2.25 \times 10^6\,\mathrm{N\,C^{-1}}$

Resultant field strength $= (9 - 2.25) \times 10^6\,\mathrm{N\,C^{-1}}$

$= 6.75 \times 10^6\,\mathrm{N\,C^{-1}}$ (to the left)

6.6 Electrical potential

Assessment statements

9.3.1 Define electric potential and electric potential energy.
9.3.2 State and apply the expression for electric potential due to a point charge.
9.3.3 State and apply the formula relating electric field strength to electric potential gradient.
9.3.4 Determine the potential due to one or more point charges.
9.3.5 Describe and sketch the pattern of equipotential surfaces due to one and two point charges.
9.3.6 State the relation between equipotential surfaces and electric field lines.
9.3.7 Solve problems involving electric potential energy and electric potential.

The concept of electric potential is very similar to that of gravitational potential; it gives us information about the amount of energy associated with different points in a field. We have already defined electric potential difference in relation to electrical circuits; it is the amount of electrical energy converted to heat when a unit charge flows through a resistor. In this section we will define the electric potential in more general terms.

Electric potential energy and potential

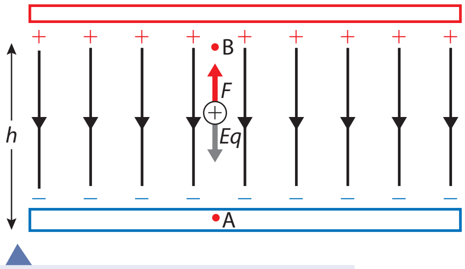


Figure 6.32

When we move a positive charge around in an electric field we have to do work on it. If we do work we must give it energy. This energy is not increasing the KE of the particle so must be increasing its PE, and so we call this *electric potential energy*. Let us first consider the uniform field shown in Figure 6.32. In order to move a charge $+q$ from A to B, we need to exert a force that is equal and opposite to the electric force, Eq. As we move the charge we do an amount of work equal to Eqh. We have therefore increased the electrical PE of the charge by the same amount, so PE $= Eqh$.

This is very similar to the room in Figure 6.9; the higher we lift the positive charge, the more PE it gets. In the same way we can define the potential of different points as being the quantity that defines how much energy a given charge would have if placed there.

Positive charge
Note that the potential is defined in terms of a positive charge.

Electric potential at a point is the amount of work per unit charge needed to take a small positive test charge from a place of zero potential to the point.

The unit of potential is $\mathrm{J\,C^{-1}}$ or volts.

Potential is a scalar quantity.

In this example if we define the zero in potential as the bottom plate, then the potential at B, $V_B = Eh$

Since the potential is proportional to h we can deduce that all points a distance h from the bottom plate will have the same potential. We can therefore draw lines of equipotential as we did in the gravitational field. Figure 6.33 shows an example with equipotentials.

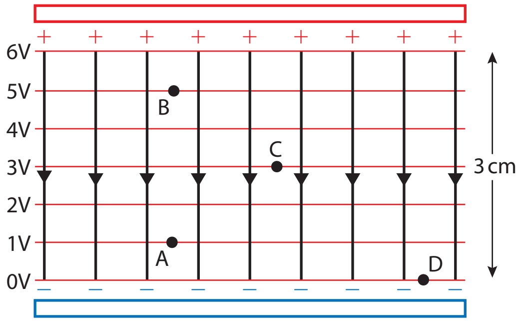


Figure 6.33

> **The electronvolt (eV)**
> This is a unit of energy used in atomic physics. 1 eV = energy gained by an electron accelerated through a p.d. of 1 V.

Exercises

Refer to Figure 6.33 for Questions 22–27.

22 What is the potential difference (p.d.) between A and C?

23 What is the p.d. between B and D?

24 If a charge of +3 C was placed at B, how much PE would it have?

25 If a charge of +2 C was moved from C to B, how much work would be done?

26 If a charge of −2 C moved from A to B, how much work would be done?

27 If a charge of +3 C was placed at B and released
   (a) what would happen to it?
   (b) how much KE would it gain when it reached A?

> **Contours**
> The lines of equipotential are again similar to contour lines but this time there is no real connection to gravity. Any hills and wells will be strictly imaginary.

Potential and field strength

In the example of a uniform field the change in potential ΔV when a charge is moved a distance Δh is given by

$$\Delta V = E\Delta h$$

Rearranging gives $E = \dfrac{\Delta V}{\Delta h}$, the field strength = the potential gradient. So in the example of Figure 6.33 the field strength is $\dfrac{6}{3 \times 10^{-2}} \dfrac{\mathrm{V}}{\mathrm{m}} = 200\ \mathrm{N\,C^{-1}}$

Potential due to point charge

The uniform field is a rather special case; a more general example would be to consider the field due to a point charge. This would be particularly useful since all bodies are made of points; so if we know how to find the field due to one point we can find the field due to many points. In this way we can find the field caused by any charged object.

Consider a point P a distance r from a point charge Q as shown in Figure 6.34. The potential at P is defined as the work done per unit charge taking a small positive test charge from infinity to P.

> **Permittivity**
> The constant $k = 9 \times 10^9\ \mathrm{Nm^2\,C^{-2}}$
> This can also be expressed in terms of the permittivity of a vacuum, ϵ_0
> $k = \dfrac{1}{4\pi\epsilon_0}$
> $\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{C^2\,N^{-1}\,m^{-2}}$
> The permittivity is different for different media but we will only be concerned with fields in a vacuum.

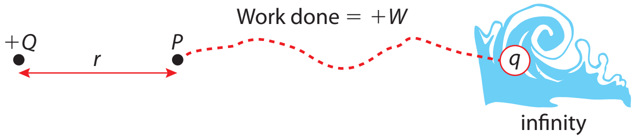


Figure 6.34 A positive charge is taken from infinity to point P.

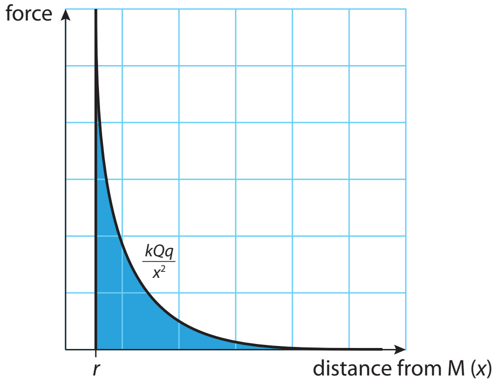


Figure 6.35 Graph of force against distance as charge $+q$ approaches $+Q$

Again we have used infinity as our zero of potential. To solve this we must find the area under the graph of force against distance, as we did in the gravity example.

Figure 6.35 shows the area that represents the work done. Notice that this time the force is positive as is the area under the graph. The area under this graph is given by the equation $\frac{kQq}{r}$ so the work done is given by:

$$W = \frac{kQq}{r}$$

The potential is the work done per unit charge so $V = \frac{kQ}{r}$.

The potential therefore varies as shown by the graph in Figure 6.36.

Potential gradient

The potential gradient is related to the field strength by the equation

$$\frac{dV}{dx} = -E$$

We can see that this is negative because the gradient of V against x is negative but the field strength is positive.

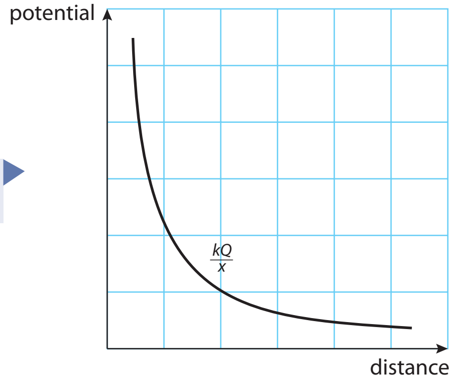
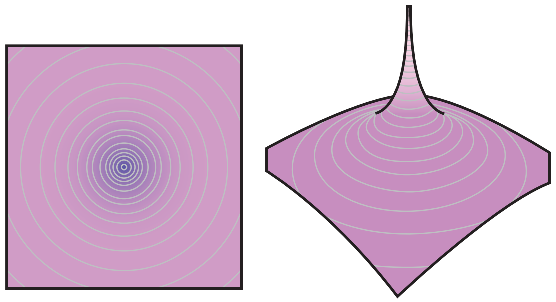


Figure 6.36 Graph of potential against distance for a positive charge

Equipotentials, wells and hills

If we draw lines of equipotential for a point charge we get concentric circles as shown in Figure 6.37. These look just the same as the gravitational equipotentials of Figure 6.15. However, we must remember that if the charge is positive, the potential increases as we get closer to it, rather than decreasing as in the case of gravitational field. These contours represent a hill not a well.

Figure 6.37 The equipotentials and potential hill of a positive point charge.



Exercises

28 Calculate the electric potential a distance 20 cm from the centre of a small sphere of charge $+50\ \mu C$.

29 Calculate the p.d. between the point in question 28 and a second point 40 cm from the centre of the sphere.

Addition of potential

Since potential is a scalar there is no direction to worry about when adding the potential from different bodies – simply add them together.

Example

At point P in Figure 6.38 the combined potential is given by:

$$V = k\frac{Q_1}{r_1} + k\frac{-Q_2}{r_2}$$

The potential of combinations of charging can be visualized by drawing lines of equipotentials. Figure 6.39 shows the equipotentials for a combination of a positive and negative charge (a dipole). This forms a hill and a well, and we can get a feeling for how a charge will behave in the field by imagining a ball rolling about on the surface shown.

● **Hint:** Zero potential is not zero field. If we look at the hill and well in Figure 6.39, we can see that there is a position of zero potential in between the two charges (where the potentials cancel). This is not, however, a position of zero field since the fields will both point towards the right in between the charges.

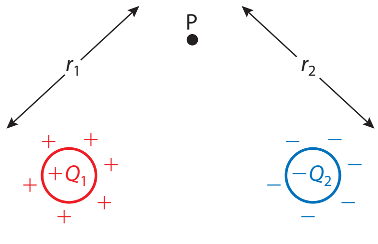


Figure 6.38

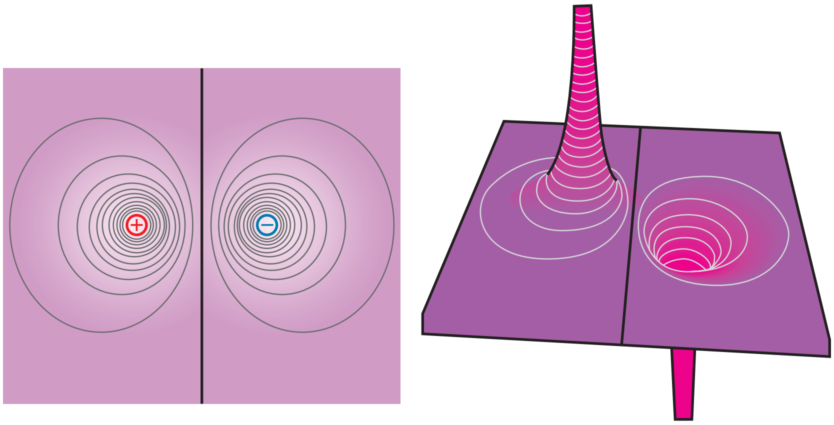


Figure 6.39 Equipotentials for a dipole.

Exercises

Refer to Figure 6.40 for questions 30–36.

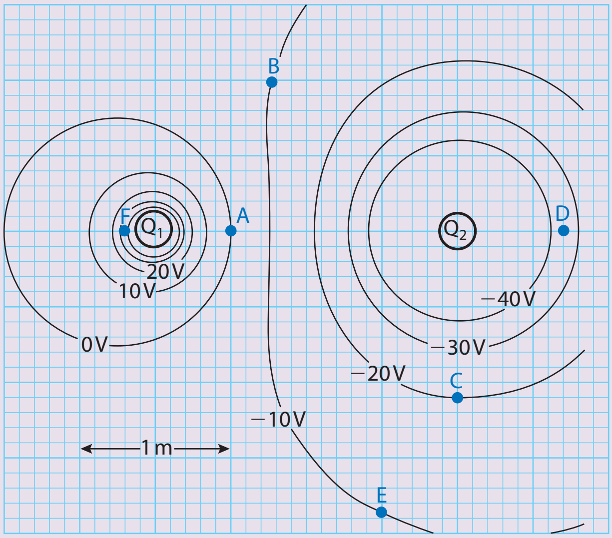


Figure 6.40 The lines of equipotential drawn every 10 V for two charges Q_1 and Q_2

30 (a) One of the charges is positive and the other is negative. Which is which?

(b) If a positive charge were placed at A, would it move, and if so, in which direction?

31 At which point A, B, C, D or F is the field strength greatest?

The electronvolt (eV)
The electronvolt is a unit of electrical PE often used in atomic and nuclear physics. 1eV is the amount of energy gained by an electron accelerated through a p.d. of 1 V.

32 What is the p.d. between the following pairs of points?
(a) A and C **(b)** C and E **(c)** B and E

33 How much work would be done taking a +2 C charge between the following points?
(a) C to A **(b)** E to C **(c)** B to E

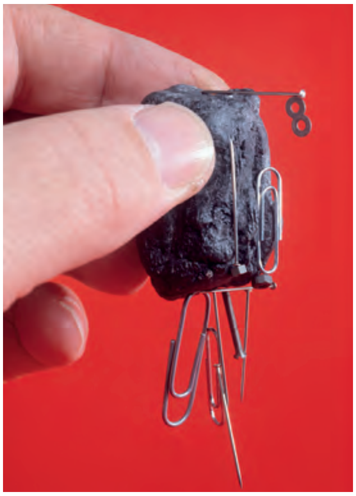
34 Using the scale on the diagram, estimate the field strength at point D. Why is this an estimate?

35 Write an equation for the potential at point A due to Q_1 and Q_2. If the charge Q_1 is 1 nC, find the value Q_2.

36 If an electron is moved between the following points, calculate the work done in eV (remember an electron is negative).
(a) E to A **(b)** C to F **(c)** A to C

6.7 Magnetic force and field

Assessment statements
6.3.1 State that moving charges give rise to magnetic fields.
6.3.2 Draw magnetic field patterns due to currents.
6.3.3 Determine the direction of the force on a current-carrying conductor in a magnetic field.
6.3.4 Determine the direction of the force on a charge moving in a magnetic field.
6.3.5 Define the magnitude and direction of a magnetic field.
6.3.6 Solve problems involving magnetic forces, fields and currents.



Not all magnets are man-made; certain rocks (for example, this piece of magnetite) are naturally magnetic.

What is a magnet?

We all know that magnets are the things that stick notes to fridge doors, but do we understand the forces that cause magnets to behave in this way?

Magnetic poles

The north-seeking pole (red) always points to the north.

Every magnet has two poles (north and south). A magnet is therefore called a *dipole*. It is not possible to have a single magnetic pole or monopole. This is not the same as electricity where you can have a dipole or monopoles. If you cut a magnet in half, each half will have both poles.

There is evidence in ancient Greek and Chinese writing, that people knew about magnets more than 2600 years ago. We do not know who discovered the first magnet, but since antiquity it was known that if a small piece of magnetite was suspended on a string and held close to another larger piece, the small one experienced a turning force, causing it to rotate. It was also found that if the small rock was held on its own, it would always turn to point toward the North Pole. The end of the rock that pointed north was named the *north-seeking pole*, the other end was named the *south-seeking pole*.

Figure 6.41 Magnets are dipoles.

Unlike poles attract

If we take two magnets and hold them next to each other, we find that the magnets will turn so that the S and N poles come together.

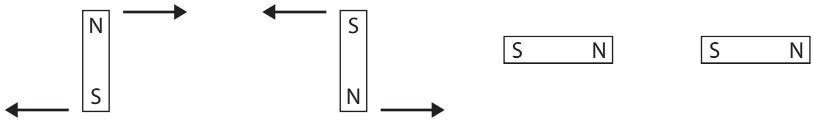


Figure 6.42 Magnets experience a turning force causing unlike poles to come together.

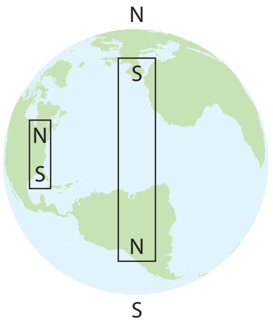
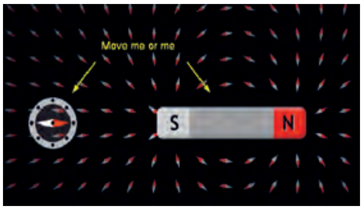


Figure 6.43 The north-seeking pole of a compass points north.

We can therefore conclude that the reason that a small magnet points toward the North Pole of the Earth is because there is a south magnetic pole there. This can be a bit confusing, but remember that the proper name for the pole of the magnet is north-seeking pole.



To plot magnetic fields on the PhET 'Faraday's electromagnetic lab', visit www.heinemann.co.uk/hotlinks, enter the express code 4426P and click on Weblink 6.5.

Magnetic field

Magnetism is similar to gravitational force and electric force in that the effect is felt even though the magnets do not touch each other; we can therefore use the concept of field to model magnetism. However, magnetism isn't quite the same; we described both gravitational and electric fields in terms of the force experienced by a small mass or charge. A small magnet placed in a field does not accelerate – it rotates, and therefore magnetic field is defined as a region of space where a small test magnet experiences a turning force.

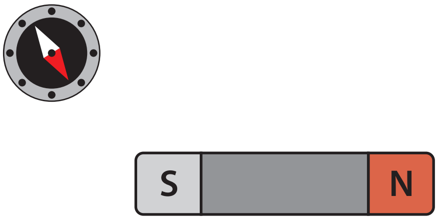


Figure 6.44 The small magnet is caused to turn, so must be in a magnetic field.

Since a small magnet rotates if held above the Earth, we can therefore conclude that the Earth has a magnetic field.

Magnetic field lines

In practice, a small compass can be used as our test magnet. Magnetic field lines are drawn to show the direction that the N pole of a small compass would point if placed in the field.

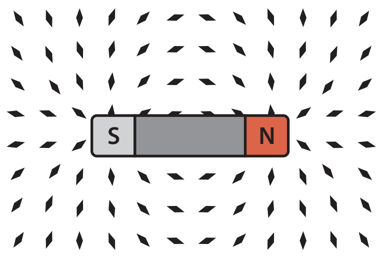


Figure 6.45 If the whole field were covered in small magnets, then they would show the direction of the field lines.

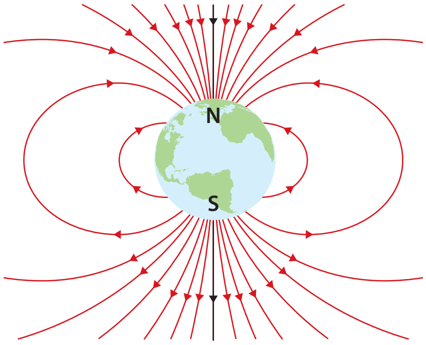


Figure 6.46 The Earth's magnetic field.

Magnetic flux density (*B*)

From what we know about fields, the strength of a field is related to the density of field lines. This tells us that the magnetic field is strongest close to the poles. The magnetic flux density is the quantity that is used to measure how strong the field is – however it is not quite the same as field strength as used in gravitational and electric fields.

The unit of magnetic flux density is the tesla (T) and it is a vector quantity.

***B* field**
Since the letter *B* is used to denote flux density, the magnetic field is often called a *B field*.

Field caused by currents

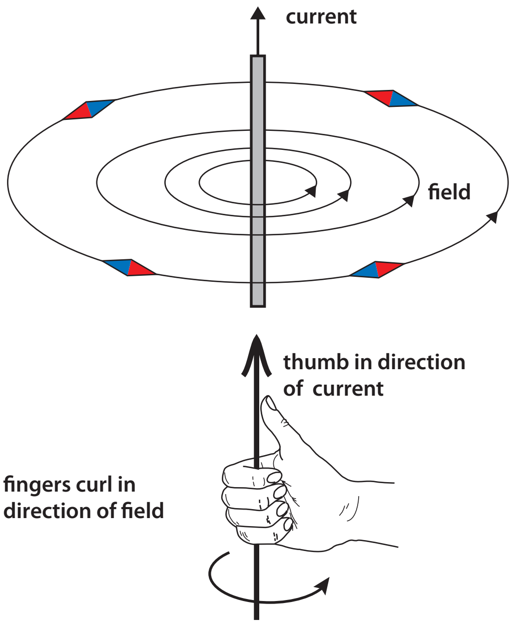


Figure 6.47 The field due to a long straight wire carrying a current is in the form of concentric circles so the field is strongest close to the wire.

If a small compass is placed close to a straight wire carrying an electric current, then it experiences a turning force that makes it always point around the wire. The region around the wire is therefore a magnetic field. This leads us to believe that magnetic fields are caused by moving charges.

Field inside a coil

When a current goes around a circular loop, the magnetic field forms circles.

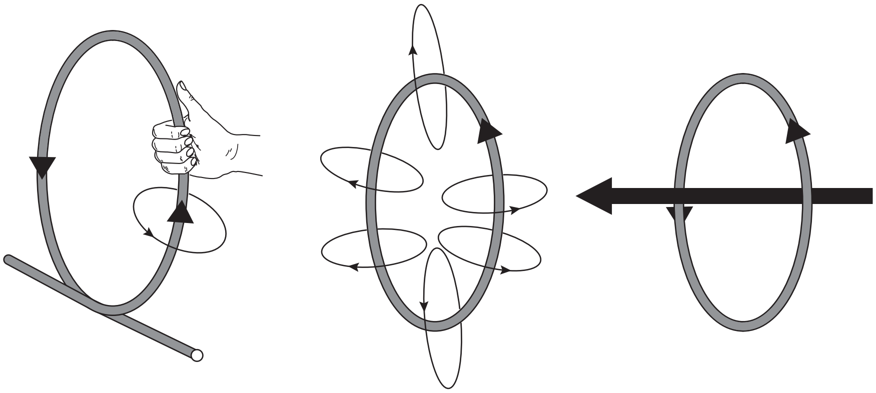


Figure 6.48 The direction of the field can be found by applying the right-hand grip rule to the wire. The circles formed by each bit of the loop add together in the middle to give a stronger field.

The field inside a solenoid

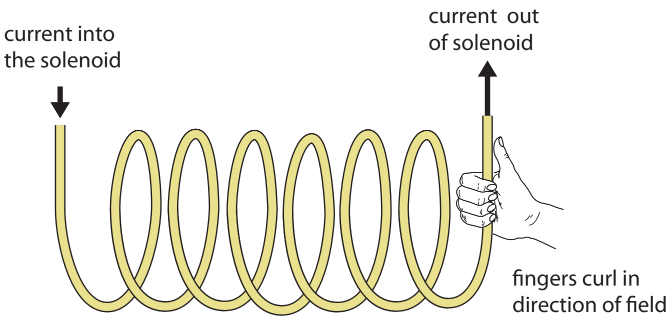


Figure 6.49 The direction of the field in a solenoid can be found using the grip rule on one coil.

The resulting field pattern is like that of a bar magnet but the lines continue through the centre.

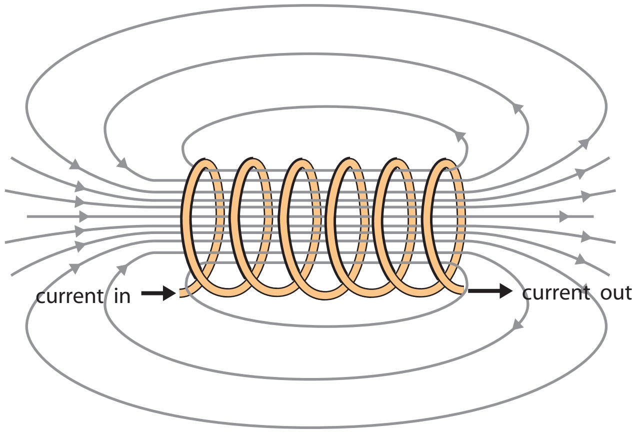
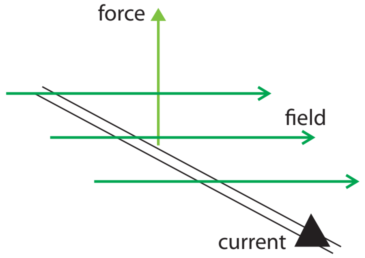


Figure 6.50 The field inside a solenoid.

Force on a current-carrying conductor

We have seen that when a small magnet is placed in a magnetic field, each end experiences a force that causes it to turn. If a straight wire is placed in a magnetic field, it also experiences a force. However, in the case of a wire, the direction of the force does not cause rotation – the force is in fact perpendicular to the direction of both current and field.

Figure 6.51 Force, field and current are at right angles to each other.



Definition of the ampere
The ampere is defined in terms of the force between two parallel current carrying conductors. A current of 1 A causes a force of 2×10^{-7} N per meter between two long parallel wires placed 1 m apart in a vacuum.

The size of this force is dependent on:

- how strong the field is – flux density B
- how much current is flowing through the wire – I
- the length of the wire – l

If B is measured in tesla, I in amps and l in metres,

$$F = BIl$$

Figure 6.52 Using Fleming's left hand rule to find the direction of the force.

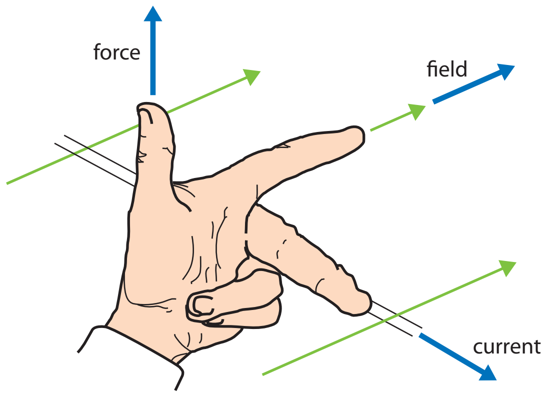
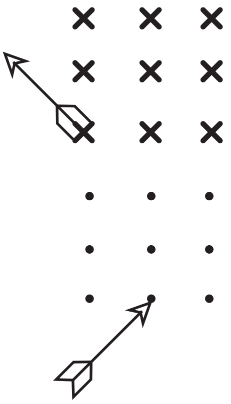


Figure 6.53 Field into the page can be represented by crosses, and field out by dots. Think what it would be like looking at an arrow from the ends.



Worked example

What is the force experienced by a 30 cm long straight wire carrying a 2 A current, placed in a perpendicular magnetic field of flux density 6 μT?

Solution

$B = 6\ \mu\text{T}$
$I = 2\ \text{A}$
$l = 0.3\ \text{m}$
$F = 6 \times 10^{-6} \times 2 \times 0.3\ \mu\text{N}$
$= 3.6\ \mu\text{N}$

• **Examiner's hint:** Use the formula $F = B \times I \times l$

Exercises

37 A straight wire of length 0.5 m carries a current of 2 A in a north–south direction. If the wire is placed in a magnetic field of 20 μT directed vertically downwards
(a) what is the size of the force on the wire?
(b) what is the direction of the force on the wire?

38 A vertical wire of length 1m carries a current of 0.5 A upwards. If the wire is placed in a magnetic field of strength 10 μT directed towards the N geographic pole
(a) what is the size of the force on the wire?
(b) what is the direction of the force on the wire?

Charges in magnetic fields

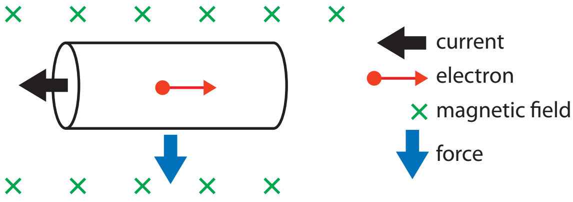


Figure 6.54 The force experienced by each electron is in the downward direction. Remember the electrons flow in the opposite direction to the conventional current.

From the microscopic model of electrical current, we believe that the current is made up of charged particles (electrons) moving through the metal. Each electron experiences a force as it travels through the magnetic field; the sum of all these forces gives the total force on the wire. If a free charge moves through a magnetic field, then it will also experience a force. The direction of the force is always perpendicular to the direction of motion, and this results in a circular path.

The force on each charge q moving with velocity v perpendicular to a field B is given by the formula $F = Bqv$.

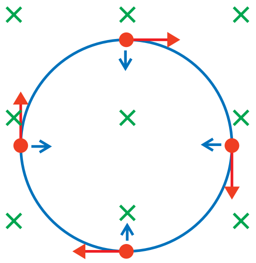


Figure 6.55 Wherever you apply Fleming's left hand rule, the force is always towards the centre.

Hint: Remember – the electron is negative, so current is in opposite direction to electron flow.

6.8 Electromagnetic induction

Assessment statements

12.1.1 Describe the inducing of an emf by relative motion between a conductor and a magnetic field.
12.1.2 Derive the formula for the emf induced in a straight conductor moving in a magnetic field.
12.1.3 Define magnetic flux and magnetic flux linkage.
12.1.4 Describe the production of an induced emf by a time-changing magnetic flux.
12.1.5 State Faraday's law and Lenz's law.
12.1.6 Solve electromagnetic induction problems.

Conductor moving in a magnetic field

We have considered what happens to free charges moving in a magnetic field, but what happens if these charges are contained in a conductor? Figure 6.56 shows a conductor of length L moving with velocity v through a perpendicular field of flux density B. We know from our microscopic model of conduction that conductors contain free electrons. As the free electron shown moves downwards through the field it will experience a force. Using Fleming's left hand rule, we can deduce that the direction of the force is to the left. (Remember, the electron is negative so if it is moving downwards the current is upwards.) This force will cause the free electrons to move to the left as shown in Figure 6.57. We can see that the electrons moving left have caused the lattice atoms on the right to become

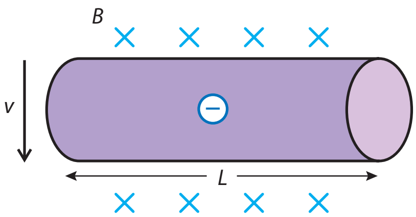
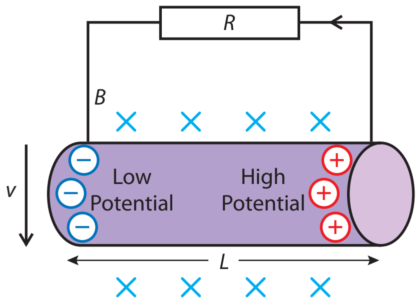


Figure 6.56 A conductor moving through a perpendicular field.

positive, and there is now a potential difference between the ends of the conductor. The electrons will now stop moving because the *B* force pushing them left will be balanced by an *E* force pulling them right.

Induced emf

Figure 6.57 Current flows from high potential to low potential.



If we connect this moving conductor to a resistor, a current would flow, as the moving wire is behaving like a battery. If current flows through a resistor then energy will be released as heat. This energy must have come from the moving wire. When a battery is connected to a resistor, the energy released comes from the chemical energy in the battery. We defined the amount of chemical energy converted to electrical energy per unit charge as the emf. We use the same quantity here but call it the induced emf.

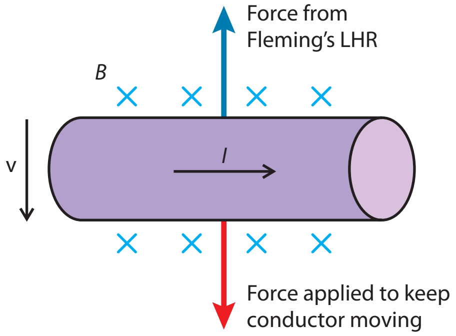
The emf is the amount of mechanical energy converted into electrical energy per unit charge.

The unit of induced emf is the volt.

Conservation of energy

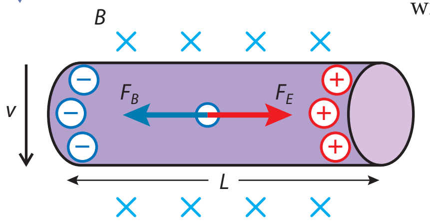
When current flows through the resistor, current will also flow from left to right through the moving conductor. We now have a current carrying conductor in a magnetic field, allowing more electrons to move to the right. There is now a current flowing through the conductor from right to left. We now have a current-carrying conductor in a magnetic field, which, according to Fleming's left hand rule, will experience a force upwards (see Figure 6.58). This means that to keep it moving at constant velocity we must exert a force downwards, which means that we are now doing work. This work increases the electrical PE of the charges; when the charges then flow through the resistor this electrical PE is converted to heat, and energy is conserved.

Figure 6.58 A constant force must be applied to the conductor to keep it moving.



Calculating induced emf

Figure 6.59 The electric and magnetic forces acting on the electron are balanced.



The maximum p.d. achieved across the conductor is when the magnetic force pushing the electrons left equals the electric force pushing them right. When the forces are balanced, no more electrons will move. Figure 6.59 shows an electron with balanced forces.

The second law of thermodynamics

You can see that when the electrons are pushed to the left of the conductor they are becoming more ordered. So that this does not break the second law, there must be some disorder taking place.

If F_B is the magnetic force and F_E is the electric force we can say that

$$F_B = F_E$$

Now we know that if the velocity of the electron is v and the field strength is B then $F_B = Bev$

The electric force is due to the electric field E which we can find from the equation $E = -\frac{dV}{dx}$ that we established in the section on electric potential. In this case, the field is uniform so the potential gradient $= \frac{V}{L}$

So, $$F_E = Ee = \frac{Ve}{L}$$

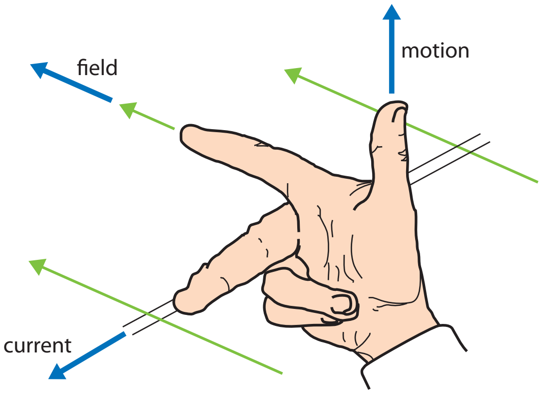
Equating the forces gives $$\frac{Ve}{L} = Bev$$

so $$V = BLv$$

• Hint:

Fleming's right hand rule

The fingers represent the same things as in the left hand rule but it is used to find the direction of induced current if you know the motion of the wire and the field. Try using it on the example in Figure 6.58.



This is the p.d. across the conductor, which is defined as the work done per unit charge, taking a small positive test charge from one side to the other. As current starts to flow in an external circuit, the work done by the pulling force enables charges to move from one end to the other, so the emf (mechanical energy converted to electrical per unit charge) is the same as this p.d.

$$\text{induced emf} = BLv$$

Non-perpendicular field

If the field is not perpendicular to the direction of motion then you take the component of the flux density that is perpendicular. In the example in Figure 6.60, this is $B\sin\theta$.

So emf $= B\sin\theta \times Lv$

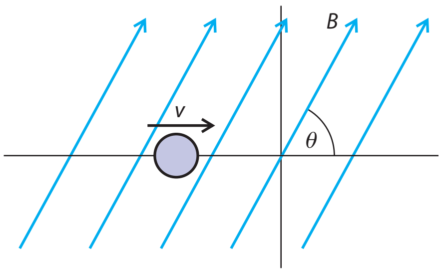


Figure 6.60 Looking along the wire, the component of the flux density that is perpendicular to the direction of motion is $B\sin\theta$.

Exercises

39 A 20 cm long straight wire is travelling at a constant 20 m s^{-1} through a perpendicular B field of flux density 50μT.

(a) Calculate the emf induced.

(b) If this wire were connected to a resistance of 2Ω how much current would flow?

(c) How much energy would be converted to heat in the resistor in 1s? (Power = I^2R)

(d) How much work would be done by the pulling force in 1s?

(e) How far would the wire move in 1s?

(f) What force would be applied to the wire?

Faraday's law

From the moving conductor example we see that the induced emf is dependent on the flux density, speed of movement and length of conductor. These three factors all change the rate at which the conductor cuts through the field lines, so a more convenient way of expressing this is:

The induced emf is equal to the rate of change of flux.

This can be written as $E = \frac{d\Phi}{dt}$

This is Faraday's law and it applies to all examples of induced emf not just wires moving through fields.

Flux and flux density

We can think of flux density as being proportional to the number of field lines per unit area, so flux is proportional to the number of field lines in a given area. If we take the example in Figure 6.61, the flux density of the field shown is B and the flux φ enclosed by the shaded area is BA.

The unit of flux is tesla metre² (Tm²).

The wire in the same diagram will move a distance v in 1s (velocity is displacement per unit time) so the area swept out per unit time = Lv. The flux cut per unit time will therefore be BLv; this is equal to the emf.

If the field is not perpendicular to the area then you use the component of the field that is perpendicular. In the example in Figure 6.62 this would give

$$\varphi = B\cos\theta \times A$$

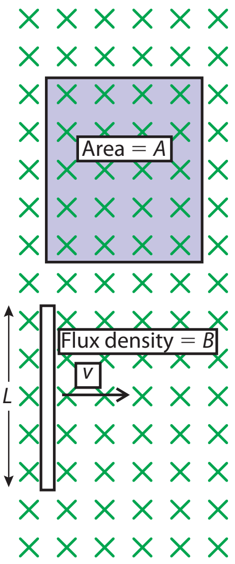


Figure 6.61 A wire is moving through a uniform B field

● **Hint:** If you think of the field lines as grass stems (they've been drawn green to help your imagination) and the conductor as a blade, then the induced emf is proportional to the rate of grass cut. This can be increased by moving the blade more quickly, having a longer blade or moving to somewhere where there is more grass.

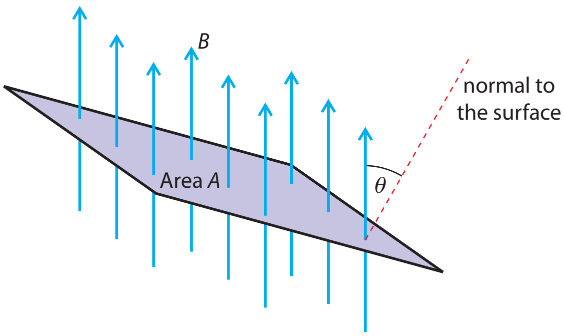


Figure 6.62 Area A not perpendicular to the field.

Lenz's law

We noticed that when a current is induced in a moving conductor, the direction of induced current causes the conductor to experience a force that opposes its motion. To keep the conductor moving will therefore require a force to be exerted in the opposite direction. This is a direct consequence of the law of conservation of energy. If it were not true, you wouldn't have to do work to move the conductor, so the energy given to the circuit would come from nowhere. Lenz's law states this fact in a way that is applicable to all examples:

The direction of the induced current is such that it will oppose the change producing it.

$$E = -\frac{dN\Phi}{dt}$$

Examples

1 **Coil and magnet**

A magnet is moved towards a coil as in Figure 6.63.

Applying Faraday's law:

As the magnet approaches the coil, the B field inside the coil increases, and the changing flux enclosed by the coil induces an emf in the coil that causes a current to flow. The size of the emf will be equal to the rate of change of flux enclosed by the coil.

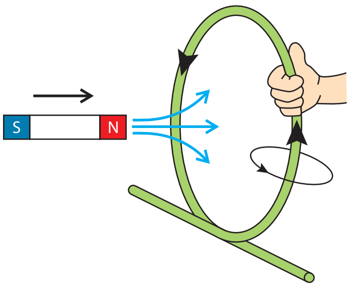


Figure 6.63 To oppose the magnet coming into the coil, the coil's magnetic field must push it out. The direction of the current is found using the grip rule.

Applying Lenz's law:
The direction of induced current will be such that it opposes the change producing it, which in this case is the magnet moving towards the coil. So to oppose this, the current in the coil must induce a magnetic field that pushes the magnet away; this direction is shown in the diagram.

2 **Coil in a changing field**
In Figure 6.64, the magnetic flux enclosed by coil B is changed by switching the current in coil A on and off.

Applying Faraday's law:
When the current in A flows, a magnetic field is created that causes the magnetic flux enclosed by B to increase. This increasing flux induces a current in coil B.

Applying Lenz's law:
The direction of the current in B must oppose the change producing it, which in this case is the increasing field from A.
So to oppose this, the field induced in B must be in the opposite direction to the field from A, as in the diagram. This is the principle behind the operation of a transformer.

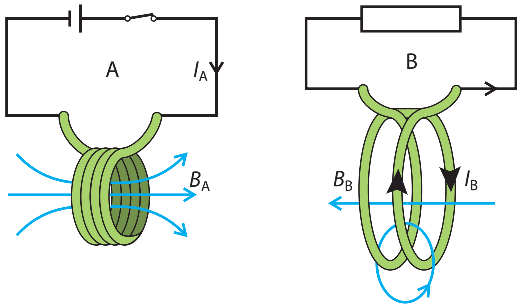


Figure 6.64 Coil A induces current in coil B. Use the grip rule to work out the direction of the fields.

Exercises

40 A coil with 50 turns and area 2 cm^2 encloses a field that is of flux density 100 μT (the field is perpendicular to the plane of the coil).
 (a) What is the total flux enclosed?
 (b) If the flux density changes to 50 μT in 2 s, what is the rate of change of flux?
 (c) What is the induced emf?

41 A rectangular coil with sides 3 cm and 2 cm and 50 turns lies flat on a table in a region of magnetic field. The magnetic field is vertical and has flux density 500 μT.
 (a) What is the total flux enclosed by the coil?
 (b) If one side of the coil is lifted so that the plane of the coil makes an angle of 30° to the table, what will the new flux enclosed be?
 (c) If the coil is lifted in 3 s, estimate the emf induced in the coil.

● **Hint:** If a coil has N turns each turn of the wire encloses the flux. If the flux enclosed by the area of the coil is φ then the total flux enclosed is $N\varphi$.

Applications of electromagnetic induction

Induction braking

Traditional car brakes use friction pads that press against a disc attached to the road wheel. Induction braking systems replace the friction pads with electromagnets. When switched on, a current is induced in the rotating discs. According to Lenz's law the induced current will oppose the change producing it, resulting in a force that slows down the car.

In an induction braking system there are no parts to wear out, which means you would not have to replace brake pads every couple of years. This is bad news for the manufacturers of brake pads. Sometimes advancements in technology are not developed because they do not benefit manufacturers.

Induction cooking

An induction hotplate uses a changing magnetic field to induce an emf in a metal saucepan; the emf causes a current to flow in the saucepan, which produces heat (I^2R). The benefit of this system is that the saucepan (and therefore the food) gets hot but not the oven; you can touch the 'hotplate' without getting burned.

6.9 Alternating current

Assessment statements

12.2.1 Describe the emf induced in a coil rotating within a uniform magnetic field.
12.2.2 Explain the operation of a basic alternating current (AC) generator.
12.2.3 Describe the effect on the induced emf of changing the generator frequency.
12.2.4 Discuss what is meant by the root mean squared (rms) value of an alternating current or voltage.
12.2.5 State the relation between peak and rms values for sinusoidal currents and voltages.
12.2.6 Solve problems using peak and rms values.
12.2.7 Solve AC circuit problems for ohmic resistors.
12.2.8 Describe the operation of an ideal transformer.
12.2.9 Solve problems on the operation of ideal transformers.

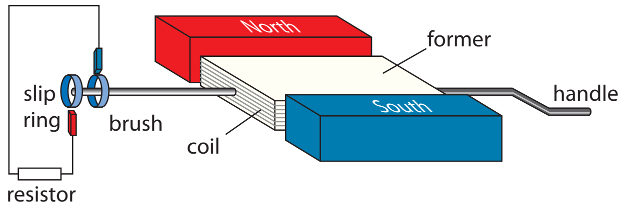
To view a simulation of an electric generator, visit www.heinemann.co.uk/hotlinks, enter the express code 4426P and click on Weblink 6.6.

The most common application of electromagnetic induction is in the production of electrical energy. There are several devices that can be used to do this, such as the dynamo, in which a coil rotates in a magnetic field, or an alternator, in which a magnet rotates in a coil. Here we will consider the simple case of a coil rotating in a uniform magnetic field.

Coil rotating in a uniform magnetic field

Consider the coil shown in Figure 6.65. This coil is being made to rotate in a uniform magnetic field by someone turning the handle. The coil is connected to a resistor, but to prevent the wires connected to the coil twisting, they are connected via two slip rings. Resting on each slip ring is a carbon brush, which makes contact with the ring whilst allowing it to slip past.

Figure 6.65 A simple AC generator.



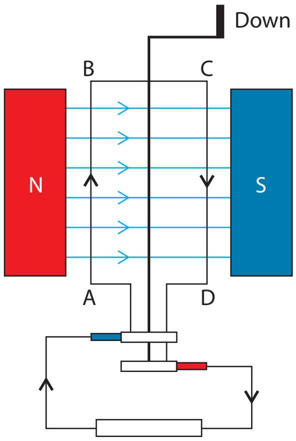


Figure 6.66 Looking at the generator from above.

To make the operation easier to understand, a simpler 2D version with only one loop of wire in the coil is shown in Figure 6.66. As the handle is turned, the wire on the left hand side (AB) moves up through the field. As it cuts the field a current will be induced. Using Fleming's right hand rule, we can deduce that the direction of the current is from A to B as shown. The direction of motion of the right hand side (CD) is opposite so the current is opposite. The result is a clockwise current through the resistor.

After turning half a revolution the coil is in the position shown in Figure 6.67. Side CD is now moving up through the field. Look carefully at how the slip ring has moved and you will see why, although the current is still clockwise in the coil, it is anticlockwise in the resistor circuit.

The size of the emf induced in a rotating coil

To find the size of the emf, we can use Faraday's law. This states that the induced emf will be equal to the rate of change of flux. The flux enclosed by the coil is related to the angle the coil makes with the field. Figure 6.68 shows a coil of N turns at time t. At this moment the normal to the plane of the coil makes an angle θ with the field.

Flux $\varphi = BA\cos\theta$

There are N turns so total flux $\quad N\varphi = BAN\cos\theta \qquad (1)$

If the angular velocity of the coil is ω then the angle $\theta = \omega t$

Substituting equation (1) gives

$$N\varphi = BAN\cos\omega t$$

This equation can be represented graphically as in Figure 6.69.

Note that the graph starts when the flux enclosed is a maximum, so $\cos\theta = 1$ meaning that $\theta = 0°$. Remember that θ is the angle between the field and the normal to the plane of the coil, so position A is when the coil in Figure 6.68 is vertical.

To find the magnitude of the emf, we need the rate of change of flux. We can calculate this from the gradient of this graph. Let us consider some specific points.

At A the gradient is zero.

At B the gradient is big and negative.

At C the gradient is zero.

At D the gradient is big and positive.

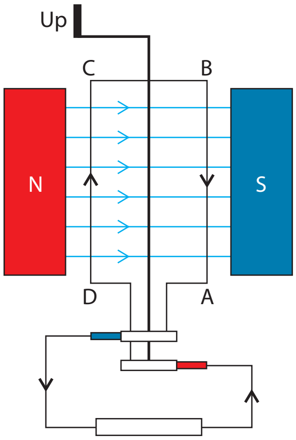


Figure 6.67 The coil after half a revolution.

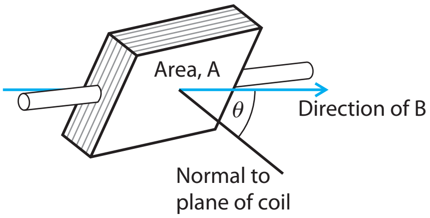


Figure 6.68 Coil at an angle to the field.

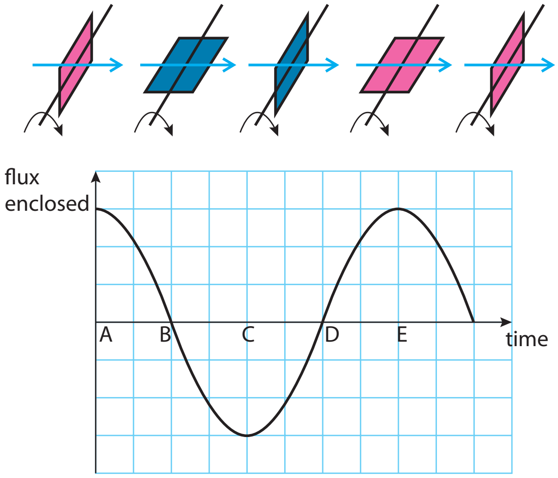
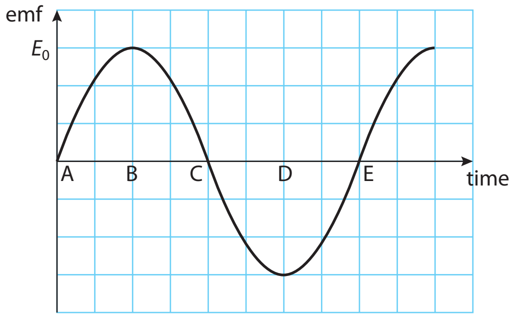


Figure 6.69 Graph of flux against time.

Figure 6.70 Graph of emf against time.



Lenz's law says that the induced current will oppose the change producing it; this means that when the rate change of flux is positive the induced emf is negative. If we plot the rate of change of flux against time we get a graph as in Figure 6.70. The equation of this line is

$$E = BAN\omega\sin\omega t$$

Where $BAN\omega$ is the peak value E_0

Note that this is a maximum when the flux enclosed is zero, that is when the coil in the position shown in Figure 6.65.

Effect of increasing angular speed

If the speed of rotation is increased, the graph of emf against time will change in two ways, as shown in Figure 6.71. Firstly, time between the peaks will be shorter, and secondly, the peaks will be higher. This is because if the coil moves faster, then the rate of change of flux will be higher and hence the emf will be greater.

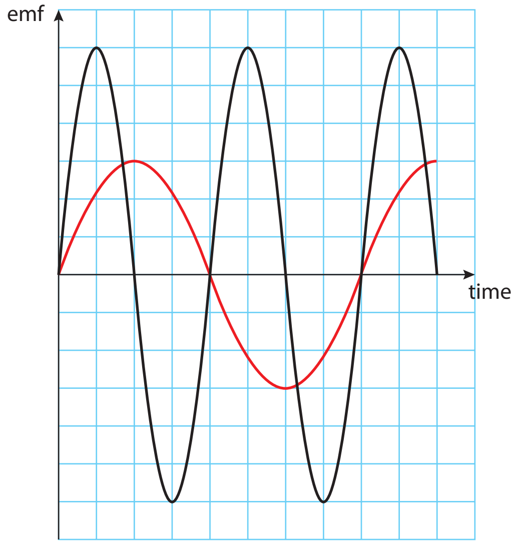


Figure 6.71 The black graph is for a coil with twice the angular speed of the red one.

Faraday's law

If you have studied differentiation in maths you will understand that Faraday's law can be written

$$\epsilon = \frac{-dN\phi}{dt}$$

Then if $N\varphi = BAN \cos \omega t$

$$\frac{-dN\phi}{dt} = BAN\omega \sin \omega t$$

Alternating current

The current delivered by the rotating coil changes in direction and size over a period of time. This is called *alternating current* (AC). A battery, however, gives a constant current called *direct current* (DC). When doing power calculations with AC (for example, calculating the heat given out per second by an electric heater), the peak value would give a result that was too big since the peak value is only attained for a very short time. In these cases we should use the root mean square value; this is a sort of average value.

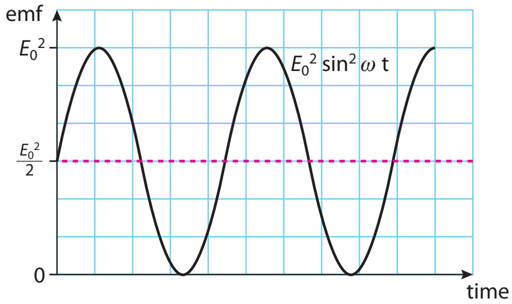
Root mean square

The root mean square or rms is the square root of the mean of the squares. Since the emf from the rotating coil varies sinusoidally then the rms emf and current will be the same as the root of the mean of the squares of the sine function.

To calculate the rms value, first we must square the function; this gives the curve shown in Figure 6.72. If we consider one complete cycle of this function we see that the mean value is $\frac{1}{2}E_0^2$. The rms value is then the square root of this.

$$E_{rms} = \sqrt{\frac{E_0^2}{2}} = \frac{E_0}{\sqrt{2}}$$

Figure 6.72



The current passing through a resistor will be proportional to the potential difference across it, so this will also be sinusoidal. If the peak current is I_0 the rms value will therefore be given by:

$$I_{rms} = \frac{I_0}{\sqrt{2}}$$

The rms values are what you need to know for power calculations, so it is these that are normally quoted. For example, the rms mains voltage in Europe = 220 V.

Power in AC circuits

If the rms current flowing through a resistor is I_{rms} and the rms potential difference across it is V_{rms} then the power dissipated in it will be $I_{rms} \times V_{rms}$.

Worked example

What will be the rms current flowing through a 100 W light bulb connected to a 220 V AC supply?

Solution

Power $= I_{rms}V_{rms}$

So $I_{rms} = \dfrac{\text{Power}}{V_{rms}} = \dfrac{100}{220} = 0.45\,\text{A}$

The electrical energy produced by a generator comes from the work done by the person turning the coil. The more current you take from the coil the harder it is to turn. This follows from Lenz's law; the current in the coil opposes the change producing it. If you don't draw any current then it's very easy to turn the coil.

Exercises

42 The rms voltage in the USA is 110 V. Calculate the peak voltage.

43 An electric oven designed to operate at 220 V has a power rating of 4 kW. What current flows through it when it is switched on?

44 A coil similar to the one in Figure 6.65 has an area of 5 cm^2 and rotates 50 times a second in a field of flux density 50 mT.

(a) If the coil has 500 turns, calculate

(i) the angular velocity, ω

(ii) the maximum induced emf

(iii) the rms emf.

(b) If the speed is reduced to 25 revolutions per second what is the new E_{rms} ?

45 Calculate the resistance of a 1000 W light bulb designed to operate at 220 V.

The transformer

A transformer consists of two coils wound on a co mmon soft iron core as shown in Figure 6.73. The primary coil is connected to an AC supply. This causes a changing magnetic field inside the coil. This field is made stronger by the presence of the soft iron core which itself becomes magnetic (temporarily). Since the secondary is wound around the same former, it will have a changing magnetic field within it, which induces an emf in its coils.

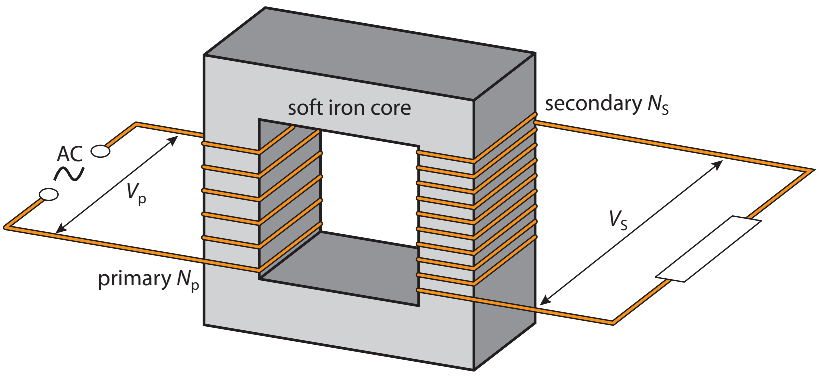


Figure 6.73 A simple transformer.

The emf induced in the secondary is directly related to the number of turns. If the supply is sinusoidal then the ratio of turns in the two coils equals the ratio of the p.d.s.

$$\frac{N_p}{N_S} = \frac{V_p}{V_S}$$

Power losses

Real transformers are not 100% efficient; power is lost in several ways:

- Heat in the wire of the coils
- Heat in the core due to induced currents in the soft iron.
- Flux leakage.
- The core retains some magnetism when the current changes.

So a transformer can make a p.d. higher or lower depending on the ratio of turns on the primary and secondary. However, since energy must be conserved, the power out cannot be bigger than the power in. Electrical power is given by $I \times V$ so if the p.d. goes up, the current must come down. An ideal transformer has an efficiency of 100%. This means that power in = power out.

$$V_p I_p = V_S I_S$$

where V_P, V_S, I_P and I_S are the rms values of p.d. and current in the primary and secondary.

Warning

The emf induced in the secondary depends on the rate of change of current in the primary. If you switch the current off then the change can be very big inducing a much bigger emf than you might have calculated.

Exercises

46 An ideal transformer steps down the 220 V mains to 4.5 V so it can be used to charge a mobile phone.

(a) If the primary has 500 turns how many turns does the secondary have?

(b) If the charger delivers 0.45 A to the phone, how much power does it deliver?

(c) How much current flows into the charger from the mains?

(d) The phone is unplugged from the charger but the charger is left plugged in. How much current flows into the charger now?

6.10 Transmission of electrical power

Assessment statements

12.3.1 Outline the reasons for power losses in transmission lines and real transformers.

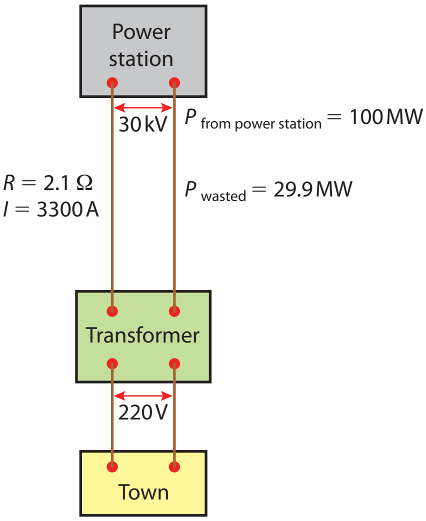
12.3.2 Explain the use of high-voltage step-up and step-down transformers in the transmission of electrical power.

12.3.3 Solve problems on the operation of real transformers and power transmission.

12.3.4 Suggest how extra-low-frequency electromagnetic fields, such as those created by electrical appliances and power lines, induce currents within a human body.

12.3.5 Discuss some of the possible risks involved in living and working near high-voltage power lines.

Figure 6.74



As outlined previously, electrical energy can be produced by moving a coil in a magnetic field. The energy required to rotate the coil can come from many sources, for example: coal, oil, falling water, sunlight, waves in the sea or nuclear fuel. The transformation of energy takes place in power stations that are often not sited close to the places where people live. For that reason, the electrical energy must be delivered via cables. These cables have resistance so some energy will be lost as the current flows through them. Let's take an example as illustrated in Figure 6.74.

The generators at a power station typically produce 100 MW of power at 30 kV. A small town 100 km from the power station requires electrical power which will be delivered via two aluminium cables (one there and one back) that have a radius of 2 cm. The houses in the town cannot use such high voltage electricity so the p.d. must be stepped down to 220 V, using a transformer that we will assume is 100% efficient.

The first thing to do is to calculate the resistance of the cables. We can do this using the formula

$$R = \frac{\rho l}{A}$$

where ρ is the resistivity of aluminium ($2.6 \times 10^{-8}\,\Omega$m), A is the cross-sectional area and l is the length.

So $$R = \frac{2.6 \times 10^{-8} \times 200 \times 10^3}{2\pi \times (2 \times 10^{-2})^2} = 2.1\,\Omega$$

Hint: 30 kV is the p.d. between the two wires not the p.d. from the power station to the transformer.

The current that must be delivered if 100 MW is produced at a p.d. of 30 kV can be found using $P = IV$

Rearranging gives $$I = \frac{1 \times 10^8}{30 \times 10^3} = 3.3 \times 10^3\,\text{A}$$

The power loss in the cables is therefore $I^2R = (3.3 \times 10^3)^2 \times 2.1 = 22.9$ MW

This is a lot of wasted power.

To reduce the power loss in the cables we must reduce the current. This can be done by stepping up the voltage before transmission, using a transformer as shown in the next example (Figure 6.75).

The p.d. between the wires is typically stepped up to 115 kV. Let us now repeat the calculation, assuming all transformers are 100% efficient.

The power is now delivered at 115 kV so the current is

$$I = \frac{P}{V} = \frac{100 \times 10^6}{115 \times 10^3} = 870\,\text{A}$$

Power loss in the cables $= I^2R = 870^2 \times 2.1 = 1.6$ MW

This is still quite a lot of wasted power but much less than before.

To reduce this further, more cables could be added in parallel, thereby reducing the resistance.

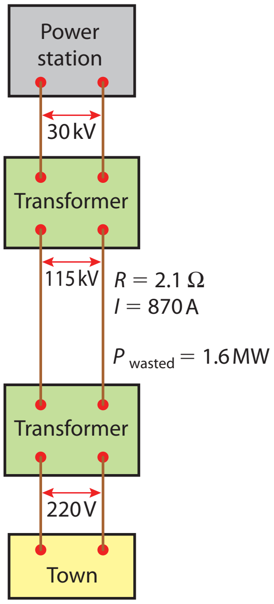


Figure 6.75

Health risks and power lines

Power lines carry large alternating currents at high potentials which produce electromagnetic fields radiating from them. The changing electric and magnetic field will induce currents in any conductors placed nearby. Your body is a conductor, so if you stand near a power line small currents will be induced in your body – is this harmful?

Unless you almost sit on the cable, these fields are very small. Typically the magnetic field you'd experience is less than the Earth's magnetic field. The frequency of the change is also very low so cannot affect the atoms of your body. It therefore seems very unlikely that any harm could be caused by these fields. In the 1970s, a study of the incidence of childhood leukaemia showed that there was a higher incidence of leukaemia in children living close to power lines. This caused a lot of concern at the time, but the study was seriously flawed. One of the main pieces of evidence against this theory is that even though the number of children living near power lines has increased significantly over the past 30 years, the incidence of leukaemia has gone down.

Power lines and leukaemia

This is a good example of how people can be influenced by the media. Even though there is no scientific evidence that power lines adversely affect one's health, people still believe what they heard on the TV 30 years ago. There are also a lot of companies marketing products that profit from this misconception.

Exercises

47 A power station that generates electricity at 50 kV produces 500 MW of power. This is delivered to a town through cables with a total resistance of 8 Ω. Before transmission, the p.d. is stepped up to 100 kV, then stepped down to 220 V at the town.

(a) How much current will flow through the cables that take the electricity away from the power station to the town?

(b) How much power is lost in the cables?

(c) What percentage of total power delivered is lost?

(d) How much power will be delivered to the town? (Assume both transformers are 100% efficient.)

(e) How much power will be available for the town to use?

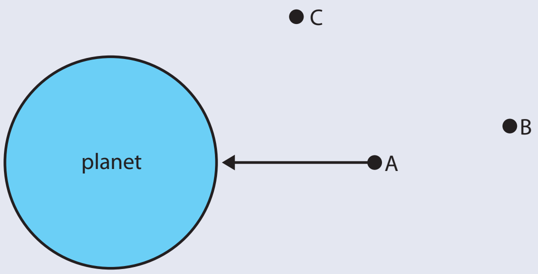
(f) How much total current will flow through the town?

Practice questions

1 This question is about gravitation and orbital motion.

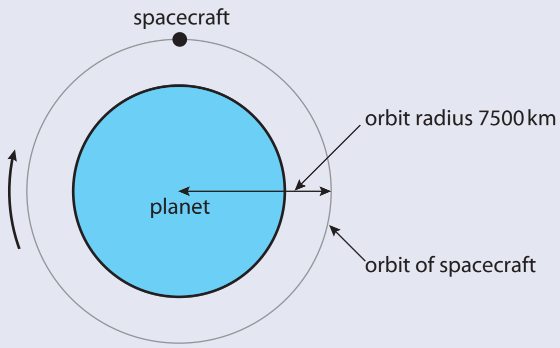
(a) Define gravitational field strength at a point in a gravitational field. (2)

The diagram below shows three points above a planet. The arrow represents the gravitational field strength at point A.



(b) Draw arrows to represent the gravitational field strength at point B and point C. (2)

A spacecraft is in a circular orbit around the planet as shown in the diagram below. The radius of the orbit is 7500 km.



(c) For the spacecraft in the position shown, draw and label arrows representing

(i) the velocity (label this arrow V). (1)

(ii) the acceleration (label this arrow A). (1)

The speed of the spacecraft is 6.5 km s^{-1}.

(d) Deduce the value of the magnitude of the gravitational field strength at a point in the spacecraft's orbit. (3)

2 This question is about gravitation and ocean tides.

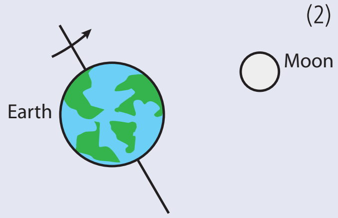
(a) State Newton's law of universal gravitation. (2)

(b) Use the following information to deduce that the gravitational field strength at the surface of the Earth is approximately 10 N kg^{-1}.

Mass of the Earth $= 6.0 \times 10^{24}$ kg

Radius of the Earth $= 6400$ km (2)

The Moon's gravitational field affects the gravitational field at the surface of the Earth. A high tide occurs at the point where the resultant gravitational field due to the Moon and to the Earth is a minimum.



(c) **(i)** On the diagram, label, using the letter P, the point on the Earth's surface that experiences the greatest gravitational attraction due to the Moon. Explain your answer. (2)

(ii) On the diagram label, using the letter H, the location of a high tide. Explain your answer. (2)

(iii) Suggest **two** reasons why high tides occur at different times of the day in different locations. (2)

3 This question is about gravitational fields.

(a) Define *gravitational field strength*. (2)

The gravitational field strength at the surface of Jupiter is 25 N kg^{-1} and the radius of Jupiter is 7.1×10^{7} m.

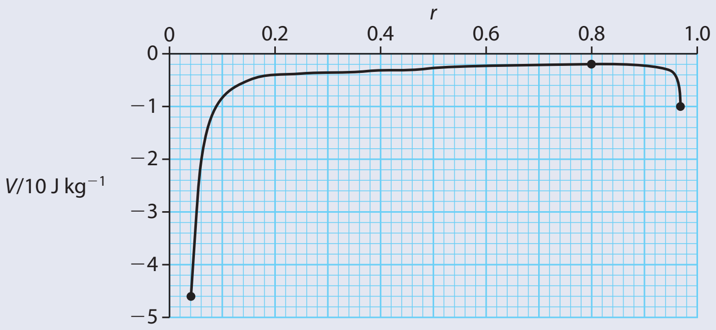
(b) **(i)** Derive an expression for the gravitational field strength at the surface of a planet in terms of its mass M, its radius R and the gravitational constant G. (2)

(ii) Use your expression in **(b) (i)** above to estimate the mass of Jupiter. (2)

4 This question is about gravitation.

(a) Define *gravitational potential* at a point. (2)

(b) The diagram below shows the variation of gravitational potential V of a planet and its moon with distance r from the centre of the planet. The unit of separation is arbitrary. The centre of the planet corresponds to $r = 0$ and the centre of the moon to $r = 1$. The curve starts at the surface of the planet and ends at the surface of the moon.



(i) At the position where $r = 0.8$, the gravitational field strength is zero. Determine the ratio

$$\frac{\text{mass of planet}}{\text{mass of moon}}$$ (3)

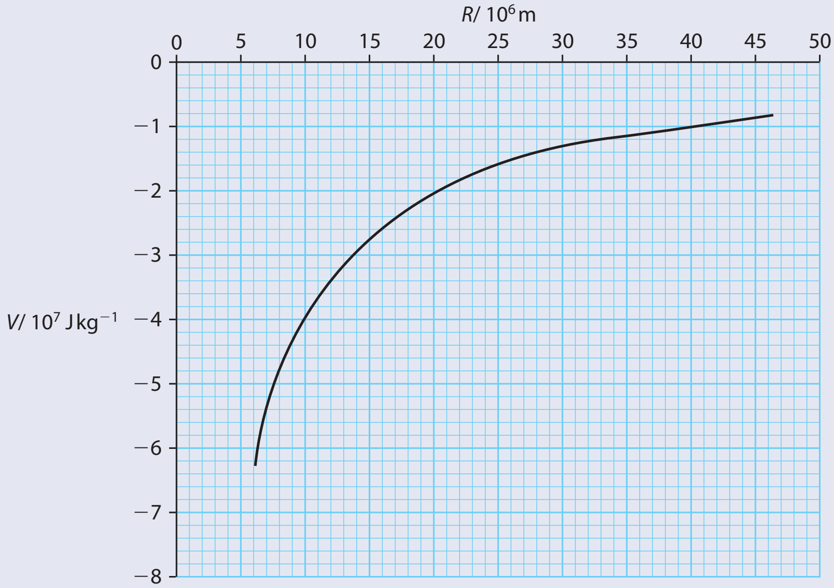
(ii) A satellite of mass 1500 kg is launched from the surface of the planet. Determine the **minimum** kinetic energy at launch the satellite must have so that it can reach the surface of the moon. (3)

(Total 8 marks)

5 This question is about gravitational potential energy.

The graph below shows the variation of gravitational potential V due to the Earth with distance R from the centre of the Earth. The radius of the Earth is 6.4×10^6 m.

The graph does not show the variation of potential V within the Earth.



(a) Use the graph to find the gravitational potential

(i) at the surface of the Earth. (1)

(ii) at a height of 3.6×10^7 m above the surface of the Earth. (2)

(b) Use the values you have found in part **(a)** to determine the minimum energy required to put a satellite of mass 1.0×10^4 kg into an orbit at a height of 3.6×10^7 m above the surface of the Earth. (3)

(c) Give **two** reasons why more energy is required to put this satellite into orbit than that calculated in **(b)** above.

(Total 8 marks)

6 This question is about the possibility of generating electrical power using a satellite orbiting the Earth.

(a) Define *gravitational field strength*. (2)

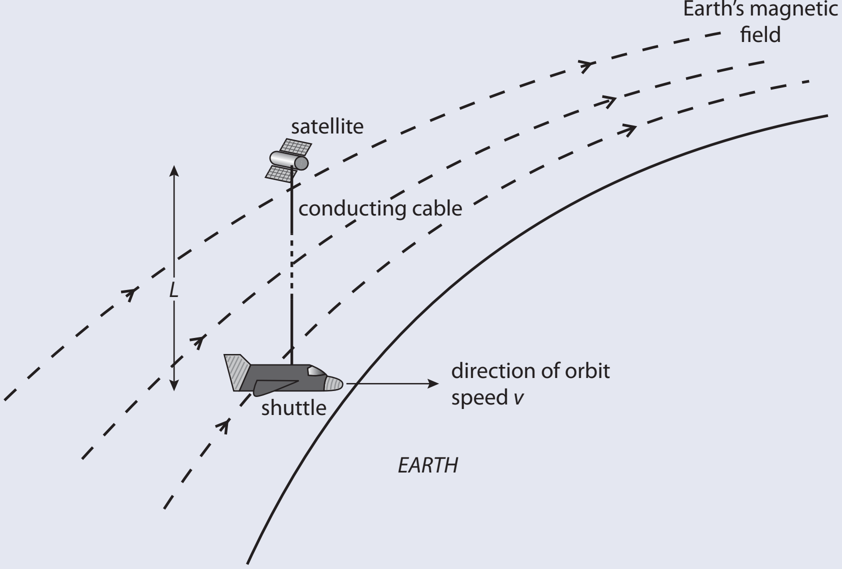
(b) Use the definition of gravitational field strength to deduce that

$$GM = g_0 R^2$$

where M is the mass of the Earth, R its radius and g_0 is the gravitational field strength at the surface of the Earth. (You may assume that the Earth is a uniform sphere with its mass concentrated at its centre.) (2)

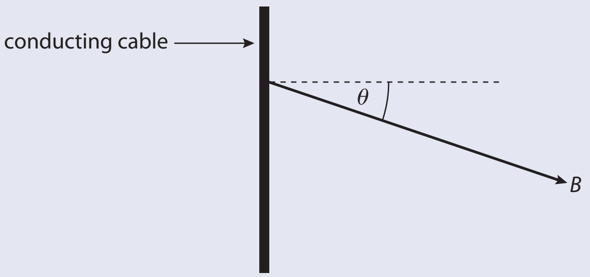
A space shuttle orbits the Earth and a small satellite is launched from the shuttle. The satellite carries a conducting cable connecting the satellite to the shuttle. When the satellite is a distance L from the shuttle, the cable is held straight by motors on the satellite.

Diagram 1



As the shuttle orbits the Earth with speed v, the conducting cable is moving at right angles to the Earth's magnetic field. The magnetic field vector $\boldsymbol{B}$ makes an angle θ to a line perpendicular to the conducting cable as shown in diagram 2. The velocity vector of the shuttle is directed out of the plane of the paper.

Diagram 2



(c) On diagram 2, draw an arrow to show the direction of the magnetic force on an electron in the conducting cable. Label the arrow F. (1)

(d) State an expression for the force F on the electron in terms of B, v, e and θ, where B is the magnitude of the magnetic field strength and e is the electron charge. (1)

(e) Hence deduce an expression for the emf E induced in the conducting wire. (3)

(f) The shuttle is in an orbit that is 300 km above the surface of the Earth. Using the expression

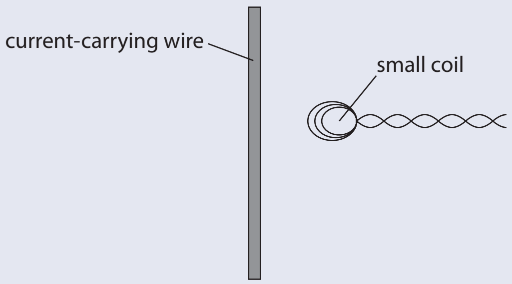
$$GM = g_0R^2$$

and given that $R = 6.4 \times 10^6$ m and $g_0 = 10\ \mathrm{N\,kg^{-1}}$, deduce that the orbital speed v of the satellite is $7.8 \times 10^3\ \mathrm{m\,s^{-1}}$. (3)

(g) The magnitude of the magnetic field strength is 6.3×10^{-6} T and the angle $\theta = 20°$. Estimate the length L of the cable required in order to generate an emf of 1 kV. (2)

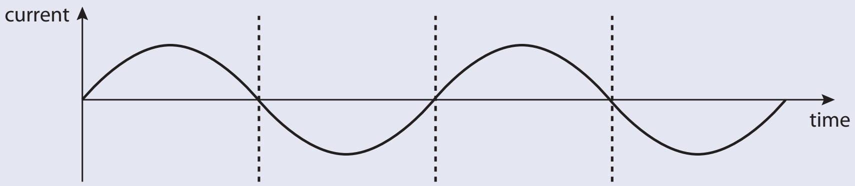
(Total 14 marks)

7 This question is about electromagnetic induction.
A small coil is placed with its plane parallel to a long straight current-carrying wire, as shown below.



(a) **(i)** State Faraday's law of electromagnetic induction. (2)
(ii) Use the law to explain why, when the current in the wire changes, an emf is induced in the coil. (1)

The diagram below shows the variation with time t of the current in the wire.



(b) **(i)** Draw, on the axes provided, a sketch-graph to show the variation with time t of the magnetic flux in the coil. (1)
(ii) Construct, on the axes provided, a sketch-graph to show the variation with time t of the emf induced in the coil. (2)
(iii) State and explain the effect on the maximum emf induced in the coil when the coil is further away from the wire. (2)
(c) Such a coil may be used to measure large alternating currents in a high-voltage cable. Identify **one** advantage and **one** disadvantage of this method. (2)
(Total 10 marks)
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8 A resistor is connected in series with an alternating current supply of negligible internal resistance. The **peak value** of the supply voltage is V_0 and the **peak value** of the current in the resistor is I_0. The **average power** dissipation in the resistor is

A $\frac{V_0 I_0}{2}$ B $\frac{V_0 I_0}{\sqrt{2}}$

C $V_0 I_0$ D $2V_0 I_0$ (1)

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9 The rms voltages across the primary and secondary coils in an ideal transformer are V_p and V_s respectively. The currents in the primary and secondary coils are I_p and I_s respectively.
Which **one** of the following statements is always true?

A $V_s = V_p$ B $I_s = I_p$

C $V_s I_s = V_p I_p$ D $\frac{V_s}{V_p} = \frac{I_s}{I_p}$ (1)

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