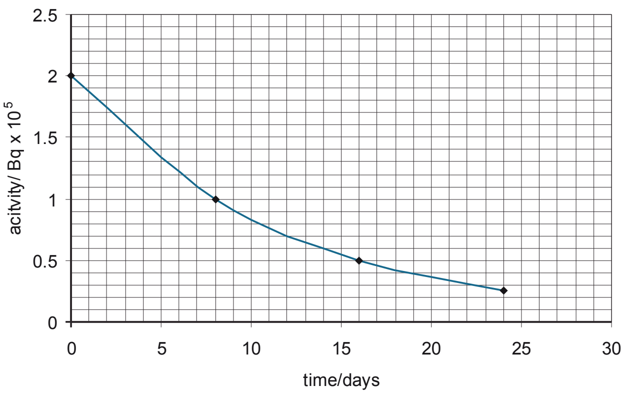
The data points for a graph showing the variation with time of the activity are shown below:

time/days	activity/Bq
0	2
8	1
16	0.5
24	0.25

So the graph is as follows

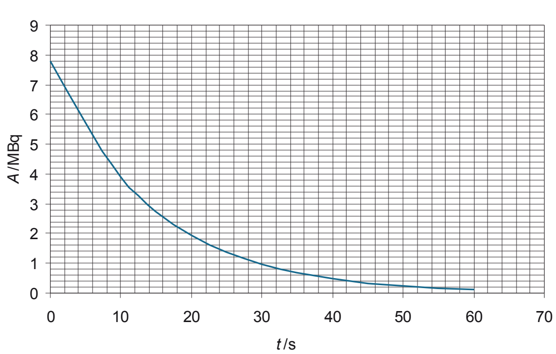


From which we see that after 12 days the activity is 7.0×10^4 Bq.

Exercise 7.2

1. The initial activity of a sample of a radioactive isotope decreases by a factor of $\frac{1}{16}$ after 90 hours. Calculate the half-life of the isotope.

2. The graph below shows the variation with time *t* of the activity *A* of a sample of the isotope xenon-114. Use the graph to determine the half-life of xenon-114.



7.3 NUCLEAR REACTIONS, FISSION AND FUSION

NUCLEAR REACTIONS

7.3.1 Describe and give an example of an artificial (induced) transmutation.

7.3.2 Construct and complete nuclear equations.

7.3.3 Define the term *unified atomic mass unit*.

7.3.4 Apply the Einstein mass–energy equivalence relationship.

7.3.5 Define the concepts of *mass defect*, *binding energy* and *binding energy per nucleon*.

7.3.6 Draw and annotate a graph showing the variation with nucleon number of the binding energy per nucleon.

7.3.7 Solve problems involving mass defect and binding energy.

7.3.1,2 ARTIFICIAL (INDUCED) TRANSMUTATION

So far only **transmutation** of elements has been discussed, i.e.the transformation of one element into another, that takes place through natural radioactivity. In 1919 Rutherford discovered that when nitrogen gas is bombarded with α-particles, oxygen and protons are produced. He surmised that the following reaction takes place:

$$^{4}_{2}He + ^{14}_{7}N \rightarrow ^{17}_{8}O + ^{1}_{1}H$$

After the discovery of this induced transformation, Rutherford working in conjunction with Chadwick, succeeded in producing artificial transmutation of all the elements from boron to potassium (excluding carbon and oxygen) by bombarding them with α-particles.

In 1934 *Irène Curie* and *F. Jolie* made the first discovery of artificial radioactive isotopes by the bombardment of aluminium with α-particles to produce a radioactive isotope of phosphorus. The nuclear reaction equation for this is

$${}^{4}_{2}\text{He} + {}^{27}_{13}\text{Al} \rightarrow {}^{30}_{15}\text{P} + {}^{1}_{0}\text{n}$$

The isotope of phosphorus is radioactive and undergoes positron decay as follows

$${}^{30}_{15}\text{P} \rightarrow {}^{30}_{14}\text{Si} + \text{e}^{+} + \bar{\nu}$$

Following the work of Curie and Joliet, extensive work was carried out into the production of artificial isotopes. It was found that neutrons are particularly effective in inducing artificial transmutation to produce artificial radioactive isotopes. Neutrons may be produced from the bombardment of beryllium with α-particles. The reaction equation for this is

$${}^{4}_{2}\text{He} + {}^{19}_{4}\text{Be} \rightarrow {}^{12}_{6}\text{C} + {}^{1}_{0}\text{n}$$

A typical neutron reaction is the bombardment of lithium to produce the radioactive isotope of hydrogen called tritium. The nuclear reaction equation for this is

$${}^{6}_{3}\text{Li} + {}^{1}_{0}\text{n} \rightarrow {}^{3}_{1}\text{H} + {}^{4}_{2}\text{He}$$

For those of you studying the Option I (Medical Physics), the importance of artificial isotopes in both therapy and diagnosis, is discussed in detail.

It is left as an exercise for you to identify the bombarding particle in following nuclear reactions:

1. ${}^{14}_{7}\text{N} + \rightarrow {}^{1}_{1}\text{H} + {}^{14}_{6}\text{C}$
2. ${}^{6}_{3}\text{Li} + \rightarrow {}^{4}_{2}\text{He} + {}^{4}_{2}\text{He}$
3. ${}^{205}_{81}\text{Tl} + \rightarrow {}^{206}_{82}\text{Pb} + {}^{1}_{0}\text{n}$

7.3.3 THE ATOMIC MASS UNIT (U)

In nuclear physics we are concerned with the interaction of different nuclei and therefore an explicit knowledge of individual isotopic masses is of fundamental importance. For this reason the old scale of atomic weights based on expressing the atomic weight of oxygen as 16, is of little use, oxygen alone having three isotopes. The atomic mass scale was therefore introduced based on the **atomic mass unit.** This unit is defined as $\frac{1}{12}$ the mass of a nucleus of carbon-12 or, to put it another way, a carbon nucleus as a mass of exactly 12 u. We know that 12 g of carbon (1 mole) has 6.02×10^{23} nuclei. Therefore 1 u is equivalent to

$$\frac{1}{12}\text{th of } \frac{12}{6.02\times10^{23}} = 1.661 \times 10^{-24}\text{ g} = 1.661 \times 10^{-27}\text{ kg.}$$

7.3.4 THE EINSTEIN MASS-ENERGY RELATION

(*This topic is discussed in more detail in Option H, topic 4*)

One consequence of *Einstein's Special Theory of Relativity* is that in order for the conservation of momentum to be conserved for observers in relative motion, the observer who considers him or herself to be at rest, will observe that the mass of an object in the moving reference frame increases with the relative speed of the frames. One of the frames of reference might be a moving object. For example, if we measure the mass of an electron moving relative to us, the mass will be measured as being greater than the mass of the electron when it is at rest relative to us, the *rest-mass*. However, an observer sitting on the electron will measure the rest mass. This leads to the famous Einstein equation $E_{tot} = mc^2$ where E_{tot} (often just written as E) is the total energy of the body, m is the measured mass and c is the speed of light in a vacuum. The total energy E_{tot} consists of two parts, 'the rest-mass energy' and the kinetic energy E_K of the object. So in general it is written:

$$E_{tot} = m_0c^2 + E_K$$

Where m_0 is the rest mass of the object.

Nuclear reactions are often only concerned with the rest-mass.

Essentially, the Einstein equation tells us that energy and mass are interchangeable. If for example, it were possible to convert 1 kg of matter completely to energy we would get 9×10^{16} J of energy. ($1 \times c^2$) Looking it another way, if a coal-fired power station produces say 9 TJ of energy a day, then if you were to measure the mass of the coal used per day and then measure the mass of all the ash and fumes produced per day, you would find that the two masses would differ by 0.1 g ($\frac{9 \times 10^{12}}{C^2}$).

In table 701 we have seen that it is usual to express particle energies in electronvolt rather than joule. Using the Einstein relation, we can express particle mass in derived energy units.

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For example, the atomic mass unit = 1.661×10^{-27} kg

which is equivalent to $1.661 \times 10^{-27} \times (2.998 \times 10^{8})^2$ J

or $\dfrac{1.661\times10^{-27} \times(2.998 \times10^{8})^2}{1.602\times10^{-19}}$eV = 931.5 MeV

(working to four significant digits).

However, bearing in mind that energy and mass cannot have the same units and that $m = \dfrac{E}{c^2}$,

we have that 1 atomic mass unit = 931.5 MeV c^{-2}.

Table 710 shows the different units used for the rest mass of the electron, proton and neutron.

(All values, apart from values in atomic mass unit, are quoted to four significant digits).

particle	Rest-mass		
	kg	u	MeV c^{-2}
electron (me)	9.109×10^{-31}	0.00054858	0.5110
proton (mp)	1.673×10^{-27}	1.007276	938.2
neutron (mn)	1.675×10^{-27}	1.008665	939.6

Figure 710 Particle rest-mass (table)

7.3.4-6 APPLYING THE MASS-ENERGY RELATION

Mass defect *Q*

Let us now examine a nuclear reaction using the idea of mass-energy conversion. For example, consider the decay of a nucleus of raduim-226 into a nucleus of radon-222. The reaction equation is

$$^{226}_{88}\mathrm{Ra} \to {}^{222}_{86}\mathrm{Rn} + {}^{4}_{2}\mathrm{He}$$

The rest masses of the nuclei are as follows

$^{226}_{88}\mathrm{Ra}$ = 226.0254 u

$^{222}_{86}\mathrm{Rn}$ = 222.0175 u

$^{4}_{2}\mathrm{He}$ = 4.0026 u

The right-hand side of the reaction equation differs in mass from the left-hand side by +0.0053 u. This mass deficiency, or **mass defect** as it is usually referred to, just as in the coal-fired power station mentioned above, represents the energy released in the reaction. If ignoring any recoil energy of the radium nucleus, then this energy is the kinetic energy of the α-particle emitted in the decay.

Using the conversion of units, we see that 0.0053 u has a mass of 4.956 MeV c^{-2}. This means that the kinetic energy of the α-particle is 4.956 MeV.

The fact that the mass defect is positive indicates that energy is released in the reaction and the reaction will take place spontaneously.

If the mass defect is negative in a reaction then this means that energy must be supplied for the reaction to take place. For example, let us postulate the following reaction:

$$^{23}_{11}\mathrm{Na} \to {}^{19}_{9}\mathrm{F} + {}^{4}_{2}\mathrm{He} + Q$$

where Q is the mass defect.

If factoring in the rest masses the equation becomes

22.9897 u → 18.9984 u + 4.0026 u + Q

This gives Q = - 0.0113 u = -10.4 MeV c^{-2}.

In other words for such a reaction to take place 10.4 MeV of energy must be supplied. $^{23}_{11}\mathrm{Na}$ is therefore not radioactive but a stable nuclide.

Binding energy

A very important quantity associated with nuclear reaction is the **nuclear binding energy.** To understand this concept, suppose we add up the individual masses of the individual nucleons that comprise the helium nucleus, then we find that this sum does not equal the mass of the nucleus as a whole. This is shown below

$$2m_p + 2\,m_n \to {}^{4}_{2}\mathrm{He} + Q$$

(2 × 938.2 + 2 × 939.6) MeV c^{-2} → 3728 MeV c^{-2} + Q

To give Q = 28.00 MeV c^{-2}.

This effectively means that when a helium nucleus is assembled from nucleons, 28 MeV of energy is released. Or looking at it another way, 28 MeV of energy is required to separate the nucleus into its individual nucleons since if we postulate, as we did above for the decay of $^{23}_{11}\mathrm{Na}$, this reaction

$$^4_2\text{He} \rightarrow 2m_p + 2\,m_n + Q$$

then $Q = -\ 28\ \text{MeV}\ c^{-2}$.

The definition of nuclear binding energy is therefore either **the energy required to separate the nucleus into it individual nucleons** or **the energy that would be released in assembling a nucleus from its individual nucleons**. Since the potential energy of a nucleus is *less* than the potential energy of its separate nucleons, some texts take the binding energy to be a **negative** quantity. In this book, however, we will regard it to be a **positive** quantity on the basis that the greater the energy required to separate a nucleus into its nucleons, the greater the difference between the potential energy of the nucleus and its individual nucleons.

Binding energy per nucleon

Rather than just refer to the binding energy of the nuclei of different isotopes, it is much more important to consider the binding energy per nucleon. The addition of each nucleon to a nucleus increases the total binding energy of the nucleus by about 8 MeV. However, the increase is not linear and if we plot the binding energy per nucleon against nucleon number N, the graph (Figure 711) shows some very interesting features.

The most stable nuclei are those with the greatest binding energy per nucleon as this means that more energy is required to separate the nucleus into its constituent nucleons. For example, much less energy is required per nucleon to "take apart" a nucleus of uranium-235 than a nucleus of helium-4. The most stable element is iron (Fe) as this has the greatest binding energy per nucleon. This graph shall be referred to again in section 7.3.9

7.3.7 Solve problems involving mass defect and binding energy

Example

1. Calculate the kinetic energy in MeV of the tritium plus the helium nucleus in the following nuclear reaction.

$$^6_3\text{Li} + ^1_0\text{n} \rightarrow ^3_1\text{H} + ^4_2\text{He}$$

mass of ^6_3Li = 6.015126 u

mass of ^3_1H = 3.016030 u

mass of ^4_2He = 4.002604 u

neutron mass = 1.008665 u

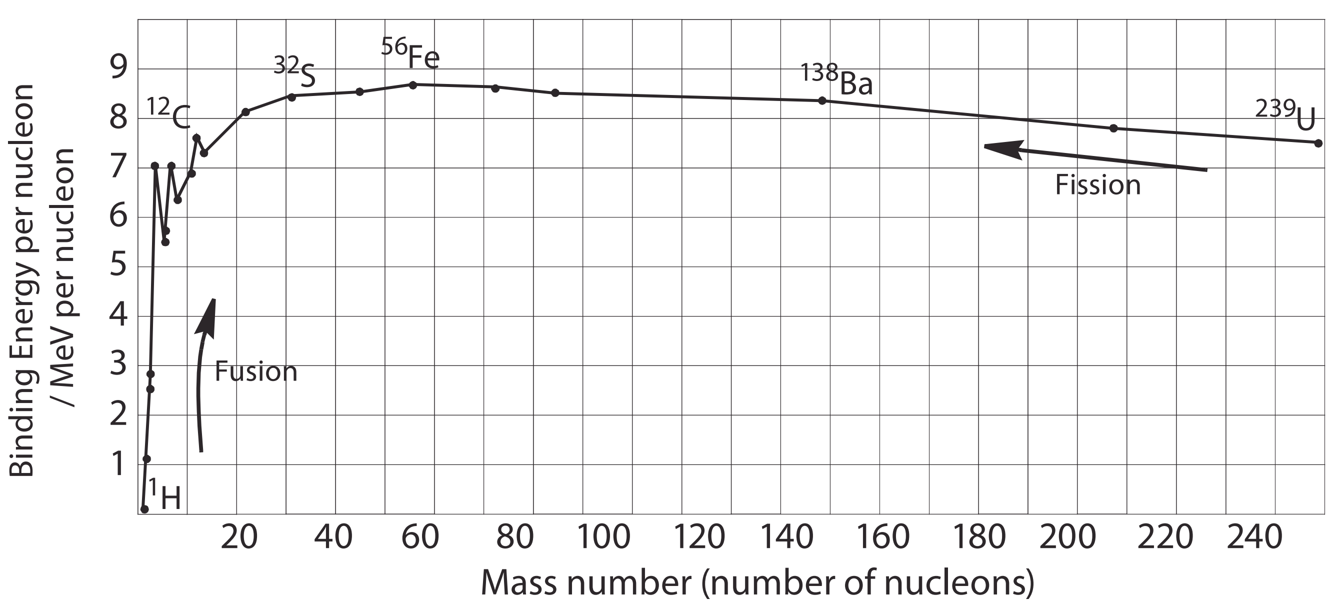

Figure 711 Binding energy per nucleon

Solution

Adding the masses of the left-hand and right-hand side of reaction equation gives

Q = 005157 u. Using 1 u = 935.1 MeV, then Q = 4.822 MeV.

Exercise 7.3

1. Calculate the energy required to separate a nucleus of lithium-6 into its constituent nucleons. Hence find the binding energy per nucleon of lithium-6.

2. Calculate the binding energy per nucleon of an α-particle

3. Deduce, whether the following reaction may take place spontaneously.

   $^{212}_{83}\mathrm{Bi} \rightarrow {}^{208}_{81}\mathrm{Tl} + \mathrm{e}^{+}$

   mass of $^{212}_{83}\mathrm{Bi}$ = 211.99127 u

   mass of $^{208}_{81}\mathrm{Tl}$ = 207.98201 u

FISSION AND FUSION

7.3.8 Describe the processes of nuclear fission and nuclear fusion.

7.3.9 Apply the graph in 7.3.6 to account for the energy release in the processes of fission and fusion.

7.3.10 State that nuclear fusion is the main source of the Sun's energy.

7.3.11 Solve problems involving fission and fusion reactions.

7.3.8 Fission and fusion

(The use of fission and fusion as world energy sources is also discussed in detail in Topic 8.4)

Fission

Nuclear reactions produce very much more energy per particle than do chemical reactions. For example, the oxidization of one carbon atom produces about 4 eV of energy whereas the decay of a uranium atom produces about 4 MeV. However, natural radioactive isotopes do not occur in sufficient quantity to be a practical source of energy. It was not until the discovery of **nuclear fission** that the possibility of nuclear reactions as a cheap and abundant source of energy became possible. In 1934 Fermi discovered that when uranium was bombarded with neutrons, radioactive products were produced. Then in 1939 Hahn and Strassman showed that one of the radioactive products was barium (Z = 56). It is now understood that a nucleus of uranium may capture a neutron to form an unstable isotope. Either of the following reactions may occur:

$$^{238}_{92}\mathrm{U} + {}^{1}_{0}\mathrm{n} \rightarrow {}^{239}_{92}\mathrm{U} \rightarrow {}^{239}_{93}\mathrm{Np} + \mathrm{e}^{-}$$

(where Np is the transuranic nuclide *neptunium)*

$$^{238}_{92}\mathrm{U} + {}^{1}_{0}\mathrm{n} \rightarrow {}^{239}_{92}\mathrm{U} \rightarrow X + Y + x\,{}^{1}_{0}\mathrm{n}$$

where X and Y are two fission elements and x is the number of neutrons produced.

Which reaction takes place is dependant on the energy of the bombarding neutron.

A typical fission reaction might be

$${}^{238}_{92}\text{U} + {}^{1}_{0}\text{n} \rightarrow {}^{90}_{38}\text{Sr} + {}^{146}_{54}\text{Xe} + 3\,{}^{1}_{0}\text{n}$$

Given the following data, it is left as an exercise for you to show that energy released in this reaction is about 150 MeV:

mass of ${}^{238}\text{U}$ = 238.050788 u

mass of ${}^{90}\text{Sr}$ = 89.907737 u

mass of ${}^{146}\text{Xe}$ = 145.947750 u

The energy released appears in the form of kinetic energy of the fission nuclei and neutrons.

The two neutrons produced is the key to using fission as a sustainable energy source as discussed in topic 8.4. Both the strontium isotope and xenon isotope produced are radioactive. Strontium-90 has a half-life of about 30 years and therein lies the main problem (as well as the large amounts of γ-radiation also produced) with nuclear fission as a sustainable energy source – the fact that the fission nuclei are radioactive often with relatively long half-lives.

The isotope uranium-235 also undergoes fission and much more readily than uranium-238. A typical fission reaction might be

$${}^{235}_{92}\text{U} + {}^{1}_{0}\text{n} \rightarrow {}^{103}_{38}\text{Sr} + {}^{131}_{54}\text{I} + 2\,{}^{1}_{0}\text{n}$$

Fusion

Energy can also be obtained from nuclear reactions by arranging for two nuclei to "fuse" together as we alluded to when we discussed nuclear binding energy above. To produce **nuclear fusion** very high temperatures and pressures are needed so that nuclei can overcome the coulomb repulsion force between them and thereby come under the influence of the strong nuclear force. A typical nuclear reaction might be ${}^{2}_{1}\text{H} + {}^{3}_{1}\text{H} \rightarrow {}^{4}_{2}\text{He} + {}^{1}_{0}\text{n}$

In this reaction a nucleus of deuterium combines with a nucleus of tritium to form a nucleus of helium and a free neutron.

Given the following data it is left as an exercise for you to show that energy released in this reaction is about 18 MeV:

mass of ${}^{2}\text{H}$ = 2.014102 u

mass of ${}^{3}\text{H}$ = 3.016049 u

mass of ${}^{4}\text{He}$ = 4.002604 u

The energy released appears in the form of kinetic energy of the helium nucleus and neutron.

The advantage that fusion has compared to fission as a source of sustainable energy is that no radioactive elements are produced. This disadvantage is obtaining and maintaining the high temperature and pressure needed to initiate fusion. Again, this is discussed in more detail in topic 8.4.

7.3.9 Applying the binding energy curve

The graph in figure 711 of binding energy per nucleon versus nucleon number shows that the nuclides with a nucleon number of about 60 are the most stable. This helps us to understand why the high nucleon number nuclides may undergo fission and the low nucleon number nuclides may under go fusion- they are trying to "reach" the nuclide that is most stable. For example consider the fission reaction

$${}^{238}_{92}\text{U} + {}^{1}_{0}\text{n} \rightarrow {}^{90}_{38}\text{Sr} + {}^{146}_{54}\text{Xe} + 3\,{}^{1}_{0}\text{n}$$

From the graph (figure 711) we have that

total binding energy of ${}^{238}\text{U}$ = 7.6×238 = 1800 MeV

total binding energy of ${}^{90}\text{Sr}$ = 8.7×90 = 780 MeV

total binding energy of ${}^{146}\text{Xe}$ = 8.2×146 = 1200 MeV

Hence the sum of the total binding energies of the fission nuclei is greater than the total binding energy of the uranium-238 nucleus. Effectively the system has become more stable by losing energy.

Similarly for the fusion reaction ${}^{2}_{1}\text{H} + {}^{3}_{1}\text{H} \rightarrow {}^{4}_{2}\text{He} + {}^{1}_{0}\text{n}$ the total binding energy of the helium nucleus is greater than the sum of binding energies of the tritium and deuterium nuclei. So, again as for fission, the system has effectively become more stable by losing energy.

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7.3.10 NUCLEAR FUSION AND THE SUN

Our Sun is an enormous factory in which hydrogen is converted into helium. At some time in its early life, due to gravitational collapse of the hydrogen making up the Sun, the pressure and temperature of the interior became high enough to initiate fusion of the hydrogen. Once started, the fusion will continue until all the hydrogen is used up, probably in about 10^{10} years from now.

One of the suggested fusion cycles that may take place in the Sun is

$$p^+ + p^+ \rightarrow {}^2_1\text{H} + e^+$$
$${}^2_1\text{H} + p^+ \rightarrow {}^3_2\text{He}$$
$${}^3_2\text{He} + {}^3_2\text{He} \rightarrow {}^4_2\text{He} + p^+$$

For the complete cycle the first two reaction must occur twice and the final result is one helium nucleus, two positrons, two protons and two neutrinos. The protons are available for further fusion.

In stars that are much more massive than our Sun, as they age fusion of elements with higher atomic numbers takes place until finally iron is reached and no further fusion can take place as seen from the binding energy graph. This evolution of stars is discussed in detail in Option E.

7.3.11 SOLVE PROBLEMS ON FISSION AND FUSION REACTIONS

The type of problems you might be expected to solve associated with fission and fusion have been looked at above. However, here are a two more.

Exercises

1. Determine the number x of neutrons produced and calculate the energy released in the following fission reaction

$${}^{235}_{92}\text{U} + {}^1_0\text{n} \rightarrow {}^{144}_{56}\text{Ba} + {}^{90}_{36}\text{Kr} + x\,{}^1_0\text{n}$$

mass of ${}^{235}\text{U}$ = 235.043929 u

mass of ${}^{146}\text{Ba}$ = 143.922952 u

mass of ${}^{90}\text{Kr}$ = 89.919516 u

2. Show that in the fusion cycle given in 7.4.10, the energy released is about 30 MeV.