The analysis of motion we have been discussing in this Chapter is basically algebraic. It is sometimes helpful to use a graphical interpretation as well; see the optional Section 2–8.

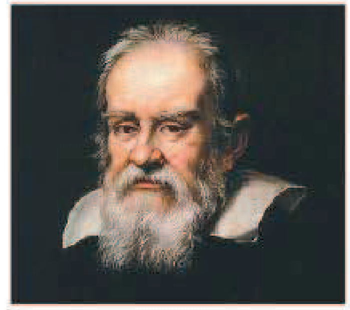


FIGURE 2–17 Galileo Galilei (1564–1642).

2–7 Falling Objects

One of the most common examples of uniformly accelerated motion is that of an object allowed to fall freely near the Earth's surface. That a falling object is accelerating may not be obvious at first. And beware of thinking, as was widely believed until the time of Galileo (Fig. 2–17), that heavier objects fall faster than lighter objects and that the speed of fall is proportional to how heavy the object is.

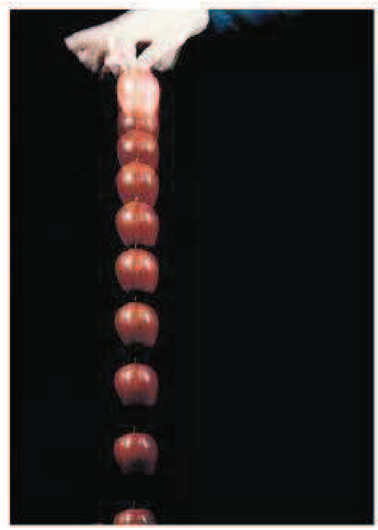
Galileo's analysis of falling objects made use of his new and creative technique of imagining what would happen in idealized (simplified) cases. For free fall, he postulated that all objects would fall with the *same constant acceleration* in the absence of air or other resistance. He showed that this postulate predicts that for an object falling from rest, the distance traveled will be proportional to the square of the time (Fig. 2–18); that is, $d \propto t^2$. We can see this from Eq. 2–11b, but Galileo was the first to derive this mathematical relation. [Among Galileo's great contributions to science was to establish such mathematical relations, and to insist on specific experimental consequences that could be quantitatively checked, such as $d \propto t^2$.]

CAUTION

The speed of a falling object is NOT proportional to its mass or weight

To support his claim that falling objects increase in speed as they fall, Galileo made use of a clever argument: a heavy stone dropped from a height of 2 m will drive a stake into the ground much further than will the same stone dropped from a height of only 0.2 m. Clearly, the stone must be moving faster in the former case.

FIGURE 2–18 Multiflash photograph of a falling apple, at equal time intervals. The apple falls farther during each successive interval, which means it is accelerating.



As we saw, Galileo also claimed that *all* objects, light or heavy, fall with the *same* acceleration, at least in the absence of air. If you hold a piece of paper horizontally in one hand and a heavier object—say, a baseball—in the other, and release them at the same time as in Fig. 2–19a, the heavier object will reach the ground first. But if you repeat the experiment, this time crumpling the paper into a small wad (see Fig. 2–19b), you will find that the two objects reach the floor at nearly the same time.

Galileo was sure that air acts as a resistance to very light objects that have a large surface area. But in many circumstances this air resistance is negligible. In a chamber from which the air has been removed, even light objects like a feather or a horizontally held piece of paper will fall with the same acceleration as any other object (see Fig. 2–20). Such a demonstration in vacuum was not possible in Galileo's time, which makes Galileo's achievement all the greater. Galileo is often called the "father of modern science," not only for the content of his science (astronomical discoveries, inertia, free fall), but also for his style or approach to science (idealization and simplification, mathematization of theory, theories that have testable consequences, experiments to test theoretical predictions).

(a) (b)

FIGURE 2–19 (a) A ball and a light piece of paper are dropped at the same time. (b) Repeated, with the paper wadded up.

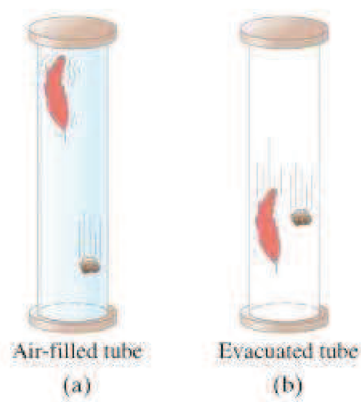


FIGURE 2–20 A rock and a feather are dropped simultaneously (a) in air, (b) in a vacuum.

Galileo's specific contribution to our understanding of the motion of falling objects can be summarized as follows:

Galileo's hypothesis: free fall is at constant acceleration g

at a given location on the Earth and in the absence of air resistance, all objects fall with the same constant acceleration.

We call this acceleration the **acceleration due to gravity** on the Earth, and we give it the symbol g. Its magnitude is approximately

Acceleration due to gravity

$$g = 9.80\ \text{m/s}^2. \qquad \text{[at surface of Earth]}$$

In British units g is about $32\ \text{ft/s}^2$. Actually, g varies slightly according to latitude and elevation, but these variations are so small that we will ignore them for most purposes. The effects of air resistance are often small, and we will neglect them for the most part. However, air resistance will be noticeable even on a reasonably heavy object if the velocity becomes large.[†] Acceleration due to gravity is a vector, as is any acceleration, and its direction is toward the center of the Earth.

When dealing with freely falling objects we can make use of Eqs. 2–11, where for a we use the value of g given above. Also, since the motion is vertical we will substitute y in place of x, and y_0 in place of x_0. We take $y_0 = 0$ unless otherwise specified. *It is arbitrary whether we choose y to be positive in the upward direction or in the downward direction; but we must be consistent about it throughout a problem's solution.*

PROBLEM SOLVING
You choose y to be positive either up or down

EXAMPLE 2–10 **Falling from a tower.** Suppose that a ball is dropped ($v_0 = 0$) from a tower 70.0 m high. How far will the ball have fallen after a time $t_1 = 1.00\ \text{s}$, $t_2 = 2.00\ \text{s}$, and $t_3 = 3.00\ \text{s}$?

"Drop" means $v_0 = 0$

APPROACH Let us take y as positive downward. We neglect any air resistance. Thus the acceleration is $a = g = +9.80\ \text{m/s}^2$, which is positive because we have chosen downward as positive. We set $v_0 = 0$ and $y_0 = 0$. We want to find the position y of the ball after three different time intervals. Equation 2–11b, with x replaced by y, relates the given quantities (t, a, and v_0) to the unknown y.

SOLUTION We set $t = t_1 = 1.00\ \text{s}$ in Eq. 2–11b:

$$y_1 = v_0 t_1 + \tfrac{1}{2}at_1^2$$
$$= 0 + \tfrac{1}{2}at_1^2 = \tfrac{1}{2}(9.80\ \text{m/s}^2)(1.00\ \text{s})^2 = 4.90\ \text{m}.$$

The ball has fallen a distance of 4.90 m during the time interval $t = 0$ to $t_1 = 1.00\ \text{s}$. Similarly, after 2.00 s ($= t_2$), the ball's position is

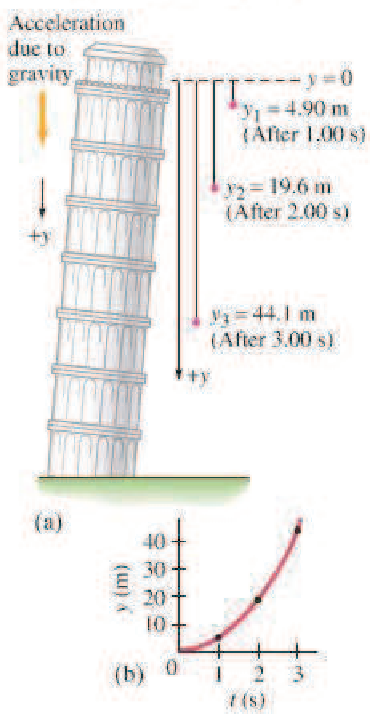
$$y_2 = \tfrac{1}{2}at_2^2 = \tfrac{1}{2}(9.80\ \text{m/s}^2)(2.00\ \text{s})^2 = 19.6\ \text{m}.$$

Finally, after 3.00 s ($= t_3$), the ball's position is (see Fig. 2–21)

$$y_3 = \tfrac{1}{2}at_3^2 = \tfrac{1}{2}(9.80\ \text{m/s}^2)(3.00\ \text{s})^2 = 44.1\ \text{m}.$$

NOTE Whenever we say "dropped," we mean $v_0 = 0$.

FIGURE 2–21 Example 2–10. (a) An object dropped from a tower falls with progressively greater speed and covers greater distance with each successive second. (See also Fig. 2–18.) (b) Graph of y vs. t.



EXAMPLE 2–11 **Thrown down from a tower.** Suppose the ball in Example 2–10 is *thrown* downward with an initial velocity of 3.00 m/s, instead of being dropped. (*a*) What then would be its position after 1.00 s and 2.00 s? (*b*) What would its speed be after 1.00 s and 2.00 s? Compare with the speeds of a dropped ball.

APPROACH We can approach this in the same way as in Example 2–10. Again we use Eq. 2–11b, but now v_0 is not zero, it is $v_0 = 3.00\ \text{m/s}$.

SOLUTION (*a*) At $t = 1.00\ \text{s}$, the position of the ball as given by Eq. 2–11b is

$$y = v_0 t + \tfrac{1}{2}at^2 = (3.00\ \text{m/s})(1.00\ \text{s}) + \tfrac{1}{2}(9.80\ \text{m/s}^2)(1.00\ \text{s})^2 = 7.90\ \text{m}.$$

At $t = 2.00\ \text{s}$, (time interval $t = 0$ to $t = 2.00\ \text{s}$), the position is

$$y = v_0 t + \tfrac{1}{2}at^2 = (3.00\ \text{m/s})(2.00\ \text{s}) + \tfrac{1}{2}(9.80\ \text{m/s}^2)(2.00\ \text{s})^2 = 25.6\ \text{m}.$$

As expected, the ball falls farther each second than if it were dropped with $v_0 = 0$.

[†] The speed of an object falling in air (or other fluid) does not increase indefinitely. If the object falls far enough, it will reach a maximum velocity called the **terminal velocity** due to air resistance.

(*b*) The velocity is obtained from Eq. 2–11a:

$$\begin{aligned} v &= v_0 + at \\ &= 3.00\,\text{m/s} + (9.80\,\text{m/s}^2)(1.00\,\text{s}) = 12.8\,\text{m/s} \qquad [\text{at } t_1 = 1.00\,\text{s}] \\ &= 3.00\,\text{m/s} + (9.80\,\text{m/s}^2)(2.00\,\text{s}) = 22.6\,\text{m/s}. \qquad [\text{at } t_2 = 2.00\,\text{s}] \end{aligned}$$

In Example 2–10, when the ball was dropped ($v_0 = 0$), the first term (v_0) in these equations was zero, so

$$\begin{aligned} v &= 0 + at \\ &= (9.80\,\text{m/s}^2)(1.00\,\text{s}) = 9.80\,\text{m/s} \qquad [\text{at } t_1 = 1.00\,\text{s}] \\ &= (9.80\,\text{m/s}^2)(2.00\,\text{s}) = 19.6\,\text{m/s}. \qquad [\text{at } t_2 = 2.00\,\text{s}] \end{aligned}$$

NOTE For both Examples 2–10 and 2–11, the speed increases linearly in time by 9.80 m/s during each second. But the speed of the downwardly thrown ball at any moment is always 3.00 m/s (its initial speed) higher than that of a dropped ball.

EXAMPLE 2–12 **Ball thrown upward, I.** A person throws a ball *upward* into the air with an initial velocity of 15.0 m/s. Calculate (*a*) how high it goes, and (*b*) how long the ball is in the air before it comes back to his hand.

APPROACH We are not concerned here with the throwing action, but only with the motion of the ball *after* it leaves the thrower's hand (Fig. 2–22) and until it comes back to his hand again. Let us choose y to be positive in the upward direction and negative in the downward direction. (This is a different convention from that used in Examples 2–10 and 2–11, and so illustrates our options.) The acceleration due to gravity will have a negative sign, $a = -g = -9.80\,\text{m/s}^2$. As the ball rises, its speed decreases until it reaches the highest point (B in Fig. 2–22), where its speed is zero for an instant; then it descends, with increasing speed.

SOLUTION (*a*) We consider the time interval from when the ball leaves the thrower's hand until the ball reaches the highest point. To determine the maximum height, we calculate the position of the ball when its velocity equals zero ($v = 0$ at the highest point). At $t = 0$ (point A in Fig. 2–22) we have $y_0 = 0$, $v_0 = 15.0\,\text{m/s}$, and $a = -9.80\,\text{m/s}^2$. At time t (maximum height), $v = 0$, $a = -9.80\,\text{m/s}^2$, and we wish to find y. We use Eq. 2–11c, replacing x with y: $v^2 = v_0^2 + 2ay$. We solve this equation for y:

$$y = \frac{v^2 - v_0^2}{2a} = \frac{0 - (15.0\,\text{m/s})^2}{2(-9.80\,\text{m/s}^2)} = 11.5\,\text{m}.$$

The ball reaches a height of 11.5 m above the hand.

(*b*) Now we need to choose a different time interval to calculate how long the ball is in the air before it returns to his hand. We could do this calculation in two parts by first determining the time required for the ball to reach its highest point, and then determining the time it takes to fall back down. However, it is simpler to consider the time interval for the entire motion from A to B to C (Fig. 2–22) in one step and use Eq. 2–11b. We can do this because y (or x) represents position or displacement, and not the total distance traveled. Thus, at both points A and C, $y = 0$. We use Eq. 2–11b with $a = -9.80\,\text{m/s}^2$ and find

$$\begin{aligned} y &= v_0 t + \tfrac{1}{2}at^2 \\ 0 &= (15.0\,\text{m/s})t + \tfrac{1}{2}(-9.80\,\text{m/s}^2)t^2. \end{aligned}$$

This equation is readily factored (we factor out one t):

$$(15.0\,\text{m/s} - 4.90\,\text{m/s}^2\,t)\,t = 0.$$

There are two solutions:

$$t = 0 \qquad \text{and} \qquad t = \frac{15.0\,\text{m/s}}{4.90\,\text{m/s}^2} = 3.06\,\text{s}.$$

The first solution ($t = 0$) corresponds to the initial point (A) in Fig. 2–22, when the ball was first thrown from $y = 0$. The second solution, $t = 3.06\,\text{s}$, corresponds to point C, when the ball has returned to $y = 0$. Thus the ball is in the air 3.06 s.

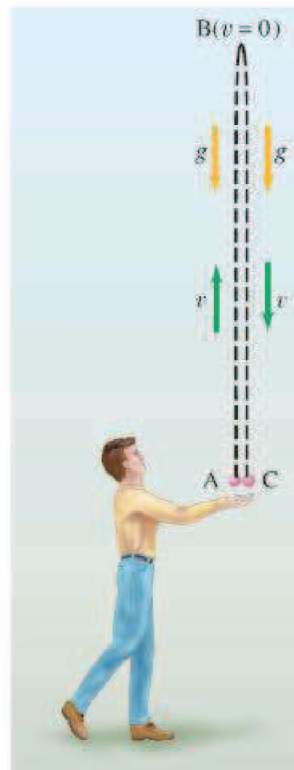


FIGURE 2–22 An object thrown into the air leaves the thrower's hand at A, reaches its maximum height at B, and returns to the original position at C. Examples 2–12, 2–13, 2–14, and 2–15.

We did not consider the throwing action in this Example. Why? Because during the throw, the thrower's hand is touching the ball and accelerating the ball at a rate unknown to us—the acceleration is *not* g. We consider only the time when the ball is in the air and the acceleration is equal to g.

CAUTION

Quadratic equations have two solutions. Sometimes only one corresponds to reality, sometimes both

Every quadratic equation (where the variable is squared) mathematically produces two solutions. In physics, sometimes only one solution corresponds to the real situation, as in Example 2–7, in which case we ignore the "unphysical" solution. But in Example 2–12, both solutions to our equation in t^2 are physically meaningful: $t = 0$ and $t = 3.06\,\text{s}$.

CONCEPTUAL EXAMPLE 2–13 **Two possible misconceptions.** Give examples to show the error in these two common misconceptions: (1) that acceleration and velocity are always in the same direction, and (2) that an object thrown upward has zero acceleration at the highest point (B in Fig. 2–22).

CAUTION

(1) Velocity and acceleration are not always in the same direction; the acceleration (of gravity) always points down

(2) $a \neq 0$ even at the highest point of a trajectory

RESPONSE Both are wrong. (1) Velocity and acceleration are *not* necessarily in the same direction. When the ball in Example 2–12 is moving upward, its velocity is positive (upward), whereas the acceleration is negative (downward). (2) At the highest point (B in Fig. 2–22), the ball has zero velocity for an instant. Is the acceleration also zero at this point? No. The velocity near the top of the arc points upward, then becomes zero (for zero time) at the highest point, and then points downward. Gravity does not stop acting, so $a = -g = -9.80\,\text{m/s}^2$ even there. Thinking that $a = 0$ at point B would lead to the conclusion that upon reaching point B, the ball would stay there: if the acceleration (= rate of change of velocity) were zero, the velocity would stay zero at the highest point, and the ball would stay up there without falling. In sum, the acceleration of gravity always points down toward the Earth, even when the object is moving up.

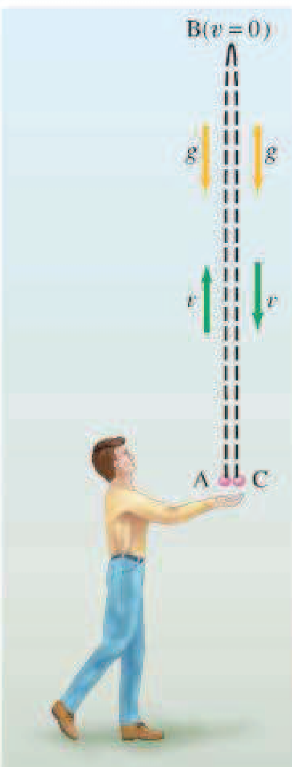


FIGURE 2–22 (Repeated for Examples 2–13, 2–14, and 2–15.)

EXAMPLE 2–14 **Ball thrown upward, II.** Let us consider again the ball thrown upward of Example 2–12, and make more calculations. Calculate (*a*) how much time it takes for the ball to reach the maximum height (point B in Fig. 2–22), and (*b*) the velocity of the ball when it returns to the thrower's hand (point C).

APPROACH Again we assume the acceleration is constant, so Eqs. 2–11 are valid. We have the height of 11.5 m from Example 2–12. Again we take y as positive upward.

SOLUTION (*a*) We consider the time interval between the throw ($t = 0$, $v_0 = 15.0\,\text{m/s}$) and the top of the path ($y = +11.5\,\text{m}$, $v = 0$), and we want to find t. The acceleration is constant at $a = -g = -9.80\,\text{m/s}^2$. Both Eqs. 2–11a and 2–11b contain the time t with other quantities known. Let us use Eq. 2–11a with $a = -9.80\,\text{m/s}^2$, $v_0 = 15.0\,\text{m/s}$, and $v = 0$:

$$v = v_0 + at;$$

setting $v = 0$ and solving for t gives

$$t = -\frac{v_0}{a} = -\frac{15.0\,\text{m/s}}{-9.80\,\text{m/s}^2} = 1.53\,\text{s}.$$

This is just half the time it takes the ball to go up and fall back to its original position [3.06 s, calculated in part (*b*) of Example 2–12]. Thus it takes the same time to reach the maximum height as to fall back to the starting point.

(*b*) Now we consider the time interval from the throw ($t = 0$, $v_0 = 15.0\,\text{m/s}$) until the ball's return to the hand, which occurs at $t = 3.06\,\text{s}$ (as calculated in Example 2–12), and we want to find v when $t = 3.06\,\text{s}$:

$$v = v_0 + at = 15.0\,\text{m/s} - (9.80\,\text{m/s}^2)(3.06\,\text{s}) = -15.0\,\text{m/s}.$$

Note the symmetry: the speed at any height is the same when going up as when coming down (but the direction is opposite)

NOTE The ball has the same magnitude of velocity when it returns to the starting point as it did initially, but in the opposite direction (this is the meaning of the negative sign). Thus, as we gathered from part (*a*), the motion is symmetrical about the maximum height.

EXERCISE C Two balls are thrown from a cliff. One is thrown directly up, the other directly down. Both balls have the same initial speed, and both hit the ground below the cliff. Which ball hits the ground at the greater speed: (*a*) the ball thrown upward, (*b*) the ball thrown downward, or (*c*) both the same? Ignore air resistance. [*Hint*: See the result of Example 2–14, part (*b*).]

The acceleration of objects such as rockets and fast airplanes is often given as a multiple of $g = 9.80\,\mathrm{m/s^2}$. For example, a plane pulling out of a dive and undergoing 3.00 g's would have an acceleration of $(3.00)(9.80\,\mathrm{m/s^2}) = 29.4\,\mathrm{m/s^2}$.

Acceleration expressed in g's

EXERCISE D If a car is said to accelerate at $0.50\,g$, what is its acceleration in $\mathrm{m/s^2}$?

Additional Example—Using the Quadratic Formula

EXAMPLE 2–15 **Ball thrown upward, III.** For the ball in Example 2–14, calculate at what time t the ball passes a point 8.00 m above the person's hand.

APPROACH We choose the time interval from the throw ($t = 0$, $v_0 = 15.0\,\mathrm{m/s}$) until the time t (to be determined) when the ball is at position $y = 8.00\,\mathrm{m}$, using Eq. 2–11b.

SOLUTION We want t, given $y = 8.00\,\mathrm{m}$, $y_0 = 0$, $v_0 = 15.0\,\mathrm{m/s}$, and $a = -9.80\,\mathrm{m/s^2}$. We use Eq. 2–11b:

$$y = y_0 + v_0 t + \tfrac{1}{2}at^2$$

$$8.00\,\mathrm{m} = 0 + (15.0\,\mathrm{m/s})\,t + \tfrac{1}{2}(-9.80\,\mathrm{m/s^2})t^2.$$

To solve any quadratic equation of the form $at^2 + bt + c = 0$, where a, b, and c are constants (a is *not* acceleration here), we use the **quadratic formula** (see Appendix A–4):

PROBLEM SOLVING
Using the quadratic formula

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

We rewrite our y equation just above in standard form, $at^2 + bt + c = 0$:

$$(4.90\,\mathrm{m/s^2})t^2 - (15.0\,\mathrm{m/s})\,t + (8.00\,\mathrm{m}) = 0.$$

So the coefficient a is $4.90\,\mathrm{m/s^2}$, b is $-15.0\,\mathrm{m/s}$, and c is 8.00 m. Putting these into the quadratic formula, we obtain

$$t = \frac{15.0\,\mathrm{m/s} \pm \sqrt{(15.0\,\mathrm{m/s})^2 - 4(4.90\,\mathrm{m/s^2})(8.00\,\mathrm{m})}}{2(4.90\,\mathrm{m/s^2})},$$

which gives us $t = 0.69\,\mathrm{s}$ and $t = 2.37\,\mathrm{s}$. Are both solutions valid? Yes, because the ball passes $y = 8.00\,\mathrm{m}$ when it goes up ($t = 0.69\,\mathrm{s}$) and again when it comes down ($t = 2.37\,\mathrm{s}$).

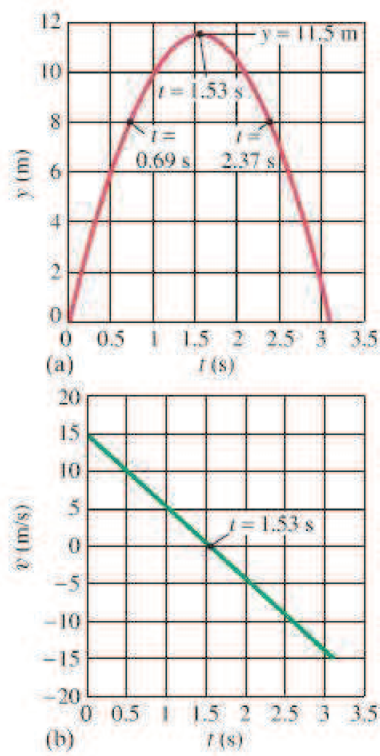


FIGURE 2–23 Graphs of (a) y vs. t, (b) v vs. t for a ball thrown upward, Examples 2–12, 2–14, and 2–15.

For some people, graphs can be a help in understanding. Figure 2–23 shows graphs of y vs. t and v vs. t for the ball thrown upward in Fig. 2–22, incorporating the results of Examples 2–12, 2–14, and 2–15. We shall discuss some useful properties of graphs in the next Section.

We will use the word "vertical" a lot in this book. What does it mean? (Try to respond before reading on.) Vertical is defined as the line along which an object falls. Or, if you put a small sphere on the end of a string and let it hang, the string represents a vertical line (sometimes called a *plumb line*).

EXERCISE E What does *horizontal* mean?