Decay Constant (λ)

- Constant of proportionality between the decay rate (activity) and the number of radioactive nuclei present.
- Probability of decay of a particular nuclei per unit time.
- Units: inverse time (s^{-1} or hr^{-1}or d^{-1} or yr^{-1})

Deriving the Radioactive Decay Law

$$A = -\frac{\Delta N}{\Delta t} = \lambda N$$

$$\frac{\partial N}{\partial t} = -\lambda N$$

$$\Downarrow$$

$$N = N_0 e^{-\lambda t}$$

$$A = A_0 e^{-\lambda t}$$

$$A_0 = \lambda N_0$$

$$\boxed{A = \lambda N_0 e^{-\lambda t}}$$

Relating the Decay Constant and Half-life

$$t = T_{\frac{1}{2}} \qquad N = \frac{1}{2} N_0$$

$$N = N_0 e^{-\lambda t}$$

$$\frac{1}{2}\cancel{N_0} = \cancel{N_0} e^{-\lambda T_{1/2}}$$

cancel out N_0

$$\left(\frac{1}{2}\right)^{-1} = \left(e^{-\lambda T_{1/2}}\right)^{-1}$$

raise both sides to the -1 power

$$2 = e^{\lambda T_{1/2}}$$

take the ln (natural log) of both sides

$$\ln 2 = \ln\left(e^{\lambda T_{1/2}}\right)$$

$$\frac{\ln 2}{\lambda} = \frac{\lambda T_{1/2}}{\lambda}$$

$$T_{\frac{1}{2}} = \frac{\ln 2}{\lambda}$$

The half-life of a certain radioactive isotope is 2.0 minutes. A particular nucleus of this isotope has not decayed within a time interval of 2.0 minutes. What is the probability of it decaying in:

a) The next two minutes
b) The next one minute
c) The next second

A sample of a radioactive isotope X has the same initial activity as a sample of the isotope Y. The sample of X contains twice the number of atoms as the sample of Y. If the half-life of X is T_X then the half-life of Y is

$A_X = A_Y$

$N_X = 2N_Y$

$A = \lambda N$

$A_X = \lambda_X N_X = \lambda_X (2N_Y)$

$A_Y = \lambda_Y N_Y = \lambda_X (2N_Y)$

$\lambda_y \cancel{N_Y} = 2\lambda_X \cancel{N_Y}$

$\therefore \lambda_x = \lambda_y/2$

$(T_{1/2})_y = \frac{\ln 2}{\lambda_y}$

$(T_{1/2})_x = \frac{\ln 2}{\lambda_x} = \frac{\ln 2}{\left(\frac{\lambda_y}{2}\right)} = \frac{2\ln 2}{\lambda_y}$

$(T_{\frac{1}{2}})_x = 2(T_{1/2})_y \Rightarrow \therefore T_Y = .5T_x$

The half-life of a radioactive isotope is 10 days. Calculate the fraction of the sample that will be left after 15 days.

$T_{1/2} = 10\,d$

$t = 15\,d$

$\frac{N}{N_0} = ?$

$\lambda =$ ____

$T_{1/2} = \frac{\ln 2}{\lambda}$

$\therefore \lambda = \frac{\ln 2}{T_{1/2}}$

$\lambda = \frac{\ln 2}{(10\,d)}$

$\lambda = .0693\,d^{-1}$

$N = N_0 e^{-\lambda t}$

$\frac{N}{N_0} = e^{-\lambda t}$

$= e^{-(.0693\,d^{-1})(15\,d)}$

$= e^{-1.0397}$

$\frac{N}{N_0} = .3536$

$\approx 35\%$

The half-life of a radioactive substance is 10 days. Initially, there are 2.00 x 10^{26} radioactive nuclei present.

a) What is the probability of any one particular nucleus decaying?
b) What is the initial activity? (A_o)
c) How many radioactive nuclei are left after 25 days? (N)
d) What is the activity of the sample after 25 days? (A)
e) How long will it take for the activity to fall to 1.0 x 10^{24} dy^{-1}? (t)

(a) 50%

(b) $\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{(10d)} = .0693\,d^{-1} = 8.0225\times10^{-7}\,s$

$A_o = \lambda N_o = (.0693\,d^{-1})(2.00\times10^{26}) = 1.39\times10^{25}$ decay/day

$= (8.0225\times10^{-7}\,s)(2.00\times10^{26}) = 1.60\times10^{20}$ Bq

(c) $N = N_o e^{-\lambda t}$

$N = (2.00\times10^{26})e^{-(.0693d^{-1})(25d)} = \underline{3.5368\times10^{25}\text{ nuclei}}$

(d) $A = \lambda N$ or $A = A_o e^{-\lambda t}$

$= (.0693d^{-1})(3.536\times10^{25})$

$A = 2.45\times10^{24}$ decays/day

$A = (1.60\times10^{20}\,Bq)e^{-(.0693)(25)}$

$A = 2.83\times10^{19}$ Bq

The half-life of a radioactive substance is 10 days. Initially, there are 2.00 x 10^{26} radioactive nuclei present.

a) What is the probability of any one particular nucleus decaying?

b) What is the initial activity? (A_0)

c) How many radioactive nuclei are left after 25 days? (N)

d) What is the activity of the sample after 25 days? (A)

e) How long will it take for the activity to fall to 1.0 x 10^{24} dy^{-1}? (t)

$$\frac{A}{A_0} = \frac{\cancel{A_0} e^{-\lambda t}}{\cancel{A_0}}$$

$$\frac{A}{A_0} = e^{-\lambda t}$$

$$\ln\left(\frac{A}{A_0}\right) = \ln\left(e^{-\lambda t}\right)$$

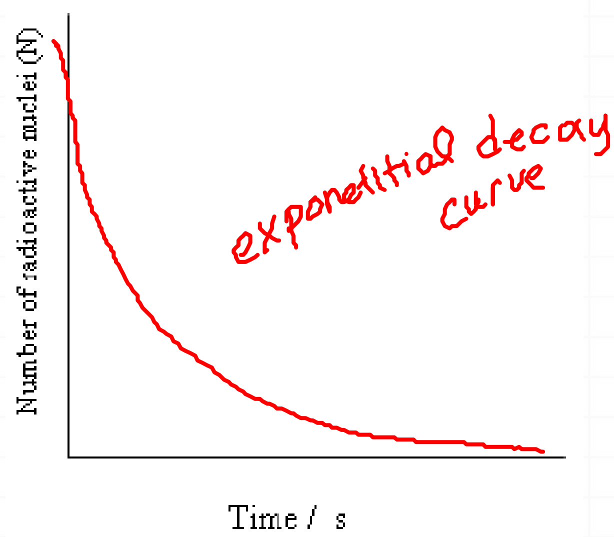
$$\frac{\ln\left(\frac{A}{A_0}\right)}{-\lambda} = \frac{-\lambda t}{-\lambda}$$

$$t = \frac{\ln\left(\frac{A}{A_0}\right)}{-\lambda}$$

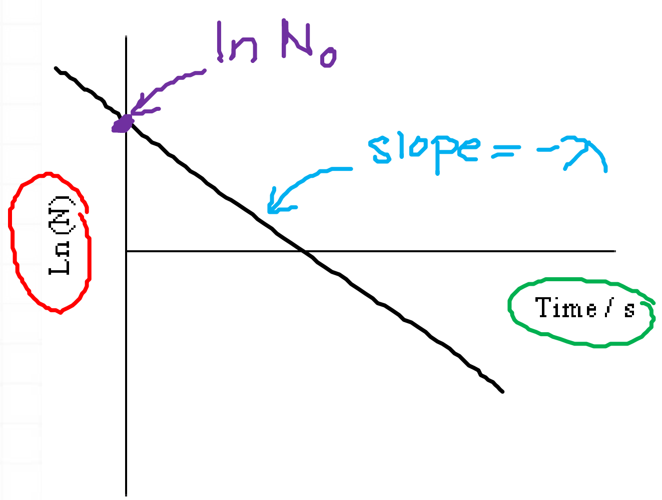
$$t = \frac{\ln\left(\frac{1.0\times10^{24}\,d^{-1}}{1.39\times10^{25}\,d^{-1}}\right)}{-(.0693\,d^{-1})} \Leftarrow \text{f/ part (b)}$$

$$t \approx 38 \text{ days}$$

Radioactive nuclei vs. time



Straightening by natural log



$$N = N_0 e^{-\lambda t}$$

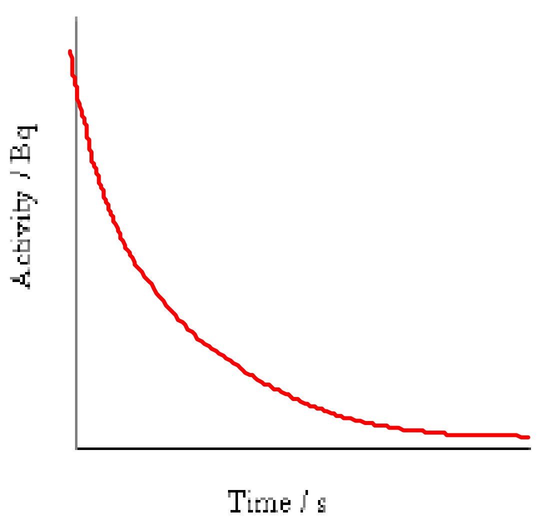
$$\ln(N) = \ln(N_0 e^{-\lambda t})$$

$$\ln N = \ln N_0 + (-\lambda t)$$

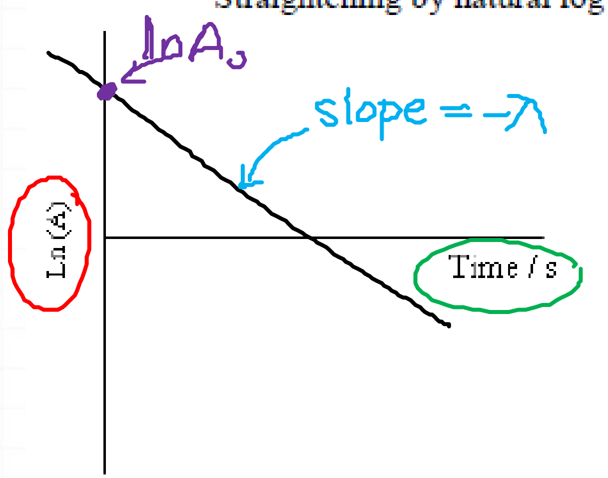
$$\ln N = -\lambda t + \ln N_0$$

$$y = mx + b$$

Activity of sample vs. time



Straightening by natural log



$$A = A_0 e^{-\lambda t}$$

$$\ln(A) = \ln\left(A_0 e^{-\lambda t}\right)$$

$$\ln A = \ln A_0 + (-\lambda t)$$

$$\ln A = -\lambda t + \ln A_0$$

$$y = m x + b$$

$$\therefore T_{1/2} = \frac{\ln 2}{\lambda}$$

then

$$T_{1/2} = \frac{\ln 2}{(-\text{slope})}$$

Determining Half-life

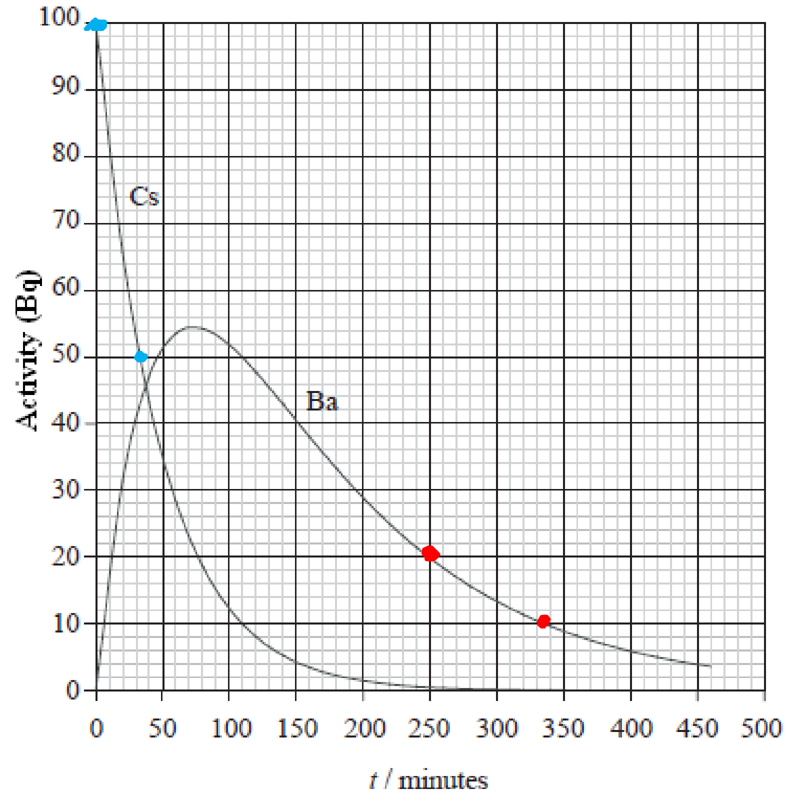
- **If the half-life is short**, then readings can be taken of activity versus time using a Geiger counter, for example. Then, either
  1. A graph of activity versus time would give the exponential shape and several values for the half-life could be read from the graph and averaged.
  2. A graph of ln (activity) versus time would be linear and the decay constant can be calculated from the slope.
- **If the half-life is long**, then the activity will be effectively constant over a period of time. If a way could be found to calculate the number of nuclei present chemically, perhaps using the mass of the sample and Avogadro's number, then the activity relation or the decay equation could be used to calculate half-life.

Cesium-138 decays into an isotope of barium. Measurements of the activity of a particular sample of cesium-138 were taken and graphed at right.

a) Suggest how the data for this graph could have been obtained.

b) Use the graph to estimate the half-life of cesium-138.

c) Use the graph to estimate the half-life of the barium isotope.



(a) Geiger counter

(b) @ $t=0$ $A = 100$ Bq

$\boxed{t \approx 35 \text{ min}}$ $A = 50$ Bq

$T_{1/2, Cs} = 35$ min

(c) 20 Bq @ $t = 250$ min

10 Bq @ $t \approx 340$ min

$(T_{1/2})_{Ba} \approx$ 90 min

You have to use a time period after 250 minutes to ensure activity is due to barium not cesium

A 2.0 mg sample of carbon-14 is measured to have an activity of 6.5×10^{10} Bq.

a) Use this information to determine the half-life of carbon-14 in years.
b) A student suggests that the half-life can be determined by taking repeated measurements of the activity and analyzing the data graphically. Use your answer to part (a) to comment on this method of determining the half-life.

The radioactive isotope potassium-40 undergoes beta decay to form the isotope calcium-40 with a half-life of 1.3 x 10^9 yr. A sample of rock contains 10 mg of potassium-40 and 42 mg of calcium-40.

a) Determine the age of the rock sample.

b) What are some assumptions made in this determination of age?

$m = m_0 e^{-\lambda t}$ (b/c mass is proportional to number of nuclei)

$(m_0 = m_K + m_{Ca} = 10\,mg + 42\,mg = 52\,mg)$

$m = m_K = 10\,mg$

$T_{1/2} = \frac{\ln 2}{\lambda}$

$\therefore \lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{(1.3\times10^9\,yr)} =$

$\frac{10}{52} = \frac{\cancel{52}\,e^{-\lambda t}}{\cancel{52}}$

$\frac{10}{52} = e^{-\lambda t}$

$\frac{\ln\left(\frac{10}{52}\right)}{-\lambda} = \frac{-\lambda t}{-\lambda}$

$\therefore t = \frac{\ln\left(\frac{10}{52}\right)}{-\frac{\ln 2}{(1.3\times10^9\,yr)}} = 3092065110\,y = 3.1\times10^9\,yr$