Units

Fundamental units

It is a fascinating fact that all physical quantities have units that can be expressed in terms of those for just 7 **fundamental** quantities (the IB syllabus uses only the first 6). The 7 fundamental quantities in the S.I. system and their units are:

1	time	second (s)
2	length	metre (m)
3	mass	kilogram (kg)
4	temperature	kelvin (K)
5	quantity of matter	mole (mol)
6	electric current	ampere (A)
7	luminous intensity	candela (cd)

Derived units

All other quantities have **derived** units i.e. combinations of the fundamental units. For example, the derived unit for force (the newton) is obtained using $F = ma$ to be $\mathrm{kg\,m\,s^{-2}}$ and that for electric potential difference (the volt) is obtained using $W = qV$ to be $\frac{\mathrm{J}}{\mathrm{C}} = \frac{\mathrm{N\,m}}{\mathrm{A\,s}} = \frac{\mathrm{kg\,m\,s^{-2}\,m}}{\mathrm{A\,s}} = \mathrm{kg\,m^2\,s^{-3}\,A^{-1}}$.

Significant figures

There is a difference in stating that the measured mass of a body is 283.63 g rather than 283.6 g. The uncertainty in the first measurement is expected to be ±0.01 g and that in the second ±0.1 g, i.e. the first measurement is more precise – it has more **significant figures** (s.f.). When we do operations with numbers (multiplication, division, powers and roots) we must express the result to as many s.f. as those in the least precisely known number in the operation.

Number		Number of s.f.	Scientific notation
34		2	3.4×10^1
3.4		2	3.4×10^0 or just 3.4
0.0340	zeros in front do not count but zeros at the end **in a decimal** do count	3	3.40×10^{-2}
340	zeros at the end **in an integer** do not count	2	3.4×10^2

Thus the kinetic energy of a mass of 2.4 kg (2 s.f.) moving at $14.6\,\mathrm{m\,s^{-1}}$ (3 s.f.) is $E_k = \frac{1}{2} \times 2.4 \times 14.6^2 = 255.792\,\mathrm{J} \approx 260 = 2.6 \times 10^2\,\mathrm{J}$. Similarly, the acceleration of a body of mass 1200 kg (2 s.f.) acted upon by a net force of 5250 N (3 s.f.) is $\frac{5250}{1200} = 4.375 \approx 4.4\,\mathrm{m\,s^{-2}}$.

Test yourself **1**

The force of resistance from a fluid on a sphere of radius r is given by $F = 6\pi\eta r v$ where v is the speed of the sphere and η is a constant. What are the units of η?

Test yourself **2**

The radius R of the fireball t seconds after the explosion of a nuclear weapon depends only on the energy E released in the explosion, the density ρ of air and the time t. Show that the quantity $\frac{Et^2}{\rho}$ has units of $\mathrm{m^5}$ and hence that $R \approx \left(\frac{Et^2}{\rho}\right)^{\frac{1}{5}}$. Calculate the energy released if the radius of the fireball is 140 m after 0.025 s. (Take $\rho = 1.0\,\mathrm{kg\,m^{-3}}$.)

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Uncertainties

Definitions

Random uncertainties Uncertainties due to the inexperience of the experimenter and the difficulty of reading instruments. They can be reduced by repeated measurements and taking an average of the measurements.

Systematic uncertainties Uncertainties mainly due to incorrectly calibrated instruments. They cannot be reduced by repeated measurements.

Accurate measurements Measurements that have a small systematic error.

Precise measurements Measurements that have a small random error.

We measure the length of the side of a cube to be 25±1 mm. The 25 mm represents the **measured value** of the length and the ±1 mm represents the **absolute uncertainty** in the measured value. The ratio $\frac{1}{25}\times 0.04$ is the **fractional uncertainty** in the length and $\frac{1}{25}\times 100\% = 4\%$ is the **percentage uncertainty** in the length.

In general for a quantity Q we have $Q = \underbrace{Q_0}_{\text{measured value}} \pm \underbrace{\Delta Q}_{\text{absolute uncertainty}}$, $\frac{\Delta Q}{Q_0}$ = fractional uncertainty, $\frac{\Delta Q}{Q_0}\times 100\%$ = percentage uncertainty.

Suppose quantities a, b and c have been measured with uncertainties, respectively, Δa, Δb and Δc. If we use these quantities to calculate another quantity Q, these uncertainties will cause uncertainties in Q. The (approximate) rules for calculating the uncertainty in Q are:

if $Q = a \pm b \pm c$ then $\Delta Q = \Delta a + \Delta b + \Delta c$.

For **both** addition and subtraction **add absolute** uncertainties.

If $Q = \frac{ab}{c}$, then $\frac{\Delta Q}{Q} = \frac{\Delta a}{a} + \frac{\Delta b}{b} + \frac{\Delta c}{c}$

For multiplication and/or division, **add fractional** or **percentage** uncertainties to get fractional or percentage uncertainty in the result.

If $Q = \frac{a^n}{b^m}$, then $\frac{\Delta Q}{Q} = n\frac{\Delta a}{a} + m\frac{\Delta b}{b}$

For square roots and other roots, **add fractional** or **percentage** uncertainties and then divide by the power of the root. This is a special case of the rule for powers.

If $Q = \sqrt{ab}$ or $Q = \sqrt{\frac{a}{b}}$, then $\frac{\Delta Q}{Q} = \frac{1}{2}\frac{\Delta a}{a} + \frac{1}{2}\frac{\Delta b}{b}$

Test yourself **3**

The resistance of a lamp is given by $R = \frac{V}{I}$. The uncertainty in the voltage is 4% and that in the current is 6%. What is the absolute uncertainty in a calculated resistance value of 24 Ω?

Test yourself **4**

Each side of a cube is measured with a fractional uncertainty of 0.02. Estimate the percentage uncertainty in the volume of the cube.

Test yourself **5**

The period of oscillation of a mass m at the end of a spring of spring constant k is given by $T = 2\pi\sqrt{\frac{m}{k}}$. What is the percentage uncertainty of the period if m is measured with a percentage uncertainty 4% and the k with a percentage uncertainty 6%?

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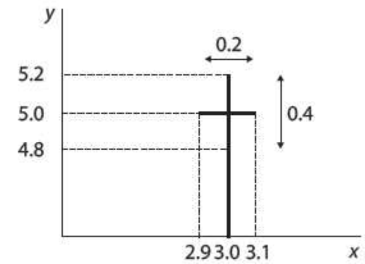
Sample

Graphical analysis

Error bars

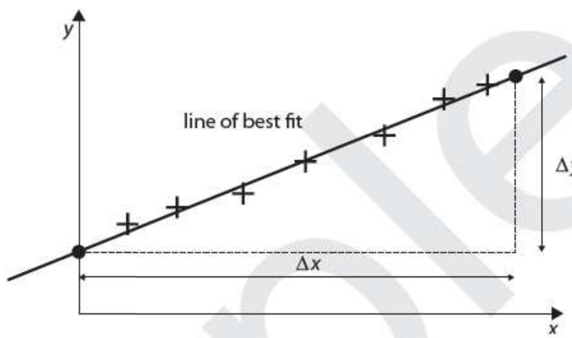
Suppose that we want to plot the point (3.0 ± 0.1, 5.0 ± 0.2) on a set of x- and y-axes. First we plot the point with coordinates (3.0, 5.0) and then show the uncertainties as error bars. The horizontal error bar will have length $2 \times 0.1 = 0.2$ and the vertical will have length $2 \times 0.2 = 0.4$.



Definition

Line of best fit The curve or straight line that goes through all the error bars.

> According to the IB, a 'line' may be straight or curved and you must remember that. Do **not** assume a 'line' to mean a straight line.

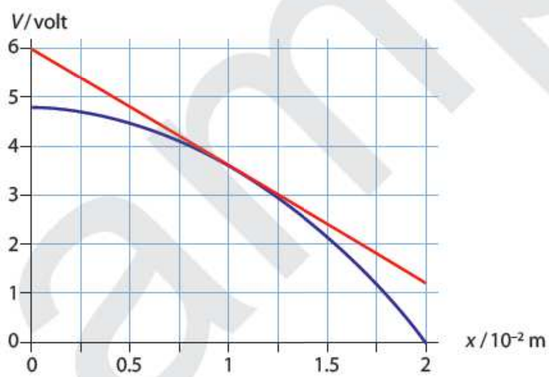


Finding slopes (gradients)

To find the slope of a curve at a particular point draw the tangent to the curve at that point. Choose two points **on the tangent** that are as far apart as possible to form the 'triangle'.

$$\text{slope} = \frac{6.0 - 1.2}{0.0 - 2.0 \times 10^{-2}} \frac{\text{volt}}{\text{m}}$$

$$= -2.4 \times 10^{2}\,\text{V m}^{-1}$$

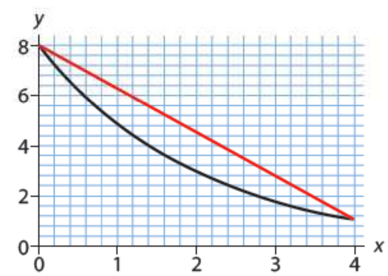


> Notice that the units on the horizontal axis in the graph are in terms of 10^{-2}. Make sure you get the right sign of the slope. Make sure you state its correct units.

Estimating areas under curves

To estimate the area under the curve in the graph, draw the straight line from the point (0, 8) to the point (4, 1.1). Each small square has area $0.2 \times 0.4 = 0.08$ square units.

It is easy to find the area of the trapezium formed as $\frac{(8+1.1)}{2} \times 4 = 18.2$. Now count squares in between the straight line and the curve and subtract. The number of squares in between is about 60 and so the required area is about $18.2 - 60 \times 0.08 = 13.4$ square units. (The exact area under the curve from $x = 0$ to $x = 4$ is 13.8 square units.)



> Use common sense – break up the area in order to reduce the counting of squares whenever possible.

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Getting straight-line graphs

If we know the relationship between two variables we can usually arrange to plot the data in such a way that we get a straight line. We must always bear in mind that the standard equation of a straight line is

$$y = \underset{\text{gradient}}{m}\, x + \underset{\text{vertical intercept}}{c}$$

where we plot the variable y on the vertical axis and the variable x on the horizontal.

> We say that y is proportional to x if $c = 0$ i.e. if the straight line goes through the origin. If the line of best fit is not straight or does not go through the origin, then **either** reason is sufficient to claim that y is **not** proportional to x.

Consider the relationship $T = 2\pi\sqrt{\frac{m}{k}}$ for the period T of a mass m undergoing oscillations at the end of a spring of spring constant k. Since:

$$T = \frac{2\pi}{\sqrt{k}} \times \sqrt{m}$$

$$\downarrow \qquad \qquad \downarrow$$

$$y = \underset{\text{constants}}{\frac{2\pi}{\sqrt{k}}} \times x$$

By identifying $T \leftrightarrow y$ and $\sqrt{m} \leftrightarrow x$ we get the equation of a straight line $y = \frac{2\pi}{\sqrt{k}}x$. So we must plot T on the vertical axis and $\sqrt{m}$ on the horizontal axis to get a straight line whose gradient will be $\frac{2\pi}{\sqrt{k}}$. Alternatively we may write

$$T^2 = \frac{4\pi^2}{k} \times m$$

$$\downarrow \qquad \qquad \downarrow$$

$$y = \underset{\text{constants}}{\frac{4\pi^2}{k}} \times x$$

By identifying $T^2 \leftrightarrow y$ and $m \leftrightarrow x$ we get the equation of a straight line $y = \frac{4\pi^2}{k}x$. So we must plot T^2 on the vertical axis and m on the horizontal axis to get a straight line whose gradient will be $\frac{4\pi^2}{k}$.

A different procedure must be followed if the variables are related through a power relation: $F = kr^n$ where the constants k and n are unknown. Taking natural logs (or logs with any base) we have

$$\ln F = \ln k + n \times \ln r$$

$$\downarrow \qquad \qquad \qquad \downarrow$$

$$y = \ln k + n \times x$$

and so plotting $\ln F$ against $\ln r$ gives a straight line with gradient n and vertical intercept $\ln k$.

A variation of this is used for an exponential equation such as, $A = A_0 e^{-\lambda t}$ where A_0 and λ are constants. Here we must take logs to get $\ln A = \ln A_0 - \lambda t$ and so

$$\ln A = \ln A_0 - \lambda \times t$$

$$\downarrow \qquad \qquad \qquad \downarrow$$

$$y = \ln A_0 - \lambda \times x$$

> Plot $\ln A$ on the vertical axis and t on the horizontal so that we get a straight line with gradient $-\lambda$ and vertical intercept $\ln A_0$.

Sample

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Test yourself **6**

Copy and complete this table.

Equation	Constants	Variables to plot to give straight line	Gradient	Vertical intercept
$P = kT$	k			
$v = u + at$	u, a			
$v^2 = 2as$	a			
$F = \frac{kq_1q_2}{r^2}$	k, q_1, q_2			
$a = -\omega^2 x$	ω^2			
$V = \frac{kq}{r}$	k, q			
$T^2 = \frac{4\pi^2}{GM}R^3$	G, M			
$I = I_0 e^{-aT}$	I_0, a			
$\lambda = \frac{h}{\sqrt{2mqV}}$	h, m, q			
$F = av + b^2$	a, b			
$E = \frac{1}{2} m\omega^2 \sqrt{A^2 - x^2}$	m, ω^2, A			
$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$	f			

Test yourself **7**

State what variables must be plotted so that we get a straight line for the relation $d = ch^{0.8}$ where c is a constant.

Estimating uncertainties in measured quantities

The general simple rule is:

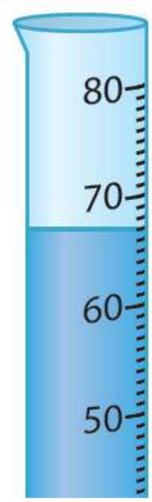
For **analogue metres**, take half of the smallest scale division. For example, for an ordinary metre rule the smallest scale division is 1 mm and so the uncertainty is ±0.5 mm. (But bear in mind that if a metre rule is to be used to measure the length of say a rod, the uncertainty of ±0.5 mm applies to the measurement of each end of the rod for a total uncertainty of ±1 mm.)

Note that this is a conservative approach. Someone may claim to be able to read to better precision than this. See the next example.

For **digital meters**, take ± of the smallest division. For example, for a digital voltmeter that can read to the nearest hundredth of a volt, the uncertainty is ±0.1V. For an ammeter that can read to the nearest tenth of an ampere take ±0.1 A.

Test yourself **8**

Estimate the reading and the uncertainty in each of the instruments in the diagrams.



Sample

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Uncertainty in the measured value of a gradient (slope)

To find the uncertainty in the gradient of the (straight) line of best fit, draw the lines of maximum and minimum gradient. Calculate these two gradients, m_{max} and m_{min}. A simple estimate of the uncertainty in the gradient is then $\frac{m_{max} - m_{min}}{2}$.

Test yourself 9

Electrons that have been accelerated by a potential difference V enter a region of magnetic field where they are bent into a circular path of radius r. Theory suggests that $r^2 = \frac{2m}{qB^2}V$. The table shows values of the radius r and potential difference V obtained in an experiment.

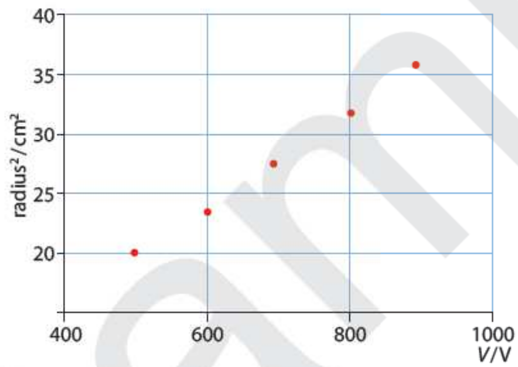
Radius r/cm ±0.1 cm	**Potential difference V/V**	**r^2/cm²**
4.5	500	±
4.9	600	±
5.3	700	±
5.7	800	±
6.0	900	±

(a) Explain why a graph of r^2 against V will result in a straight line.

(b) State the slope of the straight line in (a) in terms of the symbols m, q and B.

(c) In the right column in the table, insert values of the radius squared including the uncertainty.

The graph shows the data points plotted on a set of axes.



(d) Draw error bars for the first and the last data points.

(e) Draw a line of best fit for these data points.

(f) Calculate the gradient of the line of best fit including its uncertainty.

The magnetic field used in this experiment was $B = 1.80 \times 10^{-3}$ T.

(g) Calculate the value of the charge to mass ratio $\left(\frac{q}{m}\right)$ for the electron that this experiment gives. Include the uncertainty in the calculated value.

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Vectors

Definitions

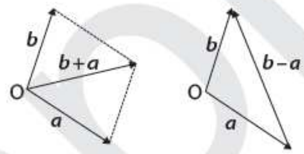
Vectors Physical quantities that have both **magnitude** and **direction**. They are represented by arrows and written in ***bold italics***. The **length** of the arrow gives the magnitude of the vector. The **direction** of the arrow is the direction of the vector. (Vectors can be positive or negative.)

Scalars Physical quantities with magnitude but **not** direction.

Vectors	Scalars
displacement	distance
velocity	time/mass
acceleration	energy/work/power
force	temperature
momentum/impulse	electric current/resistance
electric/gravitational/magnetic fields	electric/gravitational potential

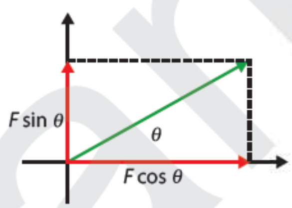
Adding vectors Have ***a*** and ***b*** start at the same point, O. Draw the parallelogram whose two sides are ***a*** and ***b***. Draw the diagonal starting at O.

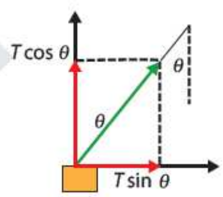
Subtracting vectors Have ***a*** and ***b*** start at the same point, O. To find ***b*** – ***a*** draw the vector from the tip of ***a*** to the tip of ***b***.

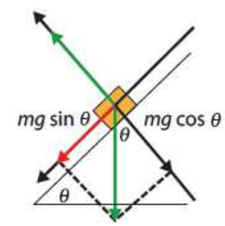


Components of vectors

As the diagrams show, the component adjacent to the angle θ involves $\cos\theta$ and that opposite to θ involves $\sin\theta$. Follow the three stages: **draw the forces, add the axes, get the components**. Choose as one of your axes the direction in which the body moves or would move if it could.

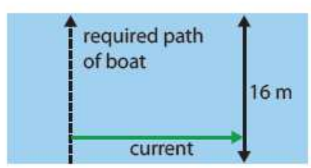






Test yourself **10**

A river is 16 m wide. The diagram shows a boat travelling at 4.0 m s^{-1} with respect to the water when the current has a speed of 3.0 m s^{-1} with respect to the shore and is directed to the right. The boat is rowed in such a way so as to arrive at the opposite shore directly across from where it started. Calculate the time taken for the trip.



Sample

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Order of magnitude estimates

The tables below give values for various distances, masses and times. You are not supposed to know these by heart but you must have a **general** idea of the size, mass and duration of various quantities and processes. Use the information in these tables to answer the questions which follow.

	Length / m
radius of the observable universe	10^{26}
distance to the Andromeda galaxy	10^{22}
diameter of the Milky Way galaxy	10^{21}
distance to Proxima Centauri (star)	10^{16}
diameter of solar system	10^{13}
distance to Sun	10^{11}
radius of the Earth	10^{7}
size of a cell	10^{-5}
size of a hydrogen atom	10^{-10}
size of an average nucleus	10^{-15}
Planck length	10^{-35}

	Mass / kg
the universe	10^{53}
the Milky Way galaxy	10^{41}
the Sun	10^{30}
the Earth	10^{24}
Boeing 747 (empty)	10^{5}
an apple	0.2
a raindrop	10^{-6}
a bacterium	10^{-15}
mass of smallest virus	10^{-21}
a hydrogen atom	10^{-27}
an electron	10^{-30}

	Time / s
age of the universe	10^{17}
time of travel by light to nearest star (Proxima Centauri)	10^{8}
one year	10^{7}
one day	10^{5}
period of a heartbeat	1
period of red light	10^{-15}
time of passage of light across an average nucleus	10^{-23}
Planck time	10^{-43}

Test yourself **11**
Estimate the weight of an apple.

Test yourself **12**
Estimate the number of seconds in a year.

Test yourself **13**
Estimate the time taken by light to travel across a nucleus.

Test yourself **14**
Estimate the time in between two of your heart beats.

Test yourself **15**
Estimate how many grains of sand can fit into the volume of the Earth.

Test yourself **16**
Estimate the number of water molecules in a glass of water.

Test yourself **17**
If the temperature of the Sun were to increase by 2% and the distance between the Earth and the Sun were to decrease by 1%, by how much would the intensity of the radiation received on Earth change?

Sample

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