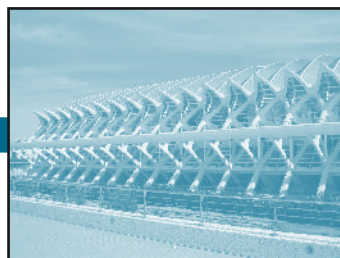


# PHYSICS AND PHYSICAL MEASUREMENT

- 1.1 The realm of Physics
- 1.2 Measurements and uncertainties
- 1.3 Vectors and scalars

1

CORE



## 1.1 THE REALM OF PHYSICS

### RANGE OF MAGNITUDES OF QUANTITIES IN OUR UNIVERSE

- 1.1.1 State and compare quantities to the nearest order of magnitude.
- 1.1.2 State the ranges of magnitude of distances, masses and times that occur in the universe, from smallest to greatest.
- 1.1.3 State ratios of quantities as differences of orders of magnitude.
- 1.1.4 Estimate approximate values of everyday quantities to one or two significant figures and/or to the nearest order of magnitude.

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### 1.1.1 ORDER OF MAGNITUDE

The **order of magnitude** of a number is the power of ten closest to that number. Often when dealing with very big or very small numbers, scientists are more concerned with the order of magnitude of a measurement rather than the precise value. For example, the number of particles in the Universe and the mass of an electron are of the orders of magnitude of  $10^{80}$  particles and  $10^{-30}$  kg. It is not important to know the exact values for all microscopic and macroscopic quantities because when you are using the order of magnitude of a quantity, you are giving an indication of size and not necessarily a very accurate value.

The order of magnitude of large or small numbers can be difficult to comprehend at this introductory stage of the course. For example,  $10^{23}$  grains of rice would cover Brazil to a depth of about one kilometre.

### 1.1.2 RANGE OF MAGNITUDES OF THE UNIVERSE

The order of magnitude of some relevant lengths in metres (m), masses in kilograms (kg) and times in seconds (s) are given in Figure 101.

Mass of Universe	$10^{50}$ kg	Height of a person	$10^0$ m
Mass of Sun	$10^{30}$ kg	1 gram	$10^{-3}$ kg
Extent of the visible Universe	$10^{25}$ m	Wavelength of visible light	$10^{-6}$ m
Mass of the earth	$10^{25}$ kg	Diameter of an atom	$10^{-10}$ m
Age of the Universe	$10^{18}$ s	Period of visible light	$10^{-15}$ s
One light year	$10^{16}$ m	Shortest lived subatomic particle	$10^{-23}$ s
Human light span	$10^9$ s	Passage of light across the nucleus	$10^{-23}$ s
One year	$10^7$ s	Mass of proton	$10^{-27}$ kg
One day	$10^5$ s	Mass of neutron	$10^{-27}$ kg
Mass of car	$10^3$ kg	Mass of electron	$10^{-30}$ kg

Figure 101 Range of magnitudes

## Examples

- The number 8 is closer to  $10^1$  (10) than  $10^0$  (1). So the order of magnitude is  $10^1$ . Similarly, 10 000 has an order of magnitude of  $10^4$ .
- However,  $4.3 \times 10^3$  has an order of magnitude of  $10^4$ . The reason for this is if you use the log button on your calculator, the value of  $4.3 \times 10^3 = 10^{3.633}$ .

Therefore the order of magnitude is  $10^4$ . So, the normal mathematical rounding up or down above or below 5 does not apply with order of magnitude values. In fact,  $100^5 = 3.16$ . This becomes our 'rounding' value in determining the order of magnitude of a quantity.

Order of magnitude, for all its uncertainty, is a good indicator of size. Let's look at two ways of calculating the order of magnitude of the number of heartbeats of a human in a lifetime. The average relaxed heart, beats at 100 beats per minute. Do you agree? Try the following activity:

Using a timing device such as a wristwatch or a stopwatch, take your pulse for 60 seconds (1 minute). Repeat this 3 times. Find the average pulse rate. Now, using your pulse, multiply your pulse per minute (say 100)  $\times$  60 minutes in an hour  $\times$  24 hours in a day  $\times$  365.25 days in a year  $\times$  78 years in a lifetime. Your answer is  $3.945 \times 10^9$ . Take the log of this answer, and you get  $10^{9.596}$ . The order of magnitude is  $10^{10}$ . Now let us repeat this but this time we will use the order of magnitude at each step:

$$10^2 \text{ beats min}^{-1} \times 10^2 \text{ min h}^{-1} \times 10^1 \text{ h day}^{-1} \times 10^3 \text{ day yr}^{-1} \times 10^2 \text{ yr}$$

The order of magnitude is  $10^{10}$ . Do the same calculations using your own pulse rate. Note that the two uncertain values here are pulse rate and lifespan. Therefore, you are only giving an estimate or indication. You are not giving an accurate value.

### 1.1.3 RATIOS OF ORDERS OF MAGNITUDE

Ratios can also be expressed as differences in order of magnitude. For example, the diameter of the hydrogen atom has an order of magnitude of  $10^{-10}$  m and the diameter of a hydrogen nucleus is  $10^{-15}$  m. Therefore, the ratio of the diameter of a hydrogen atom to the diameter of a hydrogen nucleus is  $10^{-10}/10^{-15} = 10^5$  or five orders of magnitude.

The order of magnitude of quantities in the macroscopic world are also important when expressing uncertainty in a measurement. This is covered in section 1.2 of this chapter.

### Exercise 1.1

- The order of magnitude of 4 200 000 is:
  - $10^4$
  - $10^5$
  - $10^6$
  - $10^7$
- Give the order of magnitude of the following quantities
  - 20 000
  - $2.6 \times 10^4$
  - $3.9 \times 10^7$
  - $7.4 \times 10^{15}$
  - $2.8 \times 10^{-24}$
  - $4.2 \times 10^{-30}$
- Give the order of magnitude of the following measurements.
  - The mean radius of the Earth, 6 370 000 m
  - The half-life of a radioactive isotope 0.0015 s.
  - The mass of Jupiter  
1 870 000 000 000 000 000 000 000 kg.
  - The average distance of the moon from the earth is 380 000 000 m.
  - The wavelength of red light 0.000 000 7 m.
- The ratio of the diameter of the nucleus to the diameter of the atom is approximately equal to
  - $10^{-15}$
  - $10^{-8}$
  - $10^{-5}$
  - $10^{-2}$
- What is the order of magnitude of:
  - the time in seconds in a year.
  - the time for the moon to revolve around the earth in seconds.

6. A sample of a radioactive element contains  $6.02 \times 10^{23}$  atoms. It is found that  $3.5 \times 10^{10}$  atoms decay in one day.
- Estimate the order of magnitude of the atoms that disintegrate in one second.
  - What is the ratio of the original atoms to the atoms that remain after one day in orders of magnitude?

## 1.1.4 ESTIMATES OF EVERYDAY QUANTITIES

Many problems in physics and engineering require very precise numerical measurements or calculations. The number of significant digits in a quantity measured reflect how precisely we know that quantity. When English and French engineers used their excavation machinery to dig the tunnel under the North Sea, they hoped that they would meet at a common point. The laser guidance systems used allowed for a good degree of precision in the digging process. High precision is also required in cancer radiotherapy so that the cancerous cells are killed and the good body cells are not damaged in amounts greater than necessary. Also to our amazement and sadness we have witnessed too often on television the accuracy of laser guided missiles seeking out targets with incredible accuracy.

However, in other applications estimation may be acceptable in order to grasp the significance of a physical phenomenon. For example, if we wanted to estimate the water needed to flush the toilet in your dwelling in a year, it would be reasonable to remove the lid off the toilet cistern (reservoir for storing water) and seeing whether there are graduations (or indicators) of the water capacity given on the inside on the cistern. When I removed the lid from my cistern, the water was at the 9 L ( $9 \text{ dm}^3$ ) mark and when I did a “water saving” flush, the water went to the 6 L mark. A long flush emptied the cistern. Now let’s assume there are three people in the house who are using one long flush and five short flushes a day. This makes a total of  $(3 \times 9 \text{ dm}^3) + (15 \times 3 \text{ dm}^3) = 72 \text{ dm}^3$  per day or an estimate of  $10^2 \text{ dm}^3$  per day. There are 365.25 days in a year or an estimate of  $10^3$  (using the order of magnitude) days. So the water used by this family would be  $2.6 \times 10^4 \text{ dm}^3$  per year or an estimate of  $10^5 \text{ dm}^3$ . Neither answer is accurate because both answers are only rough estimates.

With practice and experience, we will get a feel for reasonable estimates of everyday quantities. Unfortunately, students and teachers can be poor users of calculators. We should be able to estimate approximate values of everyday

quantities to the nearest order of magnitude to one or two significant digits. We need to develop a way to estimate an answer to a reasonable value.

Suppose we wanted to estimate the answer to:

$$16 \times 5280 \times 12 \times 12 \times 12$$

This can be estimated as:

$$= (2 \times 10^1) \times (5 \times 10^3) \times (5 \times 10^3) \times (1 \times 10^1) \times (1 \times 10^1) \times (1 \times 10^1)$$

$$= 250 \times 10^{10}$$

$$= 2.5 \times 10^{12}$$

The calculator answer is  $1.1562 \times 10^{12}$ . So our estimate gives a reasonable order of magnitude.

### Exercise

### 1.2

- A rough estimate of the volume of your body in  $\text{cm}^3$  would be closest to
  - $2 \times 10^3$
  - $2 \times 10^5$
  - $5 \times 10^3$
  - $5 \times 10^5$
- Estimate the
  - dimensions of this textbook in cm
  - mass of an apple in g
  - period of a heartbeat in s
  - temperature of a typical room in  $^\circ\text{C}$
- Estimate the answer to:
  - $16 \times 5280 \times 5280 \times 5280 \times 12 \times 12 \times 12$
  - $3728 \times (470165 \times 10^{-14}) \div 278146 \times (0.000713 \times 10^{-5})$
  - $47816 \times (4293 \times 10^{-4}) \div 403000$
- The universe is considered to have begun with the “Big Bang” event. The galaxies that have moved the farthest are those with the greatest initial speeds. It is believed that these speeds have been constant in time. If a galaxy  $3 \times 10^{21} \text{ km}$  away is receding from us at  $1.5 \times 10^{11} \text{ km y}^{-1}$ , calculate the age of the universe in years.

5. Give an estimate of the order of magnitude of the following:
- The length of your arm in mm.
  - The quantity of milk you drink in a year in  $\text{cm}^3$ .
  - The mass of your backpack that contains your school materials in g.
  - The diameter of a human hair in mm.
  - The time you spend at school in a year in minutes.
  - The number of people in the country where you live.

## 1.2 MEASUREMENT & UNCERTAINTIES

### THE SI SYSTEM OF FUNDAMENTAL AND DERIVED UNITS

- 1.2.1 State the fundamental units in the SI system.
- 1.2.2 Distinguish between fundamental and derived units and give examples of derived units.
- 1.2.3 Convert between different units of quantities.
- 1.2.4 State units in the accepted SI format.
- 1.2.5 State values in scientific notation and in multiples of units with appropriate prefixes.

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### 1.2.1 FUNDAMENTAL UNITS

**SI units** are those of Le Système International d'Unités adopted in 1960 by the Conférence Générale des Poids et Mesures. They are adopted in all countries for science research and education. They are also used for general measurement in most countries with the USA and the UK being the major exceptions.

Physics is the most fundamental of the sciences in that it involves the process of comparing the physical properties of what is being measured against reference or fundamental quantities, and expressing the answer in numbers and units.

Some quantities cannot be measured in a simpler form, and others are chosen for convenience. They have been selected as the basic quantities and are termed fundamental quantities. Figure 102 lists the fundamental quantities of the SI system together with their respective SI unit and SI symbol.

Quantity	SI unit	SI symbol
length	metre	m
mass	kilogram	kg
time	second	s
electric current	ampere	A
thermodynamic temperature	Kelvin	K
amount of substance	mole	mol
luminous intensity	candela	cd

Figure 102 Fundamental quantities

Scientists and engineers need to be able to make accurate measurements so that they can exchange information. To be useful, a standard of measurement must be:

1. Invariant in time. For example, a standard of length that keeps changing would be useless.
2. Readily accessible so that it can be easily compared.
3. Reproducible so that people all over the world can check their instruments.

The standard metre, in 1960, was defined as the length equal to 1 650 763.73 wavelengths of a particular orange-red line of krypton-86 undergoing electrical discharge. Since 1983 the metre has been defined in terms of the speed of light. The current definition states that 'the metre is the length of path travelled by light in a vacuum during a time interval of  $1/299\,792\,453$  second'.

The standard kilogram is the mass of a particular piece of platinum-iridium alloy that is kept in Sèvres, France. Copies of this prototype are sent periodically to Sèvres for adjustments. The standard second is the time for 9 192 631 770 vibrations of the cesium-133 atom.

Standards are commonly based upon properties of atoms. It is for this reason that the standard kilogram could be replaced at some future date. When measuring lengths, we choose an instrument that is appropriate to the order of magnitude, the nature of the length, and the sensitivity required. For example, the orders of magnitude (the factor of 10) of the radius of a gold atom, a person's height and the radius of the solar system are  $10^{-15}$ ,  $10^0$  and  $10^{12}$ .

Physical Quantity	Symbol	Name and Symbol SI Unit	Fundamental Units Involved	Derived Units involved
frequency	f or $\nu$	hertz (Hz)	$s^{-1}$	$s^{-1}$
force	F	newton (N)	$kg\ m\ s^{-2}$	$kg\ m\ s^{-2}$
work	W	joule (J)	$kg\ m^2\ s^{-2}$	Nm
energy	Q, $E_p$ , $E_k$ , $E_{elas}$	joule (J)	$kg\ m^2\ s^{-2}$	Nm
power	P	watt (W)	$kg\ m^2\ s^{-3}$	$J\ s^{-1}$
pressure	P	pascal (Pa)	$kg\ m^{-1}\ s^{-2}$	$N\ m^{-2}$
charge	Q	coulomb (C)	A s	A s
potential difference	V	volt (V)	$kg\ m^2\ s^{-3}\ A^{-1}$	$J\ C^{-1}$
resistance	R	ohm ( $\Omega$ )	$kg\ m^2\ s^{-3}\ A^{-2}$	$V\ A^{-1}$
magnetic field intensity	B	tesla (T)	$kg\ s^{-3}\ A^{-1}$	$NA^{-1}\ m^{-1}$
magnetic flux	$\Phi$	weber (Wb)	$kg\ m^2\ s^{-2}\ A^{-2}$	$T\ m^2$
activity	A	becquerel (Bq)	$s^{-1}$	$s^{-1}$
absorbed dose	W/m	gray (Gy)	$m^2\ s^{-2}$	$J\ kg^{-1}$

Figure 103 Derived Units

respectively. The nature of a person's height is different from that of the radius of a gold atom in that the person's height is macroscopic (visible to the naked eye) and can be measured with say a metre stick, whereas the diameter of the atom is microscopic and can be inferred from electron diffraction.

## 1.2.2 FUNDAMENTAL AND DERIVED UNITS

When a quantity involves the measurement of two or more fundamental quantities it is called a derived quantity, and the units of these derived quantities are called derived units. Some examples include acceleration ( $m\ s^{-2}$ ), angular acceleration ( $rad\ s^{-2}$ ) and momentum ( $kg\ m\ s^{-1}$  or  $N\ s$ ). It should be noted that the litre (L) and the millilitre (mL) are often used for measuring the volume of liquid or the capacity of a container. The litre is a derived unit but not a SI unit. The equivalent SI unit is  $dm^3$ .

Some derived units are relatively complex and contain a number of fundamental units. Figure 103 lists the common relevant derived units and associated information.

## 1.2.3 CONVERSION BETWEEN DIFFERENT UNITS

Sometimes, it is possible to express the units in different derived units. This concept will become clear as the various topics are introduced throughout the course. For example, the unit of momentum can be  $kg\ m\ s^{-1}$  or  $N\ s$ .

The unit of electrical energy could be J or Wh or kJ or kWh (kilowatt-hour). In atomic and nuclear physics the unit of energy could be J or eV (electron-volt) where  $1\ eV = 1.6 \times 10^{-19}\ J$ .

## 1.2.4 UNITS IN ACCEPTED SI FORMAT

Note the use of the accepted SI format. For example, the unit for acceleration is written as  $m\ s^{-2}$  and not  $m/s/s$ . No mathematical denominators are used but rather inverse numerators are the preferred option.

## 1.2.5 SCIENTIFIC NOTATION AND PREFIXES

Scientists tend to use scientific notation when stating a measurement rather than writing lots of figures.  $1.2 \times 10^6$  is easier to write and has more significance than 1 200 000. In order to minimise confusion and ambiguity, all quantities are best written as a value between one and ten multiplied by a power of ten.

For example, we have that,

$$0.06\ kg = 6 \times 10^{-2}\ kg$$

$140\ kg = 1.4 \times 10^2\ kg$  or  $1.40 \times 10^2\ kg$  depending on the significance of the zero in 140.

$$132.97\ kg = 1.3297 \times 10^2\ kg$$

The terms standard notation and standard form are synonymous with scientific notation. The use of prefixes

for units is also preferred in the SI system – multiple or submultiple units for large or small quantities respectively. The prefix is combined with the unit name. The main prefixes are related to the SI units by powers of three.

However, some other multiples are used.

$$1\,000\,000\,000\text{ m} = 1\text{ Gm}$$

$$1\,000\,000\text{ dm}^3 = 1\text{ Mdm}^3$$

$$0.000\,000\,001\text{ s} = 1\text{ ns}$$

$$0.000\,001\text{ m} = 1\text{ }\mu\text{m}$$

The main prefixes and other prefixes are shown in Figure 104.

Multiple	Prefix	Symbol	Multiple	Prefix	Symbol
$10^{24}$	yotta	Y	$10^{-1}$	deci	d
$10^{21}$	zetta	Z	$10^{-2}$	centi	c
$10^{18}$	exa	E	$10^{-3}$	milli	m
$10^{15}$	peta	P	$10^{-6}$	micro	$\mu$
$10^{12}$	tera	T	$10^{-9}$	nano	n
$10^9$	giga	G	$10^{-12}$	pico	p
$10^6$	mega	M	$10^{-15}$	femto	f
$10^3$	kilo	k	$10^{-18}$	atto	a
$10^2$	hecto	h	$10^{-21}$	zepto	z
$10^1$	deca	da	$10^{-24}$	yocto	y

Figure 104 Preferred and some common prefixes

### Exercise 1.3

- Which of the following isotopes is associated with the standard measurement of time?
  - uranium–235
  - krypton–86
  - cesium–133
  - carbon–12
- Which one of the following lists a fundamental unit followed by a derived unit?
  - ampere mole
  - coulomb watt
  - ampere joule
  - second kilogram
- Which one of the following is a fundamental unit?
  - Kelvin
  - Ohm
  - Volt
  - Newton
- Which of the following is measured in fundamental units?
  - velocity
  - electric charge
  - electric current
  - force
- The density in  $\text{g cm}^{-3}$  of a sphere with a radius of 3 cm and a mass of 0.54 kg is:
  - $2\text{ g cm}^{-3}$
  - $2.0 \times 10^1\text{ g cm}^{-3}$
  - $0.50\text{ g cm}^{-3}$
  - $5.0\text{ g cm}^{-3}$
- Convert the following to fundamental S.I. units.
  - 5.6 g
  - 3.5  $\mu\text{ A}$
  - 3.2 dm
  - 6.3 nm
  - 2.25 tonnes
  - 440 Hz
- Convert the following to S.I. units
  - 2.24 MJ
  - 2.50 kPa
  - $2.7\text{ km h}^{-1}$
  - $2.5\text{ mm}^2$
  - 2.4 L
  - $3.6\text{ cm}^3$
  - $230.1\text{ M dm}^3$
  - $3.62\text{ mm}^3$
- Estimate the order of magnitude for the following:
  - your height in metres
  - the mass of a 250 tonne aeroplane in kilograms
  - the diameter of a hair in metres
  - human life span in seconds.
- Calculate the distance in metres travelled by a parachute moving at a constant speed of  $6\text{ km h}^{-1}$  in 4 min.
- The force of attraction  $F$  in newtons between the earth with mass  $M$  and the moon with mass  $m$  separated by a distance  $r$  in metres from their centres of mass is given by the following equation:
 
$$F = G M m r^{-2}$$

where  $G$  is a constant called the Universal Gravitation constant

Determine the correct SI units of  $G$ .

11. Determine the SI units for viscosity  $\eta$  if the equation for the force on a sphere moving through a fluid is:

$$F = 6\pi\eta rv$$

where  $r$  is the radius of the sphere,  $v$  is the speed of the sphere in the fluid.

### UNCERTAINTY AND ERROR IN MEASUREMENT

- 1.2.6 Describe and give examples of random and systematic errors.
- 1.2.7 Distinguish between precision and accuracy.
- 1.2.8 Explain how the effects of random errors may be reduced.
- 1.2.9 Calculate quantities and results of calculations to the appropriate number of significant figures.

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## 1.2.6 RANDOM AND SYSTEMATIC ERRORS

Errors can be divided into two main classes, **random errors** and **systematic errors**.

Mistakes on the part of the individual such as:

- misreading scales.
- poor arithmetic and computational skills.
- wrongly transferring raw data to the final report.
- using the wrong theory and equations.

are definite sources of error but they are not considered as experimental errors.

A systematic error causes a random set of measurements to be spread about a value rather than being spread about the accepted value. It is a system or instrument error. Systematic errors can result from:

- badly made instruments.
- poorly calibrated instruments.
- an instrument having a zero error, a form of calibration.
- poorly timed actions.
- instrument parallax error.

Many ammeters and voltmeters have a means of adjustment to remove zero offset error. When you click a stop-watch, your reaction time for clicking at the start and the finish of the measurement interval is a systematic error. The timing instrument and you are part of the system.

Systematic errors can be eliminated or corrected before the investigation is carried out on most occasions.

Random uncertainties are due to variations in the performance of the instrument and the operator. Even when systematic errors have been allowed for, there exists error. Random uncertainties can be caused by such things as:

- vibrations and air convection currents in mass readings.
- temperature variations.
- misreadings.
- variations in the thickness of a surface being measured (thickness of a wire).
- not collecting enough data.
- using a less sensitive instrument when a more sensitive instrument is available.
- human parallax error (one has to view the scale of the meter in direct line, and not to the sides of the scale in order to minimise parallax error).

## 1.2.7 PRECISION AND ACCURACY

As well as obtaining a series of measurements with the correct units for the measurements, an indication of the experimental error or degree of uncertainty in the measurements and the solution is required. The higher the **accuracy** and **precision** in carrying out investigations, the lower the degree of uncertainty. The meanings of the words accuracy and precision are clearly defined in scientific fields.

Accuracy is an indication of how close a measurement is to the accepted value indicated by the relative or percentage

error in the measurement. An accurate experiment has a low systematic error.

Precision is an indication of the agreement among a number of measurements made in the same way indicated by the absolute error. A precise experiment has a low random error.

Suppose a technician was fine-tuning a computer monitor by aiming an electron gun at a pixel in the screen as shown in Figure 105.

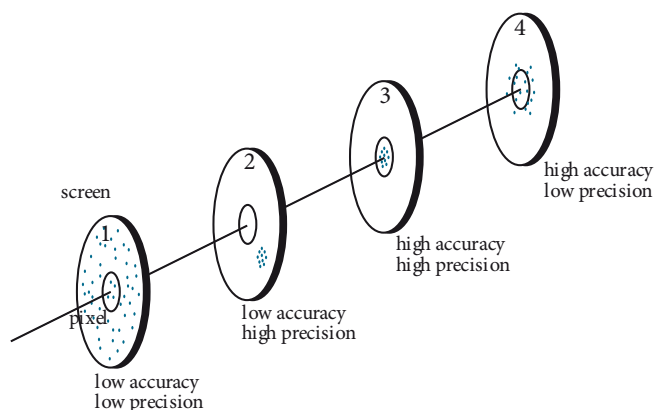


Figure 105 Precision and accuracy

In case 1 there is low accuracy and precision. The technician needs to adjust the collimator to reduce the scattering of electrons, and to change the magnetic field so the electrons hit the pixel target. In case 2, the electron gun has been adjusted to increase precision but the magnetic field still needs adjustment. In case 3, both adjustments have been made. Can you give an explanation for case four?

## 1.2.8 REDUCING RANDOM ERROR

Often the random uncertainty is not revealed until a large sample of measurements is taken. So taking a required number of readings/samples not only reveals random uncertainty but also helps to reduce it. Consistent experimental procedures can minimise random uncertainty.

Random errors can also be reduced by choosing an instrument that has a higher degree of accuracy. When measuring mass, it would be best to choose a digital balance that can read to 2 decimal points rather than a top pan balance or a digital balance that can read to 1 decimal point. Further reduction of random error can be obtained by reducing variations such as air currents, vibrations, temperature variation, loss of heat to the surroundings.

However, you should be aware that repeating measurements may reduce the random uncertainty but at the same time the systematic error will not be reduced.

## 1.2.9 SIGNIFICANT FIGURES

The concept of **significant figures** may be used to indicate the degree of accuracy or precision in a measurement. Significant figures (sf) are those digits that are known with certainty followed by the first digit which is uncertain.

Suppose you want to find the volume of a lead cube. You could measure the length  $l$  of the side of a lead cube with a vernier caliper (refer Figure 112). Suppose this length was 1.76 cm and the volume  $l^3$  from your calculator reads 5.451776. The measurement 1.76 cm was to three significant figures so the answer can only be to three significant figures. So that the volume =  $5.45 \text{ cm}^3$ .

The following rules are applied in this book.

1. All non-zero digits are significant. (22.2 has 3 sf)
2. All zeros between two non-zero digits are significant. (1007 has 4 sf)
3. For numbers less than one, zeros directly after the decimal point are not significant. (0.0024 has 2 sf)
4. A zero to the right of a decimal and following a non-zero digit is significant. (0.0500 has 3 sf)
5. All other zeros are not significant. (500 has 1 sf)

Scientific notation allows you to give a zero significance.

For example, 10 has 1 sf but  $1.00 \times 10^1$  has 3sf.

6. When adding and subtracting a series of measurements, the least accurate place value in the answer can only be stated to the same number of significant figures as the measurement of the series with the least number of decimal places.

For example, if you add 24.2 g and 0.51 g and 7.134 g, your answer is 31.844 g which has increased in significant digits. The least accurate place value in the series of measurements is 24.2 g with only one number to the right of the decimal point. So the answer can only be expressed to 3sf. Therefore, the answer is 31.8 g or  $3.18 \times 10^1$  g.



7. When multiplying and dividing a series of measurements, the number of significant figures in the answer should be equal to the least number of significant figures in any of the data of the series.

For example, if you multiply 3.22 cm by 12.34 cm by 1.8 cm to find the volume of a piece of wood your initial answer is  $71.52264 \text{ cm}^3$ . However, the least significant measurement is 1.8 cm with 2 sf. Therefore, the correct answer is  $72 \text{ cm}^3$  or  $7.2 \times 10^1 \text{ cm}^3$ .

8. When rounding off a number, if the digit following the required rounding off digit is 4 or less, you maintain the last reportable digit and if it is six or more you increase the last reportable digit by one. If it is a five followed by more digits except an immediate zero, increase the last reportable digit. If there is only a five with no digits following, increase reportable odd digits by one and maintain reportable even digits.

For example if you are asked to round off the following numbers to two significant numbers

6.42	becomes	6.4
6.46	becomes	6.5
6.451	becomes	6.5
6.498	becomes	6.5
6.55	becomes	6.6
6.45	becomes	6.4

As a general rule, round off in the final step of a series of calculations.

### Exercise 1.4

1. Consider the following measured quantities  
(a)  $3.00 \pm 0.05 \text{ m}$  (b)  $12.0 \pm 0.3 \text{ m}$

Which alternative is the best when the accuracy and precision for a and b are compared?

	a	b
A.	Low accuracy	Low precision
B.	Low accuracy	High precision
C.	High accuracy	Low precision
D.	High accuracy	High precision

2. A voltmeter has a zero offset error of 1.2 V. This fault will affect:
- neither the precision nor the accuracy of the readings.
  - only the precision of the readings.
  - only the accuracy of the readings.
  - both the precision and the accuracy of the readings.
3. A student measures the current in a resistor as 655 mA for a potential difference of 2.0 V. A calculator shows the resistance of the resistor to be  $1.310 \Omega$ . Which one of the following gives the resistance to an appropriate number of significant figures?
- $1.3 \Omega$
  - $1.31 \Omega$
  - $1.310 \Omega$
  - $1 \Omega$
4. How many significant figures are indicated by each of the following
- |                      |                           |
|----------------------|---------------------------|
| (a) 1247             | (b) 1007                  |
| (c) 0.034            | (d) $1.20 \times 10^7$    |
| (e) 62.0             | (f) 0.0025                |
| (g) 0.00250          | (h) $\sin 45.2^\circ$     |
| (i) $\tan^{-1} 0.24$ | (j) $3.2 \times 10^{-16}$ |
| (k) 0.0300           | (l) $1.0 \times 10^1$     |
5. Express the following in standard notation (scientific notation)
- 1250
  - 30007
  - 25.10
  - an area of  $4 \text{ km}^2$  in  $\text{m}^2$
  - an object of  $12.0 \text{ nm}^2$  in  $\text{m}^2$
6. Calculate the area of a square with a side of 3.2 m.
7. Add the following lengths of 2.35 cm and 7.62 m and 14.2 m.
8. Calculate the volume of a rectangular block 1.52 cm by 103.4 cm by 3.1 cm.
9. A metal has a mass of 2.0 g and a volume of  $0.01 \text{ cm}^3$ . Calculate the density of the metal in  $\text{g cm}^{-3}$ .

10. Round off the following to three significant figures:
- (a) 7.1249                      (b) 2561  
 (c) 2001                         (d) 21256  
 (e) 6.5647
11. Determine the following to the correct number of significant figures:
- (a)  $(3.74 - 1.3) \times 2.12 \times 17.65$   
 (b)  $(2.9 + 3.2 + 7.1) \div 0.134$
12. Add  $2.76 \times 10^{-6}$  cm and  $3.4 \times 10^{-5}$  cm.

### UNCERTAINTIES IN CALCULATED RESULTS

1.2.10 State uncertainties as absolute, fractional and percentage uncertainties.

1.2.11 Determine the uncertainties in results.

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## 1.2.10 ABSOLUTE, FRACTIONAL AND PERCENTAGE UNCERTAINTIES

The limit of reading of a measurement is equal to the smallest graduation of the scale of an instrument.

The maximum degree of uncertainty of a measurement is equal to half the limit of reading.

When a measuring device is used, more often than not the measurement falls between two graduations on the scale being used. In Figure 108, the length of the block is between 0.4 cm and 0.5 cm.

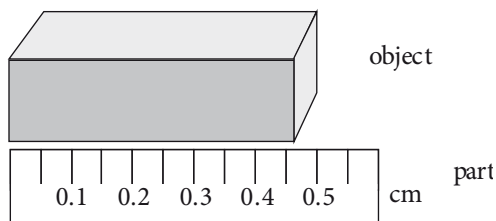


Figure 108 Linear measurement

The limit of reading is 0.05 cm and the uncertainty of the measurement is  $\pm 0.025$  cm.

The length is stated as  $0.47 \pm 0.02$  cm. (Uncertainties are given to 1 significant figure).

The smallest uncertainty possible with any measuring device is half the limit of reading. However, most investigations generate an uncertainty greater than this. Figure 109 lists the uncertainty of some common laboratory equipment.

Metre rule	$\pm 0.05$ cm
Vernier calipers	$\pm 0.005$ cm
Micrometer screw gauge	$\pm 0.005$ mm
50 cm <sup>3</sup> measuring cylinder	$\pm 0.2$ cm <sup>3</sup>
10 cm <sup>3</sup> measuring cylinder	$\pm 0.1$ cm <sup>3</sup>
Electric balance	$\pm 0.005$ g
Watch second hand	$\pm 0.5$ s
Digital timer	$\pm 0.0005$ s
Spring balance (0–20N)	$\pm 0.1$ N
Resistor	$\pm 2\%$

Figure 109 Equipment uncertainties

Absolute uncertainty is the size of an error and its units. In most cases it is not the same as the maximum degree of uncertainty (as in the previous example) because it can be larger than half the limit of reading. The experimenter can determine the absolute error to be different to half the limit of reading provided some justification can be given. For example, mercury and alcohol thermometers are quite often not as accurate as the maximum absolute uncertainty.

Fractional (relative) uncertainty equals the absolute uncertainty divided by the measurement as follows. It has no units.

$$\text{Relative uncertainty} = \frac{\text{absolute uncertainty}}{\text{measurement}}$$

Percentage uncertainty is the relative uncertainty multiplied by 100 to produce a percentage as follows

$$\text{Percentage discrepancy} = \text{relative uncertainty} \times 100\%$$

For example, if a measurement is written as  $9.8 \pm 0.2$  m, then there is a

- limit of reading = 0.1 m
- uncertainty = 0.05 m
- absolute uncertainty = 0.2 m
- relative uncertainty =  $0.2/9.8 = 0.02$
- and percentage uncertainty =  $0.02 \times 100\% = 2\%$

Percentage uncertainty should not be confused with percentage discrepancy or percentage difference which is an indication of how much your experimental answer varies from the known accepted value of a quantity.

Percentage discrepancy is often used in the conclusion of laboratory reports.

$$= \frac{\text{accepted value} - \text{experimental value}}{\text{accepted value}} \times 100$$

Note that errors are stated to only one significant figure

## 1.2.11 UNCERTAINTIES IN RESULTS DETERMINATION

### 1. THE ARITHMETIC MEAN - AVERAGING

When a series of readings are taken for a measurement, then the arithmetic mean of the readings is taken as the most probable answer, and the greatest deviation or residual from the mean is taken as the absolute error.

Study the following data in Table 110 for the thickness of a copper wire as measured with a micrometer screw gauge:

Reading/mm	5.821	5.825	5.820	5.832	5.826	5.826	5.828	5.824
Residual/mm	-0.004	0	-0.005	+0.007	+0.001	+0.001	+0.003	-0.001

Figure 110 Sample measurements

The sum of the readings = 46.602 and so the mean of the readings is 5.825.

Then, the value for the thickness is  $5.825 \pm 0.007$  mm

This method can be used to suggest an appropriate uncertainty range for trigonometric functions. Alternatively, the mean, maximum and minimum values can be calculated to suggest an appropriate uncertainty range. For example, if an angle is measured as  $30 \pm 2$ , then the mean value of  $\sin 30 = 0.5$ , the maximum value is  $\sin 32 = 0.53$  and the minimum value is  $\sin 28 = 0.47$ . The answer with correct uncertainty range is  $0.5 \pm 0.03$ .

### 2. ADDITION, SUBTRACTION AND MULTIPLICATION INVOLVING ERRORS

When adding measurements, the error in the sum is the sum of the absolute error in each measurement taken.

For example, the sum of  $2.6 \pm 0.5$  cm and  $2.8 \pm 0.5$  cm is  $5.4 \pm 1$  cm

When subtracting measurements, add the absolute errors.

If you place two metre rulers on top of each other to measure your height, remember that the total error is the sum of the uncertainty of each metre rule. ( $0.05$  cm +  $0.05$  cm). If there is a zero offset error on an instrument, say a newton balance, you will have to subtract the given reading from the zero error value.

So  $25 \pm 2.5$  N -  $2 \pm 2.5$  equals  $23 \pm 5$  N.

### 3. MULTIPLICATION AND DIVISION INVOLVING ERRORS

When multiplying and dividing, add the relative or percentage errors of the measurements being multiplied/divided. The absolute error is then the fraction or percentage of the most probable answer.

#### Example

What is the product of  $2.6 \pm 0.5$  cm and  $2.8 \pm 0.5$  cm?

#### Solution

First, we determine the product

$$2.6 \text{ cm} \times 2.8 \text{ cm} = 7.28 \text{ cm}^2$$

$$\text{Relative error 1} = 0.5/2.6 = 0.192$$

$$\text{Relative error 2} = 0.5/2.8 = 0.179$$

$$\text{Sum of the relative errors} = 0.371 \text{ or } 3.71\%$$

$$\text{Absolute error} = 0.371 \times 7.28 \text{ cm}^2 \text{ or } 3.71\% \times 7.28 \text{ cm}^2 = 2.70 \text{ cm}^2$$

Errors are expressed to one significant figure =  $3 \text{ cm}^2$

The product is equal to  $7.3 \pm 3 \text{ cm}^2$

### 4. UNCERTAINTIES AND POWERS

When raising to the  $n^{\text{th}}$  power, multiply the percentage uncertainty by  $n$ , and when extracting the  $n^{\text{th}}$  root, divide the percentage uncertainty by  $n$ .

For example, if the length  $x$  of a cube is  $2.5 \pm 0.1$  cm, then the volume will be given by  $x^3 = 15.625 \text{ cm}^3$ . The percentage uncertainty in the volume =  $3(0.1/2.5 \times 100) = 12\%$ .

$$\text{Therefore, } 12\% \text{ of } 15.625 = 1.875.$$

$$\text{Volume of the cube} = 16 \pm 2 \text{ cm}^3.$$

$$\text{If } x = 9.0 \pm 0.3 \text{ m, then } \sqrt{x} = x^{1/2} = 3.0 \pm 0.15 \text{ m} \\ = 3.0 \pm 0.2 \text{ m.}$$

### 5. MEASURING LENGTH WITH VERNIER CALIPERS OR A MICROMETER SCREW GAUGE

CORE

Two length measuring devices with lower uncertainty than the metre rule are vernier calipers and the micrometer screw gauge. The uncertainty of these instruments was given in Figure 109.

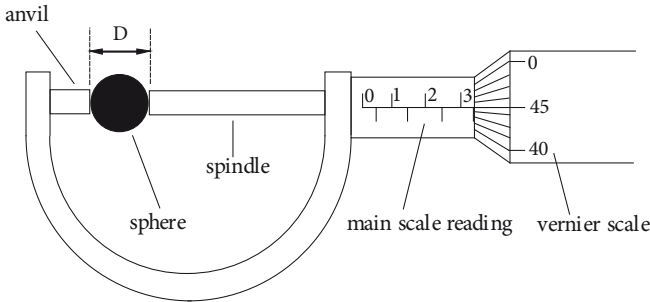


Figure 111 A micrometer screw gauge

In Figure 111, the reading on the micrometer screw gauge is 3.45 mm. You can see that the thimble (on the right of the gauge) is to the right of the 3 mm mark but you cannot see the 3.5 mm mark on the main scale. The vernier thimble scale is close to the 45 mark.

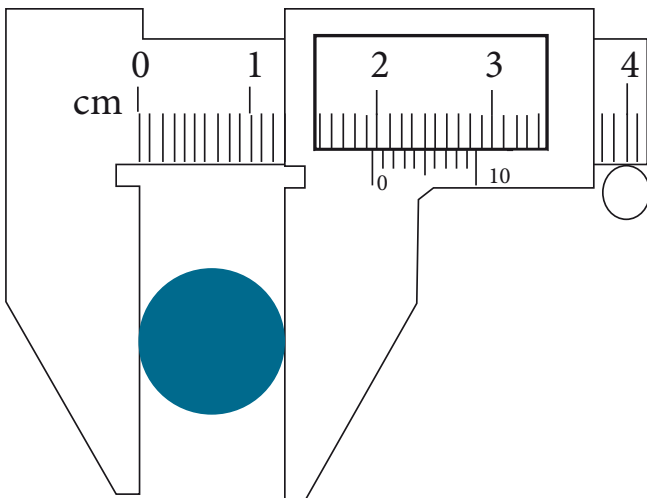


Figure 112 Vernier calipers

In Figure 112, the reading on the vernier calipers is 1.95 cm. The vertical line showing zero on the vernier scale lies between 1.9 cm and 2.0 cm. The vertical graduation on the vernier scale that matches the main scale best is the fifth graduation.

#### Exercise 1.5

- A student measures the mass  $m$  of a ball. The percentage uncertainty in the measurement of the mass is 5%. The student drops the ball from a height  $h$  of 100 m. The percentage uncertainty in the measurement of the height is 7%. The calculated value of the gravitational potential energy of the ball will have an uncertainty of ( $E_p = mgh$ )
  - 2%
  - 5%
  - 7%
  - 12%
- The electrical power dissipation  $P$  in a resistor of resistance  $R$  when a current  $I$  is flowing through it is given by the expression

$$P = I^2R.$$

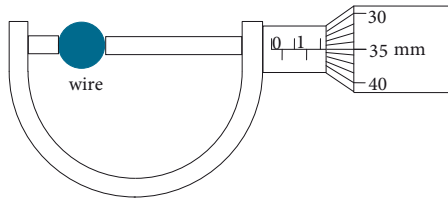
In an investigation,  $I$  was determined from measurements of  $P$  and  $R$ . The uncertainties in  $P$  and in  $R$  are as shown below.

$P$	$\pm 4\%$
$R$	$\pm 10\%$

The uncertainty in  $I$  would have been most likely

- 14 %.
  - 7 %.
  - 6 %.
  - 5 %.
- The mass of the Earth is stated as  $5.98 \times 10^{24}$  kg. The absolute uncertainty is
    - 0.005
    - 0.005 kg
    - $0.005 \times 10^{24}$  kg
    - $0.005 \times 10^{24}$
  - If  $a = 20 \pm 0.5$  m and  $b = 5 \pm 1$  m, then  $2a - b$  should be stated as
    - $35 \pm 1.5$  m
    - $35 \pm 2$  m
    - $35 \pm 0.0$  m
    - $5 \pm 2$  m

5. How should a calculation result be stated if it is found to be  $0.931940 \mu\text{m}$  with an absolute error of  $\pm 0.0005 \mu\text{m}$ .
6. This question concerns the micrometer screw gauge in the Figure shown below.



- (a) What is the reading and error on the micrometer?
- (b) The thickness of the wire being measured varies over its length. What sort of error would this be?
7. A student records the following currents in amperes A when the potential difference V across a resistor is 12V:
- 0.9 A    0.9 A    0.85 A    0.8 A    1.2 A    0.75 A  
 0.8 A    0.7 A    0.8 A    0.95 A
- (a) Would you disregard any of the readings? Justify your answer.
- (b) Calculate the current and its uncertainty
8. A spring balance reads 0.5 N when it is not being used. If the needle reads 9.5 N when masses are attached to it, then what would be the correct reading to record (with uncertainty)?
9. Five measurements of the length of a piece of string were recorded in metres as:
- 1.48    1.46    1.47    1.50    1.45
- Record a feasible length of the string with its uncertainty.
10. A metal cube has a side length of  $3.00 \pm 0.01 \text{ cm}$ . Calculate the volume of the cube.
11. An iron cube has sides  $10.3 \pm 0.2 \text{ cm}$ , and a mass of  $1.3 \pm 0.2 \text{ g}$ . What is the density of the cube in  $\text{g cm}^{-3}$ ?

12. The energy E of an  $\alpha$ -particle is  $4.20 \pm 0.03 \text{ MeV}$ . How should the value and uncertainty of  $E^{-1/2}$  be stated?
13. Suggest an appropriate answer with uncertainty range for  $\sin \theta$  if  $\theta = 60 \pm 5^\circ$ .

## UNCERTAINTIES IN GRAPHS

1.2.12 Identify uncertainties as error bars in graphs.

1.2.13 State random uncertainty as an uncertainty range ( $\pm$ ) and represent it graphically as an "error bar".

1.2.14 Determine the uncertainties in the slope and intercepts of a straight-line graph.

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## 1.2.12 UNCERTAINTIES AS ERROR BARS

When an answer is expressed as a value with uncertainty such as  $2.3 \pm 0.1 \text{ m}$ , then the uncertainty range is evident. The value lies between 2.4 ( $2.3 + 0.1$ ) and 2.2 ( $2.3 - 0.1$ ) cm. In Physics, we often determine the relationship that exists between variables. To view the relationship, we can perform an investigation and plot a graph of the dependant variable (y-axis) against the independant variable (x-axis). This aspect will be discussed fully in section 1.6.1.

Consider a spring that has various weights attached to it. As a heavier weight is attached to a spring, the spring extends further from its equilibrium position. Figure 115 shows some possible values for this weight/extension investigation.

<b>Force <math>\pm 5 \text{ N}</math></b>	100	150	200	250	300
<b>Extension <math>\pm 0.2 \text{ cm}</math></b>	3.0	4.4	6.2	7.5	9.1

Figure 115 Extension of a spring

When a graph of force versus extension is plotted, the line of best fit does not pass through every point. An **error bar** can be used to give an indication of the uncertainty range for each point as shown in Figure 116.

In the vertical direction, we draw a line up and down for each point to show the uncertainty range of the force value. Then we place a small horizontal marker line on the extreme uncertainty boundary for the point.

In the horizontal direction, we draw a line left and right for each point to show the uncertainty range of the extension value. Then we place a small vertical marker line on the extreme uncertainty boundary for the point.

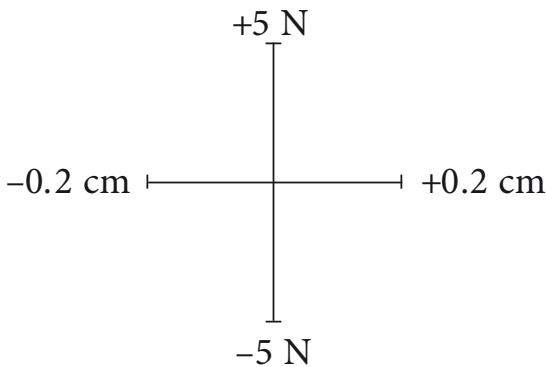


Figure 116 Error Bars

When all the points in Figure 115 are plotted on a graph, then the line of best fit with the appropriate error bars is shown in Figure 117. You can see that the line of best fit lies within the error bar uncertainty range. The line of best fit is interpolated between the plotted points. The line of best fit is extrapolated outside the plotted points.

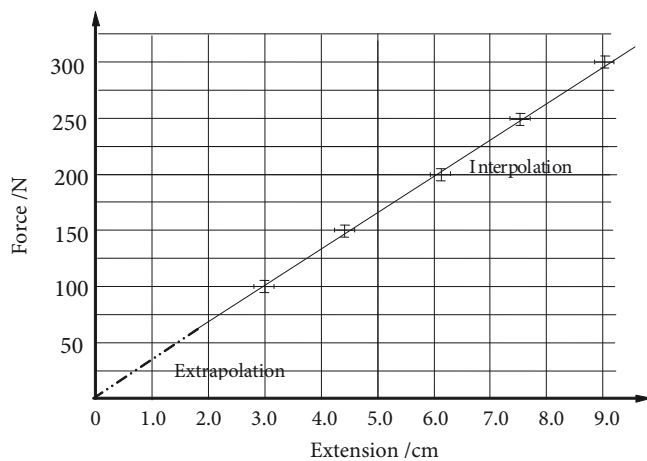


Figure 117 Example Of Error Bars

Error bars will not be expected for trigonometric or logarithmic functions in this course.

## 1.2.13,14

### RANDOM UNCERTAINTY AND UNCERTAINTIES IN THE SLOPE AND INTERCEPTS OF A STRAIGHT-LINE GRAPH

Graphs are very useful for analysing the data that is collected during investigations and is one of the most valuable tools used by physicists and physics students because

- they give a visual display of the relationship that exists between two or more variables.
- they show which data points obey the relationship and which do not.
- they give an indication of the point at which a particular relationship ceases to be true.
- they can be used to determine the constants in an equation relating two variable quantities.

Some of these features are shown in the graphs of Figure 118. Notice how two variables can be drawn on the same axis as in Figure 118 (b).

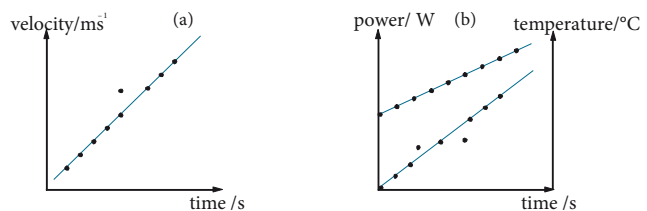


Figure 118 Examples of graphs

## 1. Choice of axes

A variable is a quantity that varies when another quantity is changed. A variable can be an independent variable, a dependent variable or a controlled variable. During an experiment, an independent variable is altered while the dependent variable is measured. Controlled variables are the other variables that may be present but are kept constant. For example, when measuring the extension of a spring when different masses are added to it, the weight force is altered and the extension from the spring's original length is measured. The force would be the independent variable plotted on the x-axis and the extension would be the dependant variable plotted on the y-axis. (The extension depends on the mass added). Possible controlled variables

would be using the same spring, the same measuring device and the same temperature control.

The values of the independent variable are plotted on the x-axis (the abscissa), and the values of the dependent variable are plotted on the y-axis (the ordinate), as shown in Figure 119.

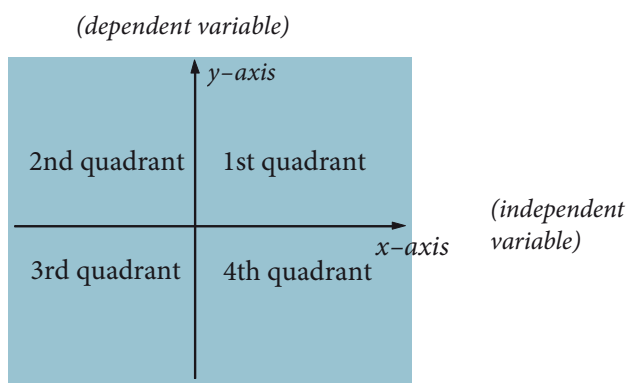


Figure 119 Use of axes

It is not always clear which variable is the dependent and which is the independent. When time is involved it is the independent variable. In many electrodynamic and electromagnetic experiments the potential difference (voltage) or the current can be varied to see what happens to the other variable – either could be the independent variable. Most experimental results will be plotted in the first quadrant. However, they can extend over the four quadrants as is the case with aspects of simple harmonic motion and waves, alternating current and the cathode ray oscilloscope to name a few.

When you are asked to plot a graph of displacement against time or to plot a graph of force versus time, the variable first mentioned is plotted on the y-axis. Therefore displacement and force would be plotted on the y-axis in the two given examples.

These days, graphs are quickly generated with graphic calculators and computer software. This is fine for quickly viewing the relationship being investigated. However, the graph is usually small and does not contain all the information that is required, such as error bars. Generally, a graph should be plotted on a piece of 1 or 2 mm graph paper and the scale chosen should use the majority of the graph paper. In the beginning of the course, it is good practice to plot some graphs manually. As the course progresses, software packages that allow for good graphing should be explored.

## 2. Scales

In order to convey the desired information, the size of the graph should be large, and this usually means making the graph fill as much of the graph paper as possible. Choose a convenient scale that is easily subdivided.

## 3. Labels

Each axis is labelled with the name and/or symbols of the quantity plotted and the relevant unit used. For example, you could write current/A or current (A). The graph can also be given a descriptive title such as ‘graph showing the relationship between the pressure of a gas and its volume at constant temperature’.

## 4. Plotting the points

Points are plotted with a fine pencil cross or as a circled dot. In many cases, error bars are required. These are short lines drawn from the plotted points parallel to the axes indicating the absolute error of the measurement. A typical graph is shown in Figure 120.

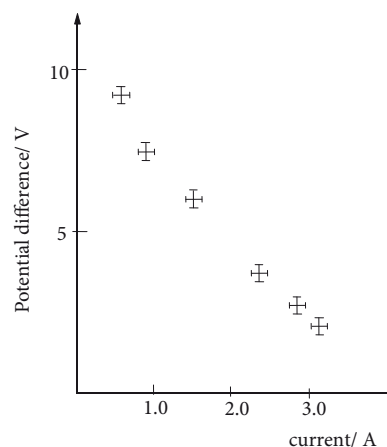


Figure 120 An example of plotting points

## 5. Lines of best fit

The line or curve of best fit is called the line of best fit. When choosing the line or curve of best fit it is practical to use a transparent ruler. Position the ruler until it lies along the ideal line. Shapes and curves can be purchased to help you draw curves. The line or curve does not have to pass through every point. Do not assume that the line should pass through the origin as lines with an x-intercept or y-intercept are common.

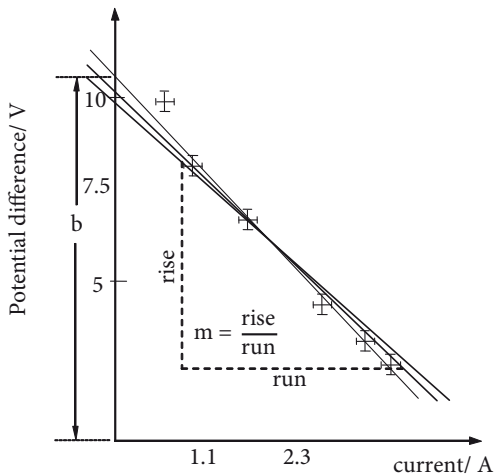


Figure 121 A graph showing the error range

Normally, the line of best fit should lie within the error range of the plotted points as shown in Figure 118. The uncertainty in the slope and intercepts can be obtained by drawing the maximum and minimum lines passing through the error bars. The line of best fit should lie in between these two lines. The uncertainty in the y-intercept can be determined as being the difference in potential difference between the best fit line and the maximum/minimum lines. The uncertainty in the slope can be obtained using the same procedure. However, do not forget that you are dividing. You will therefore have to add the percentage errors to find the final uncertainty.

In the graph, the top plotted point appears to be a data point that could be discarded as a mistake or a random uncertainty.

## AREA UNDER A STRAIGHT LINE GRAPH

The area under a straight line graph is a useful tool in Physics. Consider the two graphs of Figure 122.

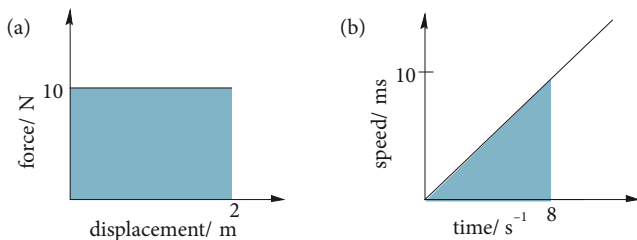


Figure 122 The area under a graph

Two equations that you will become familiar with in Chapter 2 are:

$$\text{work (J)} = \text{force (N)} \times \text{displacement (m)}$$

$$\text{distance (m)} = \text{speed (m s}^{-1}\text{)} \times \text{time (s)}$$

In these examples, the area under the straight line (Figure 1.22(a)) will give the values for the work done ( $5\text{N} \times 2\text{m} = 10\text{ J}$ ).

In Figure 1.22(b), the area enclosed by the triangle will give the distance travelled in the first eight seconds (i.e.,  $\frac{1}{2} \times 8\text{ s} \times 10\text{ m s}^{-1} = 40\text{ m}$ ).

## GRAPHICAL ANALYSIS AND DETERMINATION OF RELATIONSHIPS

### STRAIGHT-LINE EQUATION

The ‘straight line’ graph is easy to recognise and analyse. Often the relationships being investigated will first produce a parabola, a hyperbola or an exponential growth or decay curve. These can be transformed to a straight line relationship (as we will see later).

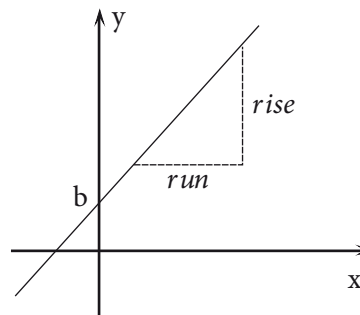


Figure 123 A Straight Line Graph

The general equation for a straight line is

$$y = mx + c$$

where  $y$  is the dependent variable,  $x$  is the independent variable,  $m$  is the slope or gradient given by

$$\frac{\text{vertical run}}{\text{horizontal run}} = \frac{\Delta y}{\Delta x}$$

and  $b$  is the point where the line intersects the y-axis.



In short, an ‘uphill’ slope is positive and a ‘downhill’ slope is negative. The value of  $m$  has units.

Consider Figure 124 below. The slope of the graph shown can be determined. Note that only a small portion of the line of best fit is used.

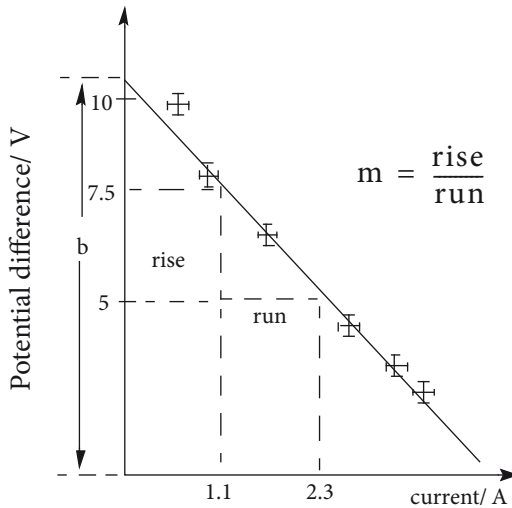


Figure 124 Determining the slope of the graph

$$m = \text{rise/run} = \frac{\Delta y}{\Delta x} = \frac{5.0 - 7.5}{2.3 - 1.1} = -2.08 \text{ V A}^{-1}$$

The equation for the graph shown is generally given as

$$V = \epsilon - Ir \text{ or } V = -Ir + \epsilon$$

Because  $V$  and  $I$  are variables, then  $m = -r$  and  $b = \epsilon$ .

If  $T = 2\pi\sqrt{l/g}$  where  $T$  and  $l$  are the variables, and  $2\pi$  and  $g$  are constants, then  $T$  plotted against  $l$  will not give a straight-line relationship. But if a plot of  $T$  against  $\sqrt{l}$

or  $T^2$  against  $l$  is plotted, it will yield a straight line. These graphs are shown in below.

- (i)  $T$  vs  $l$                       (ii)  $T$  vs  $\sqrt{l}$                       (iii)  $T^2$  vs  $l$

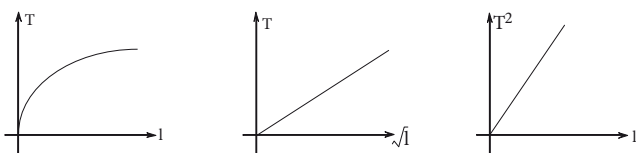
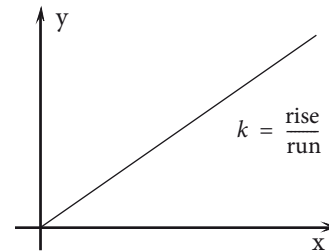


Figure 125 Some different relationships

## STANDARD GRAPHS

### 1. Linear

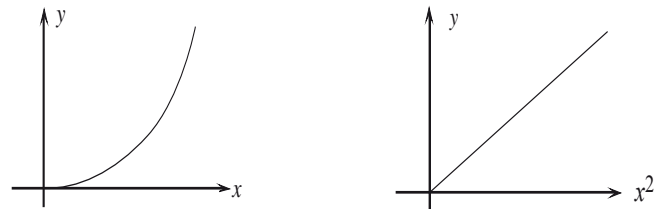
The linear graph shows that  $y$  is directly proportional to  $x$



i.e.,  $y \propto x$  or  $y = kx$  where  $k$  is the constant of proportionality.

### 2. Parabola

The parabola shows that  $y$  is directly proportional to  $x^2$ . That is,  $y \propto x^2$  or  $y = kx^2$  where  $k$  is the constant of proportionality.



In the equation  $s = ut + \frac{1}{2}at^2$ , where,

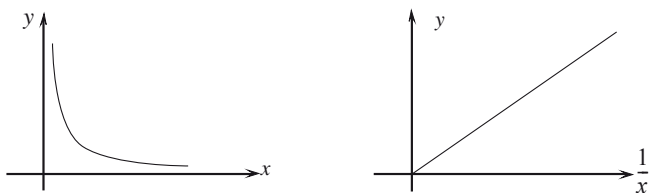
- $s$  = displacement in m
- $u$  = initial velocity in  $\text{m s}^{-1}$
- $a$  = acceleration in  $\text{m s}^{-2}$
- $t$  = time in s

then,  $s \propto t^2$ ,  $k = \frac{1}{2}a/\text{m s}^{-2}$  and  $u = y$ -intercept

### 3. Hyperbola

The hyperbola shows that  $y$  is inversely proportional to  $x$  or  $y$  is directly proportional to the reciprocal of  $x$ .

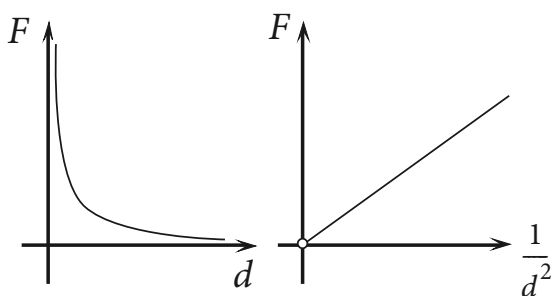
i.e.,  $y \propto 1/x$  or  $xy = k$



An example of an inverse proportionality is found in relating pressure,  $P$ , and volume,  $V$ , of a fixed mass of gas at constant temperature

$$P \propto \frac{1}{V} \Rightarrow P = \frac{k}{V}$$

or  $PV = k$  (= constant)



An inverse square law graph is also a hyperbola. The force  $F$  between electric charges at different distances  $d$  is given by:

$$F = \frac{k q_1 q_2}{r^2}$$

A graph of  $F$  versus  $d$  has a hyperbolic shape, and a graph of  $F$  versus  $1/d^2$  is a straight line.

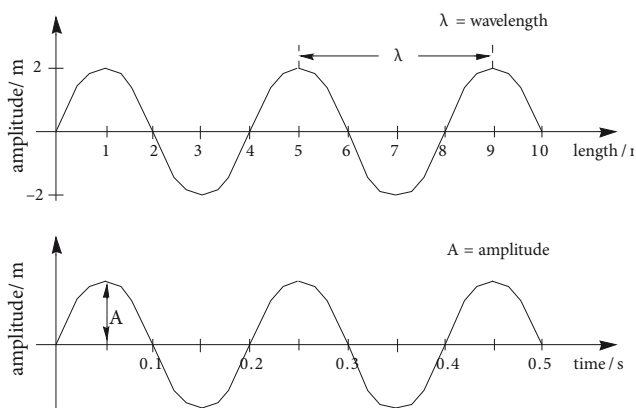
## 4. Sinusoidal

A sinusoidal graph is a graph that has the shape of a sine curve and its mathematics is unique. It can be expressed using degrees or radians.

The wavelength  $\lambda$  is the length of each complete wave in metres and the amplitude  $A$  is the maximum displacement from the x-axis. In the top sinusoidal graph the wavelength is equal to 5m and the amplitude is equal to 2m.

The frequency  $f$  of each wave is the number of waves occurring in a second measured in hertz (Hz) or  $s^{-1}$ . The period  $T$  is the time for one complete wave. In the bottom sinusoidal wave, the frequency is 5 Hz, and the period is 0.2 s.

Sinusoidal Graph



The equations for these graphs will be explored in Chapter 4 when you will study oscillations and simple harmonic motion.

## LOGARITHMIC FUNCTIONS (AHL)

### Exponential and logarithmic graphs

If the rate of change of a quantity over time depends on the original amount of matter, the rate of change may well be exponential. Certain elements undergo exponential decay when they decay radioactively. When bacteria reproduce, the change of bacteria over time is given by an exponential growth.

Consider a sample of a material with an original number of atoms  $N_0$  that undergo radioactive decay as shown in Figure 131. It can be shown that the number of atoms  $N$  left to decay after a period of time  $t$  is given by

$$N = N_0 e^{-kt}$$

From the logarithmic equations given in Appendix 1, it can be shown that

$$\ln N = -kt + \ln N_0$$

Therefore when  $\ln N$  is plotted against time the slope of the straight line produced is equal to  $-k$ .

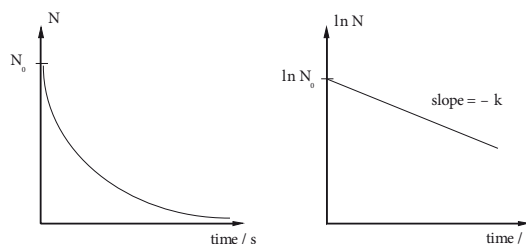


Figure 131 Logarithmic Graphs

Now let us examine a logarithmic function. In thermodynamics, the pressure  $p$  versus volume  $V$  curve for an adiabatic change at constant temperature is given by the equation

$$pV^\gamma = k \quad (\text{where } \gamma \text{ and } k \text{ are constants})$$

If we take the log of both sides then the equation will be  $\log p + \gamma \log V = \log k$

Now if the equation is rearranged into the straight line for  $y = mx + b$ , we get

$$\log p = -\gamma \log V + \log k$$

If a graph of  $\log p$  versus  $\log V$  is plotted, a straight line is obtained with the gradient being equal to  $\gamma$  and the  $y$ -intercept being equal to  $\log k$ .

### Exercise 1.6

- It can be shown that the pressure of a fixed mass of gas at constant temperature is inversely proportional to the volume of the gas. If a graph of pressure versus volume was plotted, the shape of the graph would be
  - a straight line
  - a parabola
  - an exponential graph
  - a hyperbola
- Newton showed that a force of attraction  $F$  of two masses  $m$  and  $M$  separated by a distance  $d$  was given by  $F \propto Mm/d^2$ . If  $m$  and  $M$  are constant, a graph of  $F$  versus  $d^{-2}$  would have which shape?
  - a parabola
  - a straight line
  - a hyperbola
  - an exponential shape
- The resistance of a coil of wire  $R_\theta$  increases as the temperature  $\theta$  is increased. The resistance  $R_\theta$  at a temperature can be expressed as  $R_\theta = R_0(1 + \mu\theta)$  where  $\mu$  is the temperature coefficient of resistance. Given the following data, plot a graph that will allow you to determine  $R_0$  and  $\mu$ .

$R_\theta / \Omega$	23.8	25.3	26.5	28.1	29.7	31.7
$\theta / ^\circ\text{C}$	15	30	45	60	80	100

- Given that  $s = \frac{1}{2}gt^2$  where  $s$  is the distance travelled by a falling object in time  $t$ , and  $g$  is a constant. The following data is provided:

$s \text{ (m)}$	5.0	20	45	80
$T^2 \text{ (s}^2\text{)}$	1.0	4.0	9.0	16

Plot a relevant graph in order to determine the value of the constant  $g$ .

### (AHL)

- It can be shown that  $V = \frac{RE}{(R+r)}$  where  $E$  and  $r$  are constants.

In order to obtain a straight line graph, one would plot a graph of

- $\frac{1}{V}$  against  $R$
  - $V$  against  $R$
  - $\frac{1}{V}$  against  $\frac{1}{R}$
  - $V$  against  $\frac{1}{R}$
- The magnetic force  $F$  between 2 magnets and their distance of separation  $d$  are related by the equation  $F = kd^n$  where  $n$  and  $k$  are constants.
    - What graph would you plot to determine the values of the two constants?
    - From the graph how could you determine  $n$  and  $k$ ?
  - The intensity  $I$  of a laser beam passing through a cancer growth decreases exponentially with the thickness  $x$  of the cancer tissue according to the equation  $I = I_0 e^{-\mu x}$ , where  $I_0$  is the intensity before absorption and  $\mu$  is a constant for cancer tissue.

What graph would you draw to determine the values of  $I_0$  and  $\mu$ ?

## UNCERTAINTIES IN GRAPHS (EXTENSION)

Students must be able to determine the uncertainties in the slope and intercepts of a straight line graph. In order to cover this skill, it is best to use an example.

Example

The schematic diagram in Figure 134 demonstrates an experiment to determine Planck's Constant. The wavelength ( $\lambda$ ) of light from the light source incident on a metal photoemissive plate of a photoelectric cell is varied, and the stopping voltage  $V_s$  applied across the photoelectric cell is measured.

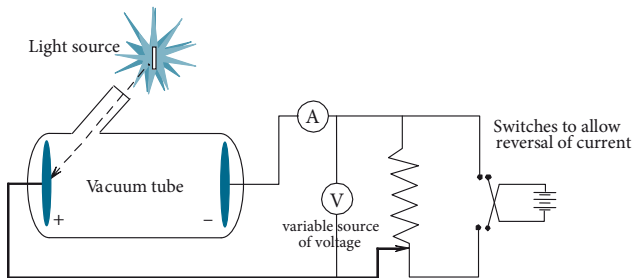


Figure 134 Determining Planck's Constant

The following values were obtained for different light radiation colours

Light Radiation Colour	Stopping Voltage $V_s$ $\pm 0.05$ V	$\lambda$ $\pm 0.3 \times 10^{-7}$ m
Red	1.20	6.1
Orange	1.40	5.5
Yellow	1.55	5.2
Green	1.88	4.6
Blue	2.15	4.2
Violet	2.50	3.8

Figure 135 Data For Planck's Constant

It can be shown that for this experiment  $hc / \lambda = h f = \Phi + eV_s$  where  $h$  is Planck's Constant

$c$  is the speed of light constant  $3 \times 10^8$  m s<sup>-1</sup>

$\lambda$  is the wavelength in m and  $f$  is the frequency in Hz

$\Phi$  is the work function.

$e$  is the charge on an electron ( $1.6 \times 10^{-19}$ C)

- (a) Copy Figure 135, add 2 more columns and complete the frequency and the uncertainties columns for each colour of light radiation in the table.

Because the wavelength is given to two significant figures, the frequency can only be given to two significant figures.

For division, to find the frequency from  $hc / \lambda$ , the relative uncertainty in the frequency has to be calculated for each wavelength. For example, for dark red:

$$\text{the relative uncertainty} = 0.3 \times 10^{-7} \div 6.1 \times 10^{-7} = 0.0492$$

$$\text{the absolute uncertainty} = 0.0492 \times 1.6 \times 10^{14}$$

$$= \pm 0.07 \times 10^{14} \text{ Hz}$$

In this case, the absolute uncertainty is not half the limit of reading as the absolute uncertainty of the wavelength was given as  $\pm 0.3 \times 10^{-7}$  m. Remember that the minimum possible absolute uncertainty is half the limit of reading which would be  $\pm 0.05 \times 10^{-7}$  m.

Light Radiation Colour	Stopping Voltage $V_s$ $\pm 0.05$ V	$\lambda$ $\pm 0.3 \times 10^{-7}$ m	Frequency $\times 10^{14}$ Hz	Uncertainty $\pm 10^{14}$ Hz
Red	1.20	6.1	1.6	0.07
Orange	1.40	5.5	1.8	0.09
Yellow	1.55	5.2	1.9	0.1
Green	1.88	4.6	2.2	0.1
Blue	2.15	4.2	2.4	0.2
Violet	2.50	3.8	2.6	0.2

Figure 136 Data showing uncertainties

- (b) Plot a fully labelled graph graph paper with stopping voltage on the vertical axis against the reciprocal of the wavelength on the horizontal axis. Allow for a possible negative y-intercept.

Now can you put in the error bars for each point and label the axis. There will be a negative y-intercept.

Mark in the gradient and the y-intercept.

The required graph is shown in Figure 137. Note the maximum and minimum lines and the line of best fit, the gradient of the straight line of best fit and the value of the negative y-intercept.

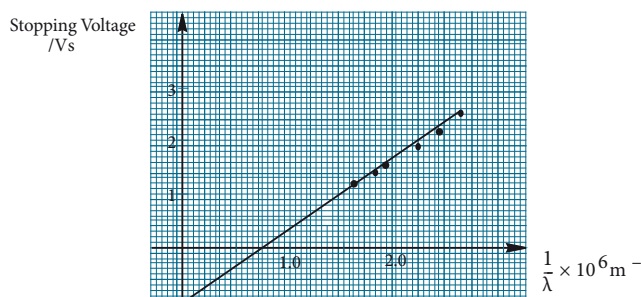


Figure 137 Data for Planck's Constant

- (c) Calculate Planck's Constant by graphical means and compare your value with the theoretical value of  $6.63 \times 10^{-34} \text{ J s}$ .

The equation given at the start of this example was:

$$hc/\lambda = hf = \Phi + eV_s$$

If we rearrange this equation in the form  $y = mx + c$ , the equation becomes:

$$V_s = hf/e - \Phi/e$$

Therefore, the gradient  $= h/e = 2.07 \text{ V} / 4.62 \times 10^{14} \text{ s}^{-1}$

$$= 4.5 \times 10^{-15} \text{ Vs}$$

$$h = \text{gradient}/e = 4.5 \times 10^{-15} \text{ Vs} \times 1.6 \times 10^{-19} \text{ C}$$

$$= 7.2 \times 10^{-34} \text{ Js}$$

The accepted value of Planck's constant is  $6.63 \times 10^{-34} \text{ Js}$ .

The percentage discrepancy  $= 7.2 - 6.63 / 6.63 \times 100\%$

$$= 8.6 \%$$

- (d) Determine the minimum frequency of the photoelectric cell by graphical means.

The threshold frequency is the x-intercept

$$= 2.2 \pm 0.6 \times 10^{14} \text{ Hz}$$

- (e) From the graph, calculate the work function of the photoemissive surface in the photoelectric cell in joules and electron-volts.

The y-intercept is equal to  $-\Phi/e$

$$\text{Work function, } \Phi = e \times (\text{y-intercept}) = 1.6 \times 10^{-19} \text{ C} \times -1 \text{ V} \\ = 1.6 \times 10^{-19} \text{ J}$$

## Exercise

## 1.7

1. An investigation was undertaken to determine the relationship between the length of a pendulum  $l$  and the time taken for the pendulum to oscillate twenty times. The time it takes to complete one swing back and forth is called the period  $T$ . It can be shown that

$$T = 2\pi\sqrt{\frac{l}{g}}$$

where  $g$  is the acceleration due to gravity.

The data in the table below was obtained.

- Copy the table and complete the period column for the measurements. Be sure to give the uncertainty and the units of  $T$ .
- Calculate the various values for  $T^2$  including its units.
- Determine the absolute error of  $T^2$  for each value.
- Draw a graph of  $T^2$  against  $l$ . Make sure that you choose an appropriate scale to use as much of a piece of graph paper as possible. Label the axes, put a heading on the graph, and use error bars. Draw the curve of best fit.
- What is the relationship that exists between  $T^2$  and  $l$ ?
- Are there any outliers?
- From the graph determine a value for  $g$ .

Length of pendulum $\pm 0.05 \text{ m}$	Time for 20 oscillations $\pm 0.2 \text{ s}$	Period $T$	$T^2$	Absolute error of $T^2$
0.21	18.1			
0.40	25.5			
0.62	31.5			
0.80	36.8			
1.00	40.4			

# 1.3 VECTORS AND SCALARS

- 1.3.1 Distinguish between vector and scalar quantities, and give examples of each.
- 1.3.2 Determine the sum or difference of two vectors by a graphical method.
- 1.3.3 Resolve vectors into perpendicular components along chosen axes.

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## 1.3.1 VECTORS AND SCALARS - EXAMPLES

**Scalars** are quantities that can be completely described by a magnitude (size). Scalar quantities can be added algebraically. They are expressed as a positive or negative number and a unit. Some scalar quantities, such as mass, are always positive, whereas others, such as electric charge, can be positive or negative. Figure 139 lists some examples of scalar and vector quantities.

Scalars	Vectors
distance ( $s$ )	displacement ( $s$ )
speed	velocity ( $v$ )
mass ( $m$ )	area ( $A$ )
time ( $t$ )	acceleration ( $a$ )
volume ( $V$ )	momentum ( $p$ )
temperature ( $T$ )	force ( $F$ )
charge ( $Q$ )	torque ( $\tau$ )
density ( $\rho$ )	angular momentum ( $L$ )
pressure ( $P$ )	flux density ( $\Phi$ )
energy ( $E$ )	electric field intensity ( $E$ )
power ( $P$ )	magnetic field intensity ( $B$ )

Figure 139 Examples of scalar and vector quantities

**Vectors** are quantities that need both magnitude and direction to describe them. The magnitude of the vector is always positive. In this textbook, vectors will be represented in heavy print. However, they can also be represented by underlined symbols or symbols with an arrow above or below the symbol. Because vectors have both magnitude and direction, they must be added, subtracted and multiplied in a special way.

The basic mathematics of vector analysis will be outlined hereunder, and no mention will be made of  $i$ ,  $j$  and  $k$  unit vectors.

When vectors are graphed, the system of coordinates is called a rectangular coordinate system or a Cartesian coordinate system, or simply, a coordinate plane. Vectors in the same plane are said to be co-planar.

## 1.3.2 THE SUM OR DIFFERENCE OF TWO VECTORS

### Addition of vectors

From simple arithmetic it is known that  $4 \text{ cm} + 5 \text{ cm} = 9 \text{ cm}$

However, in vector context, a different answer is possible when 4 and 5 are added.

For example,  $4 \text{ cm north (N)} + 5 \text{ cm south (S)} = 1 \text{ cm south}$

Suppose you move the mouse of your computer 4 cm up your screen (N), and then 5 cm down the screen (S), you move the mouse a total distance of 9 cm. This does not give the final position of the arrow moved by the mouse. In fact, the arrow is 1 cm due south of its starting point, and this is its displacement from its original position. The first statement adds scalar quantities and the second statement adds two vector quantities to give the resultant vector R.

The addition of vectors which have the same or opposite directions can be done quite easily:

$$1 \text{ N east} + 3 \text{ N east} = 4 \text{ N east (newton force)}$$

$$200 \mu\text{m north} + 500 \mu\text{m south} = 300 \mu\text{m south (micrometre)}$$

$$300 \text{ m s}^{-1} \text{ north-east} + 400 \text{ m s}^{-1} \text{ south-west} = 100 \text{ m s}^{-1} \text{ south west (velocity)}$$

The addition of co-planar vectors that do not have the same or opposite directions can be solved by using scale drawings or by calculation using Pythagoras' theorem and trigonometry.

Vectors can be denoted by boldtype, with an arrow above the letter, or a tilde, i.e.,  $\mathbf{a}$ ,  $\vec{a}$  or  $\underline{a}$  respectively. They are represented by a straight line segment with an arrow at the end. They are added by placing the tail of one to the tip of the first (placing the arrow head of one to the tail of the other). The resultant vector is then the third side of the triangle and the arrowhead points in the direction from the 'free' tail to the 'free' tip. This method of adding is called the **triangle of vectors** (see Figure 140).

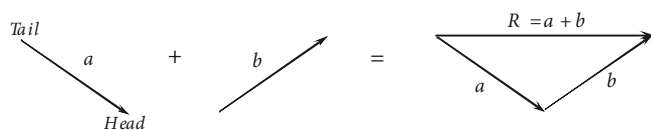


Figure 140 Addition Of Vectors

The parallelogram of vectors rule for adding vectors can also be used. That is, place the two vectors tail to tail and then complete a parallelogram, so that the diagonal starting where the two tails meet, becomes the resultant vector. This is shown in Figure 118.

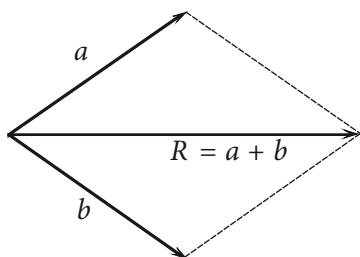


Figure 141 Addition of vectors using parallelogram rule

If more than two co-planar vectors are to be added, place them all head to tail to form a polygon. Consider the three vectors,  $a$ ,  $b$  and  $c$  shown in Figure 142. Adding the three vectors produces the result shown in Figure (b).

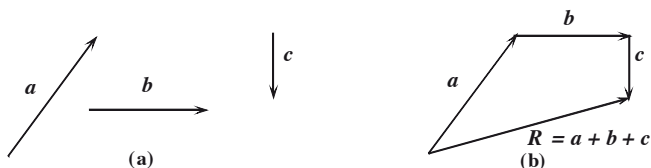


Figure 142 Addition of more than two vectors

Notice then that  $a + b + c = a + c + b = b + a + c = \dots$ . That is, vectors can be added in any order, the resultant vector remaining the same.

### Example

On an orienteering expedition, you walk 40 m due south and then 30 m due west. Determine how far and in what direction are you from your starting point.

### Solution

#### Method 1 By scale drawing

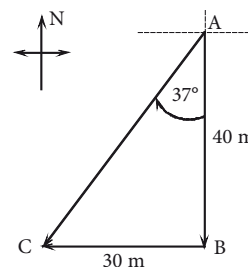


Figure 143 Orienteering

Draw a sketch of the two stages of your journey.

From the sketch make a scale drawing using 1 cm equal to 10 m (1 cm∶10m).

If you then draw the resultant AC, it should be 5 cm in length. Measure  $\angle CAB$  with a protractor.

The angle should be about  $37^\circ$ .

Therefore, you are 50 m in a direction south  $37^\circ$  west from your starting point (i.e., S  $37^\circ$  W).

#### Method 2 By calculation

Using Pythagoras' theorem, we have

$$AC^2 = 40^2 + 30^2 \therefore AC = \sqrt{40^2 + 30^2} = 50$$

(taking the positive square root).

From the tan ratio,

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}} \quad \text{we have} \quad \tan \theta = \frac{BC}{AB} = \frac{30}{40} = 0.75$$

$$\therefore \tan^{-1}(0.75) = 36.9^\circ$$

You are 50 m in a direction south  $37^\circ$  west from your starting point (i.e. S  $37^\circ$  W).

## Subtraction of vectors

In Chapter 2, you will describe motion – kinematics. You will learn that change in velocity,  $\Delta v$ , is equal to the final velocity minus the initial velocity,  $v - u$ . Velocity is a vector quantity so  $\Delta v$ ,  $v$  and  $u$  are vectors. To subtract  $v - u$ , you reverse the direction of  $u$  to obtain  $-u$ , and then you add vector  $v$  and vector  $-u$  to obtain the resultant  $\Delta v$ .

That is,  $\Delta v = v + (-u)$ . Vectors  $v$  and  $u$  are shown. For  $v - u$ , we reverse the direction of  $u$  and then add head to tail

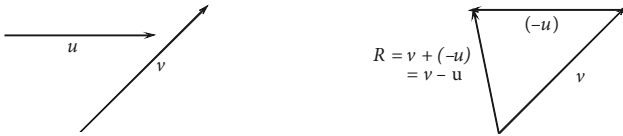


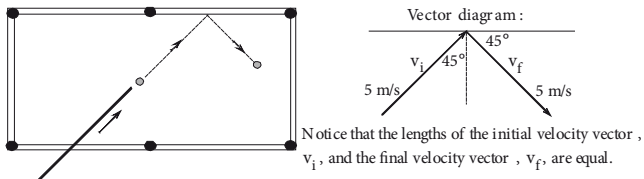
Figure 144 Subtraction of vectors

### Example

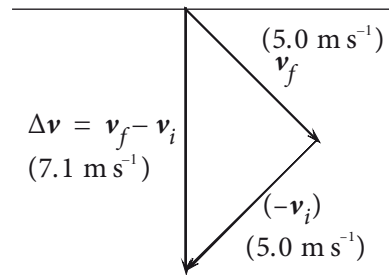
A snooker ball is cued and strikes the cushion of the snooker table with a velocity of  $5.0 \text{ m s}^{-1}$  at an angle of  $45^\circ$  to the cushion. It then rebounds off the cushion with a velocity of  $5.0 \text{ m s}^{-1}$  at an angle of  $45^\circ$  to the cushion. Determine change in velocity? (Assume the collision is perfectly elastic with no loss in energy).

### Solution

You can solve this problem by scale drawing or calculation. Draw a sketch before solving the problem, then draw the correct vector diagram.



Using the vector diagram above we can now draw a vector diagram to show the change in velocity.



Using the same scale as that used for the  $5.0 \text{ m s}^{-1}$  velocity vector, the change in velocity is  $7.1 \text{ m s}^{-1}$  at right angles to the cushion.

We could also use Pythagoras' theorem to determine the length (or magnitude) of the change in velocity vector,  $\Delta v$ :

$$|\Delta v|^2 = |v_f|^2 + |v_i|^2,$$

$$\text{so that } |\Delta v|^2 = 5^2 + 5^2 = 50 \therefore |\Delta v| = \sqrt{50} \approx 7.1 \text{ m s}^{-1}$$

## Multiplying vectors and scalars

Scalars are multiplied and divided in the normal algebraic manner, for example:

$$5 \text{ m} \div 2 \text{ s} = 2.5 \text{ m s}^{-1} \quad 2 \text{ kW} \times 3 \text{ h} = 6 \text{ kW h (kilowatt-hours)}$$

A vector multiplied by a scalar gives a vector with the same direction as the vector and magnitude equal to the product of the scalar and the vector.

For example:  $3 \times 15 \text{ N east} = 45 \text{ N east}$ ;

$2 \text{ kg} \times 15 \text{ m s}^{-1} \text{ south} = 30 \text{ kg m s}^{-1} \text{ south}$

The vector analysis of a vector multiplied by a vector is not required for the syllabus. However, you will encounter these situations when you study work, energy and electromagnetism. Two points will be made in an oversimplified manner:

1. Vectors can be multiplied to give a scalar answer.

For example, force can be multiplied by a displacement to give work which is a scalar. Finding the product in this manner is called the dot product, i.e.,  $U \cdot V = |U| |V| \cos \theta$  where  $\theta$  is the angle between the directions of  $V$  and  $U$ .





Figure 147 Multiplying vectors

2. The product of two vectors can also give a vector answer. For example, the force exerted on a proton moving with a velocity in a magnetic field is given by the equation  $F = qv \times B$  where  $q$  is the charge in coulombs,  $v$  is the velocity in metres per second, and  $B$  is the magnetic field strength in teslas.  $q$  is a scalar and  $v$  and  $B$  are vectors.

The answer  $F$  is a vector. Finding the product in this manner is called the **cross product**,  $V \times U$ .

The magnitude of the cross product,  $V \times U$  is given by  $|V \times U| = |U| |V| \sin \theta$

The direction of the answer,  $V \times U$  is at right angles to both  $V$  and  $U$  and can be found by curling the fingers of your right hand in the direction of  $V$  so that they curl towards  $U$  when you bend them. Your thumb is then pointing in the required direction.

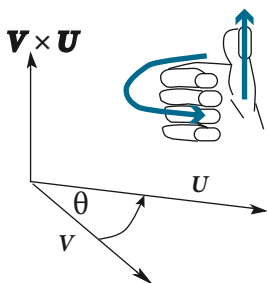


Figure 148 Right Hand Rule

In the Figure 148, the direction of  $V \times U$  is 'up' the page.

## Exercise

1.8

1. Which of the following lines best represents the vector 175 km east (1 cm : 25 km)?

- A.
- B.
- C.
- D.

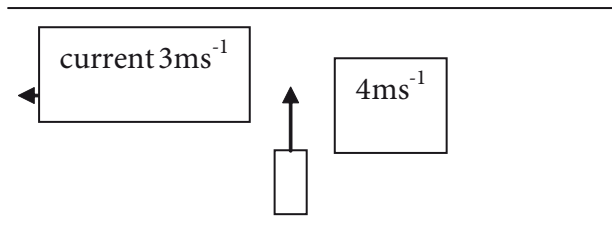
2. Which one of the following is a vector quantity?

- A. Work  
B. Speed  
C. Acceleration  
D. Pressure

3. Which one of the following is a scalar quantity?

- A. Force  
B. Velocity  
C. Momentum  
D. Energy

4. The diagram below shows a boat crossing a river with a velocity of  $4 \text{ m s}^{-1}$  north. The current flows at  $3 \text{ m s}^{-1}$  west.



The resultant magnitude of the velocity of the boat will be

- A.  $3 \text{ m s}^{-1}$   
B.  $4 \text{ m s}^{-1}$   
C.  $5 \text{ m s}^{-1}$   
D.  $7 \text{ m s}^{-1}$

5. Two vectors with displacements of 10 m north-west and 10 m north-east are added. The direction of the resultant vector is

- A. south      B. north-east  
C. north      D. north-west

6. Add the following vectors by the graphical method

- (a) 4 m south and 8 m north,  
(b) 5m north and 12 m west,  
(c) 6.0 N west and 6.0 N north,  
(d)  $9.0 \text{ m s}^{-1}$  north +  $4.0 \text{ m s}^{-1}$  east +  $6.0 \text{ m s}^{-1}$  south.

7. Subtract the following vectors by either the graphical method or by calculation
  - (a) 2 m east from 5 m east (i.e., 5 m east – 2m east),
  - (b) 9 m s<sup>-2</sup> north from 4 m s<sup>-2</sup> south,
  - (c) 4.0 N north from 3.0 N east,
  - (d) 3.2 T east from 5.1 T south.
8. Calculate the following products
  - (a) 20 m s<sup>-1</sup> north by 3
  - (b) 12 by 5 N s north 12° east
9. If a cyclist travelling east at 40 m s<sup>-1</sup> slows down to 20 m s<sup>-1</sup>, what is the change in velocity?
10. Find the resultant of a vector of 5 m north 40° west added to a vector of 8 m east 35° north

### 1.3.3 RESOLUTION OF VECTORS

The process of finding the components of vectors is called resolving vectors. Just as two vectors can be added to give a resultant vector, a single vector can be split into two components or parts.

The vector 5m south has a vertical component of 5 m south and a zero horizontal component just as the vector 10 N east has a zero vertical component and a horizontal component of 10 N east.

Suppose you have a vector that is at an angle to the horizontal direction. Then that vector consists of measurable horizontal and vertical components. In Figure 151, the vector *F* is broken into its components. Note that the addition of the components gives the resultant *F*.

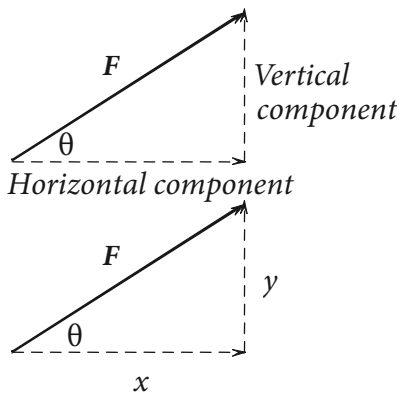


Figure 151 Resolution of vectors

From trigonometry

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{y}{|F|}$$

$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{x}{|F|}$$

and  $\cos \theta = \text{adjacent/hypotenuse} = x/F$

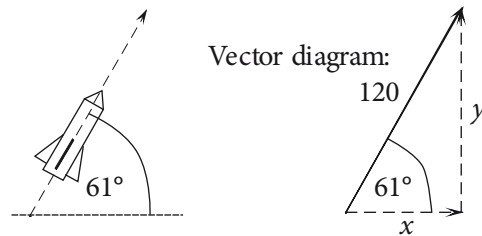
This means that the magnitude of the vertical component =  $y = F \sin \theta$

and the magnitude of the horizontal component =  $x = F \cos \theta$

#### Example

A sky rocket is launched from the ground at an angle of 61.0° with an initial velocity of 120 m s<sup>-1</sup>. Determine the components of this initial velocity?

#### Solution



From the vector diagram we have that

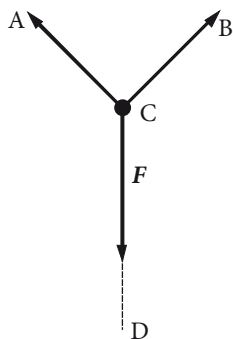
$$\begin{aligned} \sin 61^\circ &= \frac{y}{120} \Rightarrow y = 120 \sin 61^\circ \\ &= 104.954... \\ &= 1.05 \times 10^2 \end{aligned}$$

$$\begin{aligned} \text{and } \cos 61^\circ &= \frac{x}{120} \Rightarrow x = 120 \cos 61^\circ \\ &= 104.954... = 1.05 \times 10^2 \text{ and} \\ &= 58.177... \\ &= 58.2 \end{aligned}$$

That is, the magnitude of the vertical component is  $1.1 \times 10^2 \text{ m s}^{-1}$  and the magnitude of the horizontal component is  $58 \text{ m s}^{-1}$ .

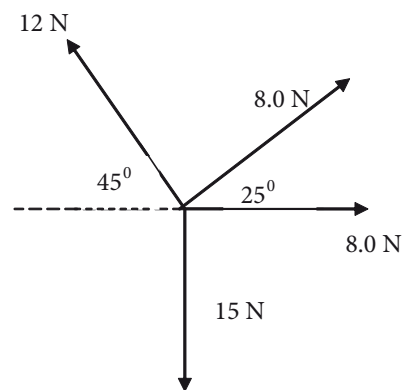
## Exercise 1.9

- The vertical component of a vector of a 4.0 N force acting at  $30^\circ$  to the horizontal is
  - 4.3 N
  - 2 N
  - 4 N
  - 8.6 N
- Calculate the horizontal component of a force of 8.4 N acting at  $60.0^\circ$  to the horizontal.
- Calculate the vertical and horizontal components of the velocity of a projectile that is launched with an initial velocity of  $25.0 \text{ m s}^{-1}$  at an angle of elevation of  $65^\circ$  to the ground.
- Calculate the easterly component of a force of 15 N south-east.
- Calculate the vector whose components are 5.0 N vertically and 12 N horizontally.
- Calculate  $F$  in the diagram below if the sum of all the forces in the is zero.



$$AC = 2 \text{ N} \quad BC = 2 \text{ N} \text{ and} \\ \angle ACD = 135^\circ \quad \angle BCD = 135^\circ$$

- Calculate the acceleration of a small object down a frictionless plane that is inclined at  $30.0^\circ$  to the horizontal. Take the acceleration due to gravity  $g$  equal to  $9.81 \text{ ms}^{-2}$ .
- Calculate the resultant force of all the forces acting on a point object in the diagram below.



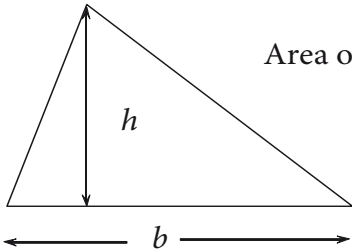
# APPENDIX 1

## MATHEMATICAL REQUIREMENTS

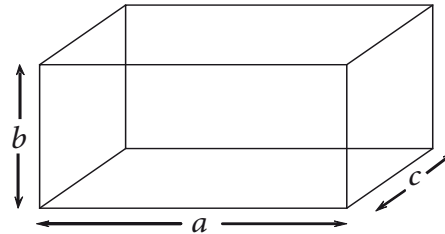
During this course you should experience a range of mathematical techniques. You will be required to develop mathematical skills in the areas of arithmetic and computation, algebra, geometry and trigonometry, and graphical and vector analysis for both external and internal assessment.

### MATHEMATICAL SENTENCES

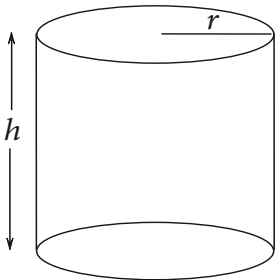
- = is equal to
- / divided by or in units of
- < is less than
- > is greater than
- ∞ is proportional to
- ≈ is approximately equal to
- Δx a small difference between two values of x
- |x| the absolute value of x



Area of any triangle =  $\frac{1}{2}bh$



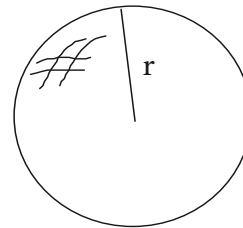
Surface area of a cuboid =  $2(ab + bc + ac)$



Area of a hollow cylinder =  $2\pi h$

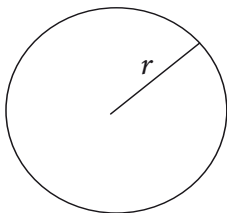
Surface area of a cylinder =  $2\pi r(h + r)$

Volume of a cylinder =  $\pi r^2 h$



Circumference =  $2\pi r$

Area of a circle =  $\pi r^2$



## GEOMETRY

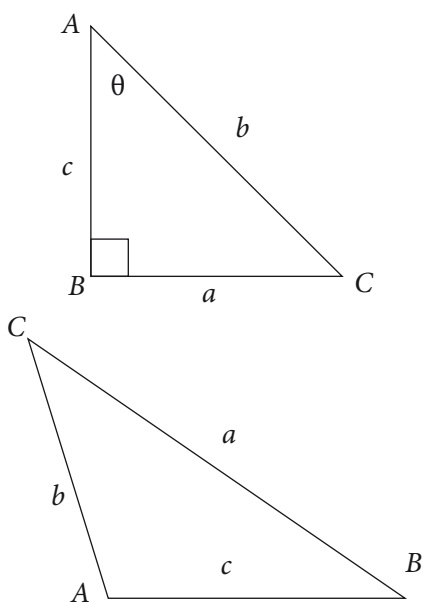
### INDICES

1.  $a^x \times a^y = a^{x+y}$
2.  $a^x \div a^y = \frac{a^x}{a^y} = a^{x-y}$
3.  $(a^x)^y = a^{x \times y}$
4.  $a^x \times b^x = (a \times b)^x$
5.  $a^0 = 1, 1^x = 1, 0^x = 0 (x \neq 0), \sqrt[x]{a} = a^{1/x}$

### LOGARITHMS

1.  $\log x + \log y = \log(x \times y), x > 0, y > 0.$
2.  $\log x - \log y = \log\left(\frac{x}{y}\right), x > 0, y > 0.$
3.  $x \log y = \log y^x, y > 0.$
4.  $a^x = y \Leftrightarrow x = \log_a y$

## TRIGONOMETRY



$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{a}{b}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{c}{b}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{a}{c}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}, \cos\theta \neq 0$$

For very small angles,  $\sin\theta \approx \tan\theta \approx \theta$ ,  $\cos\theta \approx 1$

$$\text{Sine rule: } \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\text{Cosine rule: } a^2 = b^2 + c^2 - 2bc \times \cos A$$

$$\text{Area of triangle: } A = \frac{1}{2}ab \times \sin C$$

Identities:

$$\sin^2 A + \cos^2 A = 1$$

$$\sin(A - B) + \sin(A + B) = 2 \sin A \sin B$$

$$\sin A + \sin B = 2 \sin[(A + B)/2] \times \cos[(A - B)/2]$$

## ANGULAR MEASURE

Angles are measured in radians. One radian is the angle subtended by an arc with length equal to the radius. If  $s = r$ , then  $\theta = s/r$ .

Note then, that  $2\pi \text{ rad} = 360^\circ$ , and  $1 \text{ rad} = 57.3^\circ$

## Exercise

1.10

- Convert  $13/17$  to a decimal and to a percentage.
- Use a calculator to find  $3.6^3$  and  $\log 120$ .
- Make  $y$  the subject of the equation if  $x = 2y - 6$ .
- Make  $v$  the subject of the equation given that  $F = mv^2/r$
- Make  $g$  the subject of the equation given that  $T = 2\pi \sqrt{1/g}$
- Solve for  $x$  and  $y$  in the following simultaneous equations
 
$$2x + 4y = 18$$

$$x - y = -1$$
- Calculate the following
  - $16^2 + 16^3$
  - $25^{1.5}$
  - $(\sqrt{2})^4$
  - $(\sqrt{3})^{-2}$
- Evaluate the following
  - $\log_4 64$
  - $\log_{10} 0.01$
- Find the circumference and area of a circle of radius 0.8 cm.
- Calculate the volume and surface area of a sphere of radius 0.023 m.
- How many radians are there in
  - $270^\circ$
  - $45^\circ$
- If  $\sin 2\theta = 1$  then what is  $\theta$  equal to?

## GREEK SYMBOLS

The Greek alphabet is commonly used in Physics for various quantities and constants. The capital and small letters and their names are given here for your convenience:

Letters		Name
A	$\alpha$	alpha
B	$\beta$	beta
$\Gamma$	$\gamma$	gamma
$\Delta$	$\delta$	delta
E	$\epsilon$	epsilon
Z	$\zeta$	zeta
H	$\eta$	eta
$\Theta$	$\theta$	theta
I	$\iota$	iota
K	$\kappa$	kappa
M	$\mu$	mu
N	$\nu$	nu
$\Xi$	$\xi$	xi
O	$\omicron$	omicron
$\Pi$	$\pi$	pi
P	$\rho$	rho
$\Sigma$	$\sigma$	sigma
T	$\tau$	tau
$\Phi$	$\varphi$	phi
X	$\chi$	chi
$\Psi$	$\psi$	psi
$\Omega$	$\omega$	omega