Alpha Decay

General equation:

$$ {}^{A}_{Z}X \longrightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He + \text{energy} $$

$$ ({}^{4}_{2}\alpha) $$

Where does the kinetic energy come from?

rest mass of nucleons ($m_{nucleons} = \Delta m + m_{nucleus}$)

Result: nucleus is in a more stable state with higher binding energy and higher binding energy per nucleon.

Practice Problem

A radium nucleus, initially at rest, decays by the emission of an alpha particle into radon in the reaction described above. The mass of ${}_{88}^{226}\text{Ra}$ is 226.025402 u and the mass of ${}_{86}^{222}\text{Rn}$ is 222.017571 u and the mass of the alpha particle is 4.002602 u.

a) Calculate the energy released in this decay.

$\Delta m = m_{\text{products}} - m_{\text{reactant}} = (m_{\alpha} + m_{Rn}) - m_{Ra}$

$\Delta m = .005229\text{ u} \Rightarrow (.005229\text{ u} * 1.661\times10^{-27}\text{ kg/u})$

$\Delta E = \Delta mc^2 = 4.87\text{ MeV}$

$(4.87\times10^{6}\text{ eV})$

$\underline{* \; 1.6\times10^{-19}\text{ J} = 1\text{ eV}}$

Practice Problem

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b) Compare the momenta, speeds, and kinetic energies of the two particles produced by this reaction

Practice Problem

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- c) If the kinetic energy of the alpha particle is 4.77 MeV, calculate its speed.

$$KE_\alpha = \frac{1}{2} m_\alpha V_\alpha^2$$

$$\therefore V_\alpha = \sqrt{\frac{2KE_\alpha}{m_\alpha}} = \sqrt{\frac{2(4.77\times10^6\,\mathrm{eV})(1.6\times10^{-19}\,\mathrm{J/eV})}{(4.002602\,\mathrm{u})(1.661\times10^{-27}\,\mathrm{kg/u})}}$$

$$V_\alpha = 1.5\times10^7\,\mathrm{ms^{-1}}$$

Practice Problem

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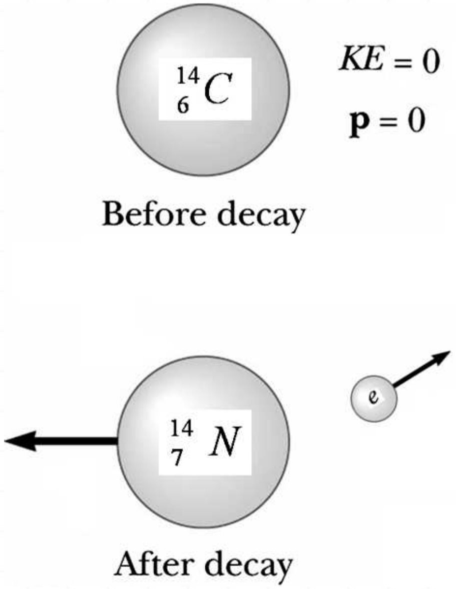
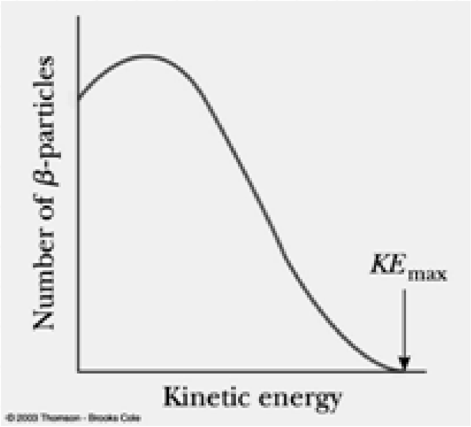
d) Calculate the recoil speed of the radon nucleus.

Beta Decay

- Beta-minus particle:
  electron, β^-, ${}_{-1}^{0}e$
- Beta-plus particle:
  positron, β^+, ${}_{+1}^{0}e$





Continuous spectrum of beta particles in beta decay

Beta Decay

- Conclusion:

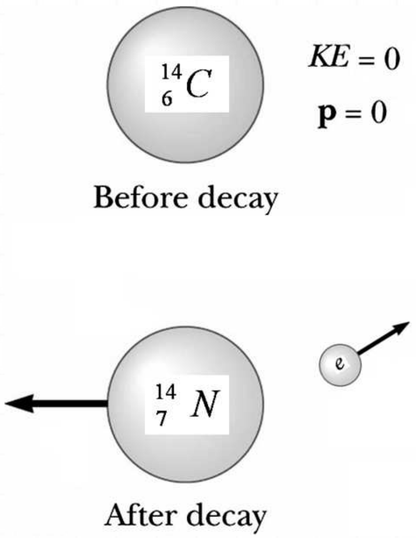
  there is third particle involved with beta decay that carries away some KE and momentum – virtually undetectable

- Neutrino and anti-neutrino:

  fundamental particles

  no charge

  very small mass



Beta-minus decay

Example reaction:

$$ {}^{14}_{6}C \rightarrow {}^{14}_{7}N + {}^{0}_{-1}e + {}^{0}_{0}\bar{\nu} + energy $$

antineutrino

General equation:

$$ {}^{A}_{Z}X \rightarrow {}^{A}_{Z+1}Y + {}^{0}_{-1}e + {}^{0}_{0}\overline{\nu} + \text{energy} $$

$$ \left({}^{0}_{-1}\beta\right) $$

How does this happen? Weak nuclear force

$$ {}^{1}_{0}n \rightarrow {}^{1}_{1}p + {}^{0}_{-1}e + {}^{0}_{0}\overline{\nu} + \text{energy} $$

Beta-plus decay

Example reaction:

$$ {}^{12}_{7}N \rightarrow {}^{12}_{6}C + \underbrace{{}^{0}_{+1}e}_{\text{positron}} + \underbrace{{}^{0}_{0}\nu}_{\text{neutrino}} + energy $$

General equation:

$$ {}^{A}_{Z}X \rightarrow {}^{A}_{Z-1}Y + {}^{0}_{+1}e + {}^{0}_{0}\nu + \text{energy} $$

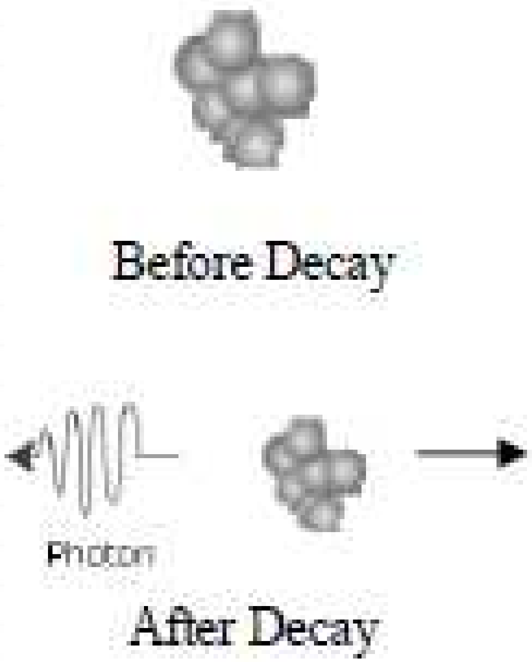
How does this happen? Weak nuclear force

$$ {}^{1}_{1}p \rightarrow {}^{1}_{0}n \quad {}^{0}_{+1}e + {}^{0}_{0}\nu + \text{energy} $$

Gamma Decay

- Gamma particle:
  - high energy photon, γ
- Example reaction:

$$ {}^{12}_{6}C^{*} \rightarrow {}^{12}_{6}C + \Upsilon + energy $$



Gamma Decay

General equation:

$${}^{A}_{Z}X^{*} \longrightarrow {}^{A}_{Z}X + \gamma + \text{energy}$$

Where does the photon (energy) come from?

Rest mass of the nucleus

Energy Spectra of Radiation

The nucleus itself, like the atom as a whole, is a quantum system with allowed states and discrete energy levels. The nucleus can be in any one of a number of discrete allowed excited states or in its lowest energy relaxed state. When it transitions between a higher energy level and a lower one, it emits energy in the form of alpha, beta, or gamma radiation. When an alpha particle or a gamma photon is emitted from the nucleus, only discrete energies are observed. *These discrete energy spectra give evidence that a nucleus has energy levels*. (However, the spectrum of energies emitted as beta articles is continuous due to its sharing the energy with a neutrino or antineutrino in any proportion.)

Energy Spectra of Radiation

- Importance:

  discrete energy spectra give evidence for nuclear energy levels

Alpha spectra	**Beta spectra**	**Gamma spectra**
discrete	continuous	discrete

Ionizing Radiation

- **Ionizing Radiation** – As this radiation passes through materials, it “knocks off” electrons from neutral atoms thereby creating an ion pair: **free electrons** and **a positive ion**. This **ionizing property** allows the radiation to be detected but is also dangerous since it can lead to mutations in biologically important molecules in cells, such as DNA.

Ionizing Radiation

	α	β	γ
Particle	helium nucleus	electron or positron	high-energy photon
Penetration ability	low	medium	high
Material needed to absorb it	sheet of paper (a few cm of air)	aluminum foil (1 mm)	lead (≈10 cm)
Path length in air	a few cm	less than 1m	∞

Radioactive Decay

- **Random process**: It cannot be predicted when a particular nucleus will decay, only the probability that it will decay.
- **Spontaneous process:** It is not affected by external conditions. For example, changing the pressure or temperature of a sample will not affect the decay process.
- **Rate of decay decreases exponentially with time:** Any amount of radioactive nuclei will reduce to half its initial amount in a constant time, independent of the initial amount.

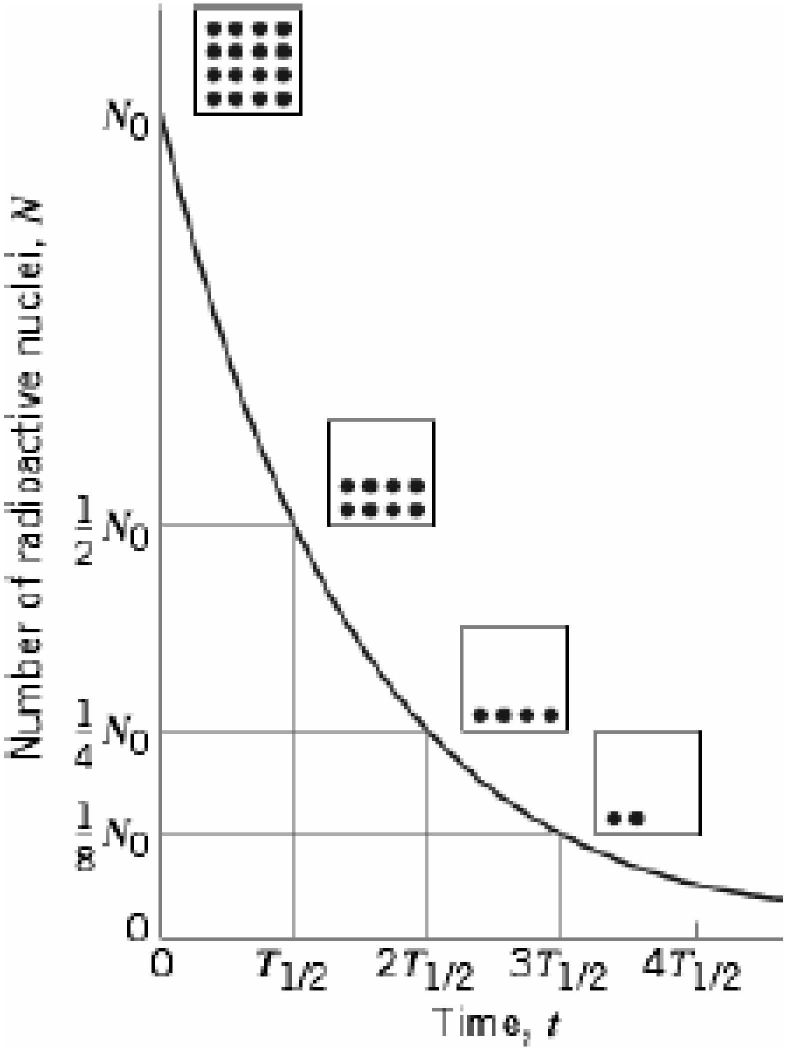
Half-life ($T_{1/2}$)

- the time taken for ½ of the radioactive nuclides in a sample to decay
- the time taken for the activity of a sample to decrease to ½ of its initial value
- Units:

  time (s or hr or d or yr)

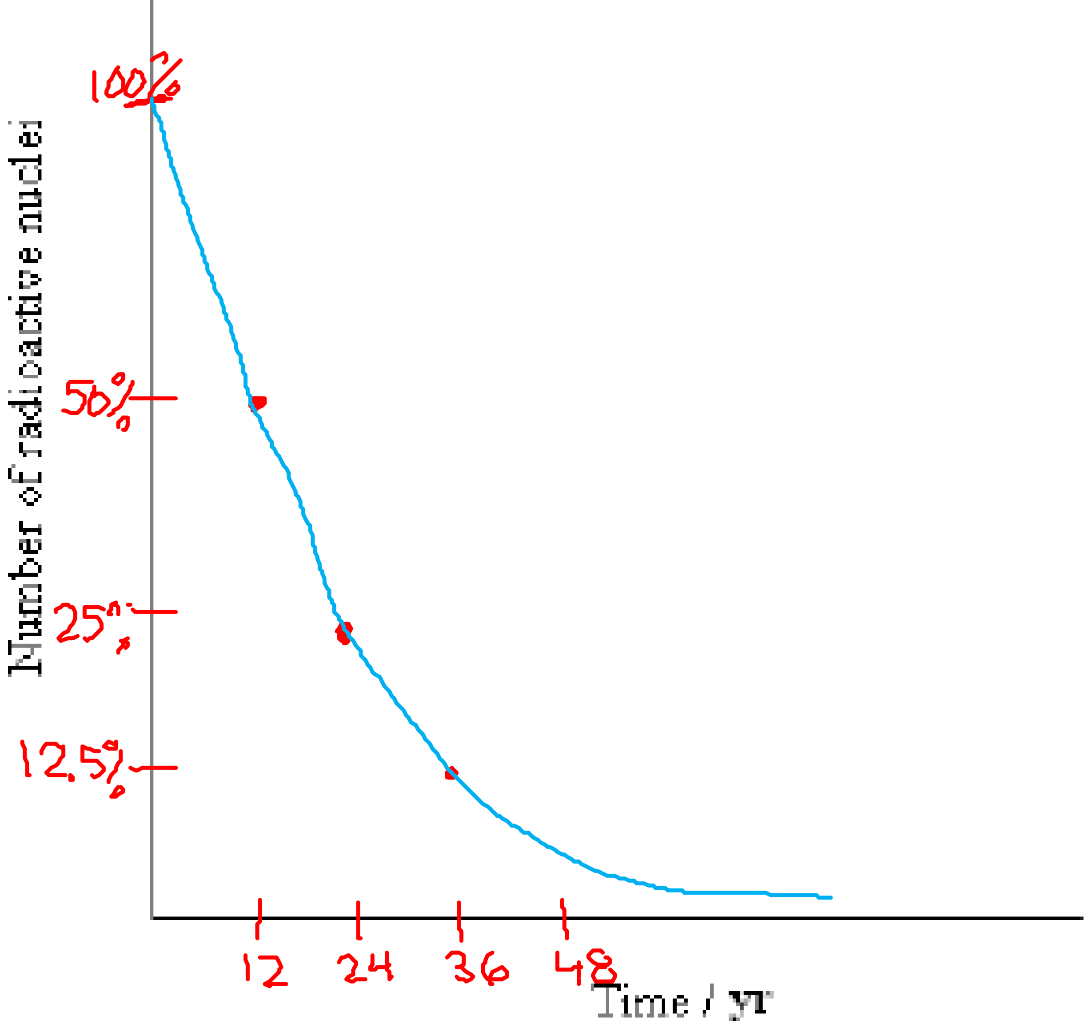
N_0 = Number of nuclei originally present

N = Number of nuclei present at any one time



Your Turn

Radioactive tritium has a half-life of about 12 years. Complete the graph below.



A nuclide X has a half-life of 10 s. On decay the stable nuclide Y is formed. Initially a sample contains only atoms of X. After what time will 87.5% of the atoms in the sample have decayed into nuclide Y?

t	%X	%Y
0	100	0
10	50	50
20	25	75
30	12.5	87.5
40	6.25	93.75

Activity

Activity (A) –

the number of radioactive disintegrations (decays) per unit time

Formula:

$$A = -\frac{\Delta N}{\Delta t}$$

Units:

$\frac{\text{decays}}{\text{time}}$ (s^{-1}, hr^{-1}, d^{-1}, yr^{-1})

Standard units:

Becquerel (Bq)

1 Bq = 1 decay per second

1. A sample originally contains 8.0 x 10^{12} radioactive nuclei and has a half-life of 5.0 seconds. Calculate the activity of the sample and its half-life after:
   a) 5.0 s
   b) 10. s
   c) 15 s

(a) $A = \frac{\Delta N}{\Delta t}$

$\Delta N = N_0 - N$

$= (8.0\times10^{12} - 4.0\times10^{12})$

$= 4.0\times10^{12}$ nuclides

$\Delta t = 5$ s

$A = \frac{4.0\times10^{12} \text{ decays}}{5.0 \text{ s}}$

$A = 8.0\times10^{11}$ Bq

(b) $A = \frac{\Delta N}{\Delta t}$

# of nuclei after 10s

$\Delta N = N_0 - N$

$\Delta N = (8.0\times10^{12}) - (2.0\times10^{12})$

$\Delta N = 6.0\times10^{12}$ decays

$\Delta t = 10$ s

$A = 6.0\times10^{11}$ Bq

(c) $\Delta N = (8.0\times10^{12}) - (1.0\times10^{12})$

$A = \frac{\Delta N}{\Delta t} = \frac{7.0\times10^{12} \text{ decays}}{15 \text{ s}}$

$A = 4.7\times10^{11}$ Bq

2. Samples of two nuclides X and Y initially contain the same number of radioactive nuclei, but the half-life of nuclide X is greater than the half-life of nuclide Y. Compare the initial activities of the two samples.

# $T_{1/2}$	% Y	% X
0	100%	100%
1	50%	—
2	25%	50%
3	12.5%	—
4	6.25	25%

$\frac{1}{2}T_{1/2}(X) = T_{1/2}(Y)$

$N_X > N_Y$

$\therefore A_X > A_Y$

Activity of Y is greater

Activity

$A \propto N$

$A = \lambda N$

The Radioactive Decay Law: The rate at which radioactive nuclei in a sample decay (the activity) is proportional to the number of radioactive nuclei present in the sample at any one time.

[As the number of radioactive nuclei decreases, so does the average rate of decay (the activity).]

Initial Activity

$A_0 = \lambda N_0$

The initial activity (A_0) is directly related to the number of radioactive nuclei originally present (N_0) in the sample.

3. The isotope Francium-224 has a half-life of 20 minutes. A sample of the isotope has an initial activity of 800 disintegrations per second. What is the approximate activity of the sample after 1 hour?

N.

$A_0 = 800\ Bq$

$N_0 =$ not given

$T_{1/2} = 20\ min$

$t = 1\ h \Rightarrow 3T_{1/2}$

$N = \frac{1}{8} N_0$

$$\boxed{N = \frac{1}{2^n} N_0}$$

$A_0 \propto N_0$

$\frac{1}{8} A_0 \propto \frac{1}{8} N_0$

$\frac{1}{8}(800) = \boxed{100\ Bq}$

Decay Constant (λ)

- Constant of proportionality between the decay rate (activity) and the number of radioactive nuclei present.
- Probability of decay of a particular nuclei per unit time.
- <u>Units</u>: inverse time (s^{-1} or hr^{-1}or d^{-1} or yr^{-1})

Deriving the Radioactive Decay Law

$$A = \frac{-\Delta N}{\Delta t} = \lambda N$$

$$\frac{\partial N}{\partial t} = -\lambda N$$

$$\Downarrow$$

$$N = N_0 e^{-\lambda t}$$

$$A = A_0 e^{-\lambda t}$$

$$A_0 = \underline{\lambda N_0}$$

$$\boxed{A = \lambda N_0 e^{-\lambda t}}$$

Relating the Decay Constant and Half-life