6 Fields and forces

6.1 Gravitational force and field

Assessment statements

6.1.1 State Newton's universal law of gravitation.

6.1.2 Define *gravitational field strength*.

6.1.3 Determine the gravitational field due to one or more point masses.

6.1.4 Derive an expression for gravitational field strength at the surface of a planet, assuming that all its mass is concentrated at its centre.

6.1.5 Solve problems involving gravitational forces and fields.

Gravitational force and field

We have all seen how an object falls to the ground when released. Newton was certainly not the first person to realize that an apple falls to the ground when dropped from a tree. However, he did recognize that the force that pulls the apple to the ground is the same as the force that holds the Earth in its orbit around the Sun; this was not obvious - after all, the apple moves in a straight line and the Earth moves in a circle. In this chapter we will see how these forces are connected.

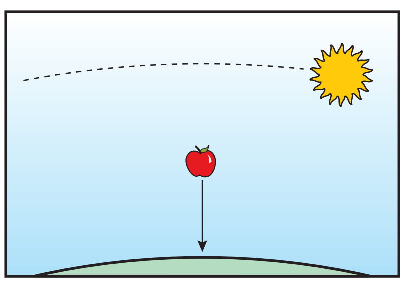


Figure 6.1 The apple drops and the Sun seems to move in a circle, but it is gravity that makes both things happen.

Newton's universal law of gravitation

Newton extended his ideas further to say that every single particle of mass in the universe exerts a force on every other particle of mass. In other words, everything in the universe is attracted to everything else. So there is a force between the end of your nose and a lump of rock on the Moon.

The modern equivalent of the apparatus used by Cavendish to measure *G* in 1798.

Was it reasonable for Newton to think that his law applied to the whole universe?

Newton's universal law of gravitation states that:

> every single point mass attracts every other point mass with a force that is directly proportional to the product of their masses and inversely proportional to the square of their separation.

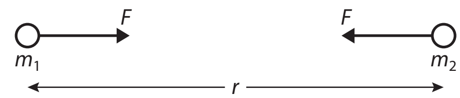


Figure 6.2 The gravitational force between two point masses.

If two point masses with mass m_1 and m_2 are separated by a distance r then the force, F, experienced by each will be given by:

$$F \propto \frac{m_1 m_2}{r^2}$$

The constant of proportionality is the universal gravitational constant G.

$$G = 6.6742 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$$

Therefore the equation is $$F = G\frac{m_1 m_2}{r^2}$$

Spheres of mass

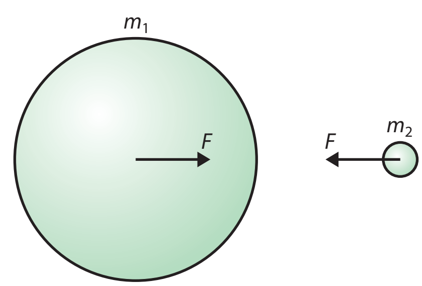


Figure 6.3 Forces between two spheres. Even though these bodies don't have the same mass, the force on them is the same size. This is due to Newton's third law – if mass m_1 exerts a force on mass m_2 then m_2 will exert an equal and opposite force on m_1.

By working out the total force between every particle of one sphere and every particle of another, Newton deduced that spheres of mass follow the same law, where the separation is the separation between their centres. Every object has a centre of mass where the gravity can be taken to act. In regularly-shaped bodies, this is the centre of the object.

How fast does the apple drop?

If we apply Newton's universal law to the apple on the surface of the Earth, we find that it will experience a force given by

$$F = G\frac{m_1 m_2}{r^2}$$

where:

m_1 = mass of the Earth = 5.97×10^{24} kg

m_2 = mass of the apple = 250 g

r = radius of the Earth = 6378 km (at the equator)

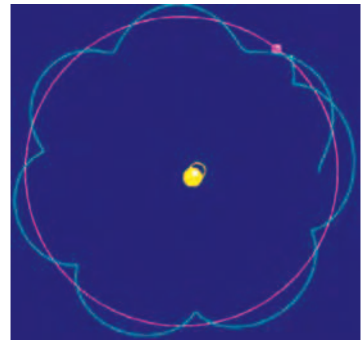
So $F = 2.43$ N

From Newton's 2nd law we know that $F = ma$.

So the acceleration (a) of the apple $= \frac{2.43}{0.25}\,\mathrm{m\,s^{-2}}$

$a = 9.79\,\mathrm{m\,s^{-2}}$

This is very close to the average value for the acceleration of free fall on the Earth's surface. It is not exactly the same, since 9.82 m s^{-2} is an average for the whole Earth, the radius of the Earth being maximum at the equator.



Exercise

1 The mass of the Moon is 7.35×10^{22} kg and the radius 1.74×10^{3} km. What is the acceleration due to gravity on the Moon's surface?

How often does the Earth go around the Sun?

To build your own solar system with the 'solar system' simulation from PhET, visit www.heinemann.co.uk/hotlinks, enter the express code 4426P and click on Weblink 6.1.

Applying Newton's universal law, we find that the force experienced by the Earth is given by:

$$F = G\frac{m_1 m_2}{r^2}$$

where

m_1 = mass of the Sun = 1.99×10^{30} kg

m_2 = mass of the Earth = 5.97×10^{24} kg

r = distance between the Sun and Earth = 1.49×10^{11} m

So $F = 3.56 \times 10^{22}$ N

The planets orbit the Sun.

We know that the Earth travels in an elliptical orbit around the Sun, but we can take this to be a circular orbit for the purposes of this calculation. From our knowledge of circular motion we know that the force acting on the Earth towards the centre of the circle is the centripetal force given by the equation $F = \frac{mv^2}{r}$

Here the law is used to make predictions that can be tested by experiment.

So the velocity $v = \sqrt{\frac{Fr}{m}}$

$= 29\,846$ m s^{-1}

The circumference of the orbit = $2\pi r = 9.38 \times 10^{11}$ m

Time taken for 1 orbit = $\frac{9.38 \times 10^{11}}{29\,846}$

$= 3.14 \times 10^{7}$ s

This is equal to 1 year.

This agrees with observation. Newton's law has therefore predicted two correct results.

Gravitational field

The fact that both the apple and the Earth experience a force without being in contact makes gravity a bit different from the other forces we have come across. To model this situation, we introduce the idea of a *field*. A field is simply a region of space where something is to be found. A potato field, for example, is a region where you find potatoes. A gravitational field is a region where you find gravity. More precisely, gravitational field is defined as a region of space where a mass experiences a force because of its mass.

So there is a gravitational field in your classroom since masses experience a force in it.

Field strength on the Earth's surface:
Substituting M = mass of the Earth $= 5.97 \times 10^{24}$ kg
r = radius of the Earth = 6367 km
gives $g = Gm_1M/r^2$
$= 9.82\ \text{N kg}^{-1}$
This is the same as the acceleration due to gravity, which is what you might expect, since Newton's 2nd law says $a = F/m$.

Gravitational field strength (*g*)

This gives a measure of how much force a body will experience in the field. It is defined as the force per unit mass experienced by a small test mass placed in the field.

So if a test mass, m, experiences a force F at some point in space, then the field strength, g, at that point is given by $g = \frac{F}{m}$.

g is measured in N kg^{-1}, and is a vector quantity.

Note: The reason a small test mass is used is because a big mass might change the field that you are trying to measure.

Gravitational field around a spherical object

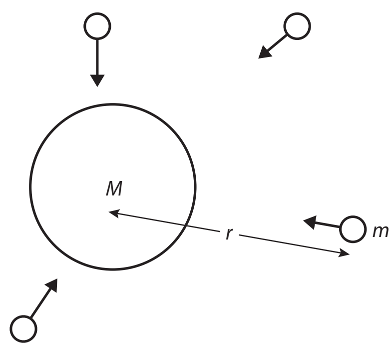


Figure 6.4 The region surrounding M is a gravitational field since all the test masses experience a force.

The force experienced by the mass, m is given by;

$$F = G\frac{Mm}{r^2}$$

So the field strength at this point in space, $g = \frac{F}{m}$

So
$$g = G\frac{M}{r^2}$$

Exercises

2 The mass of Jupiter is 1.89×10^{27} kg and the radius 71 492 km.
What is the gravitational field strength on the surface of Jupiter?

3 What is the gravitational field strength at a distance of 1000 km from the surface of the Earth?

Field lines

Field lines are drawn in the direction that a mass would accelerate if placed in the field – they are used to help us visualize the field.

The field lines for a spherical mass are shown in Figure 6.5.

The arrows give the direction of the field.

The field strength (g) is given by the density of the lines.

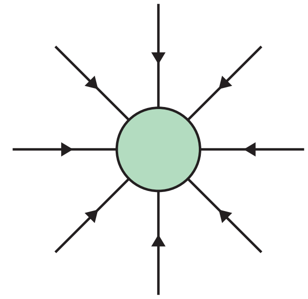


Figure 6.5 Field lines for a sphere of mass.

Gravitational field close to the Earth

When we are doing experiments close to the Earth, in the classroom for example, we assume that the gravitational field is uniform. This means that wherever you put a mass in the classroom it is always pulled downwards with the same force. We say that the field is *uniform*.

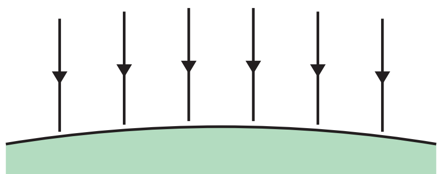


Figure 6.6 Regularly spaced parallel field lines imply that the field is uniform.

Addition of field

Since field strength is a vector, when we add field strengths caused by several bodies, we must remember to add them vectorially.

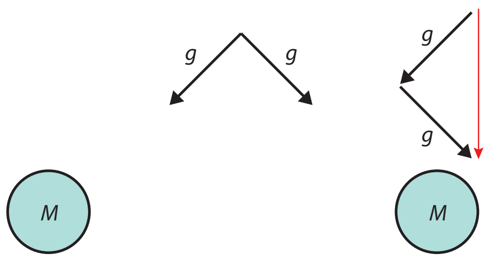


Figure 6.7 Vector addition of field strength.

In this example, the angle between the vectors is 90°. This means that we can use Pythagoras to find the resultant.

$$g = \sqrt{g_1{}^2 + g_2{}^2}$$

Worked example

Calculate the gravitational field strength at points A and B in Figure 6.8.

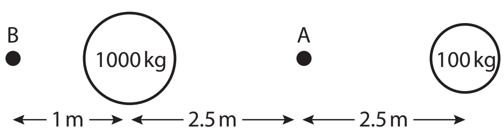


Figure 6.8

Solution

The gravitational field strength at A is equal to the sum of the field due to the two masses.

Field strength due to large mass $= G \times 1000/2.5^2 = 1.07 \times 10^{-8}\,\mathrm{N\,kg^{-1}}$

Field strength due to small mass $= G \times 100/2.5^2 = 1.07 \times 10^{-9}\,\mathrm{N\,kg^{-1}}$

Field strength $= 1.07 \times 10^{-8} - 1.07 \times 10^{-9}$
$= 9.63 \times 10^{-9}\,\mathrm{N\,kg^{-1}}$

● **Examiner's hint:** Since field strength g is a vector, the resultant field strength equals the vector sum.

Exercises

4 Calculate the gravitational field strength at point B.

5 Calculate the gravitational field strength at A if the big mass were changed for a 100 kg mass.