FIELDS AND FORCES

6

CORE

6.1 GRAVITATIONAL FORCE AND FIELD

6.1.1 State Newton's universal law of gravitation.

6.1.2 Define *gravitational field strength*.

6.1.3 Determine the gravitational field due to one or more point masses.

6.1.4 Derive an expression for gravitational field strength at the surface of a planet, assuming that all its mass is concentrated at its centre.

6.1.5 Solve problems involving gravitational forces and fields.

6.1.1 NEWTON'S UNIVERSAL LAW OF GRAVITATION

In the *Principia* Newton stated his Universal **Law of Gravitation** as follows:

'Every material particle in the Universe attracts every other material particle with a force that is directly proportional to the product of the masses of the particles and that is inversely proportional to the square of the distance between them'.

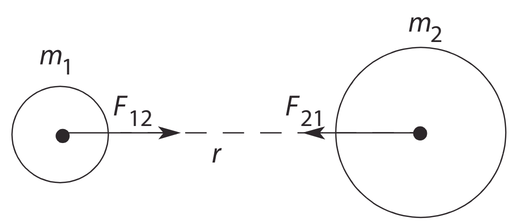

Figure 601 Newton's law of gravitation

We can write this law mathematically as:

$$\boldsymbol{F}_{12} = \frac{G m_1 m_2}{r^2}\hat{\boldsymbol{a}}_{12} = -\boldsymbol{F}_{21}$$

$\boldsymbol{F}_{12}$ is the force that particle 1 exerts on particle 2 and $\boldsymbol{F}_{21}$ is the force that particle 2 exerts on particle 1.

$\hat{\boldsymbol{a}}_{12}$ is a unit vector directed along the line joining the particles.

m_1 and m_2 are the masses of the two particles respectively and r is their separation.

G is a constant known as the *Universal Gravitational Constant* and its accepted present day value is 6.67×10^{-11} N m^{-2} kg^{-2}.

There are several things to note about this equation. The forces between the particles obey Newton's third law as discussed in Section 2.7. That is, the forces are equal and opposite. The mass of the particles is in fact their gravitational mass as discussed in Section 2.3.3.

Every particle in the Universe, according to Newton, obeys this law and this is why the law is known as a 'universal' law. This is the first time in the history of physics that we come across the idea of the universal application of a physical law. It is now an accepted fact that if a physical law is indeed to be a law and not just a rule then it must be universal. Newton was also very careful to specify the word particle. Clearly any two objects will attract each other because of the attraction between the respective particles of each object. However, this will be a very complicated force and will depend on the respective shapes of the bodies. Do not be fooled into thinking that for objects we need only specify the distance r as the distance between their respective centres of mass. If this were the case it would be impossible to peel an orange. The centre of mass of the orange is at its centre and the centre of the mass of the peel is also at this point. If we think that r in the Newton Law refers to the distance between the centres of mass of objects the distance between the two centres of mass is zero. The force therefore between the peel and the orange is infinite.

You will almost invariably in the IB course and elsewhere, come across the law written in its scalar form as

$$F = \frac{Gm_1m_2}{r^2}$$

However, do not forget its vector nature nor that it is a force law between particles and not objects or "masses".

6.1.2 Gravitational Field Strength

The Law of Universal Gravitation is an inverse square law and in this sense is very similar to the Coulomb force law discussed in topic 6.2. In fact if you replace m with q in the Newton law and G with $\frac{1}{4\pi\varepsilon_0}$ then we have the Coulomb law (except of course we can have negative charge but as far as we know there is no negative mass). So all that follows in the rest of this section is very similar to that to be discussed in topic 6.2 in connection with Coulomb's Law.

Any particle in the Universe exerts a gravitational force on any other particle in the Universe. In this sense we can think of the effect that a particle P for example produces on other particles without even knowing the location of P. We can think of a "field of influence radiating out" from P. This influence we call the **Gravitational Field** and by introducing this concept we are essentially moving our attention from the source of the field to the effect that the source produces.

In Figure 602 a particle of mass m is placed at point X somewhere in the Universe.

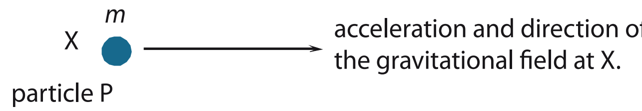

Figure 602 Force and acceleration

The particle is observed to accelerate in the direction shown. We deduce that this acceleration is due to a gravitational field at X. We do not know the source of the field but that at this stage does not matter. We are only concerned with the effect of the field. If the mass of P is small then it will not effect the field at X with its own field. We define the gravitational field strength I at X in terms of the force that is exerted on P as follows

$$\boldsymbol{I} = \frac{\boldsymbol{F}}{m}$$

That is 'the gravitational field strength at a point is the force exerted per unit mass on a particle of small mass placed at that point'.

From Newton's 2nd law F = ma we see that the field strength is actually equal to the acceleration of the particle. For this reason the gravitational field strength is often given the symbol 'g' So we can express the magnitude of the field strength in either N kg^{-1} or m s^{-2}. However, if we are dealing explicitly with field strengths then we tend to use the unit N kg^{-1}.

6.1.3,4 The Gravitational Field Strength of Point Masses and Sphere

Figure 603(a) shows the "field pattern" for an isolated particle of mass M.

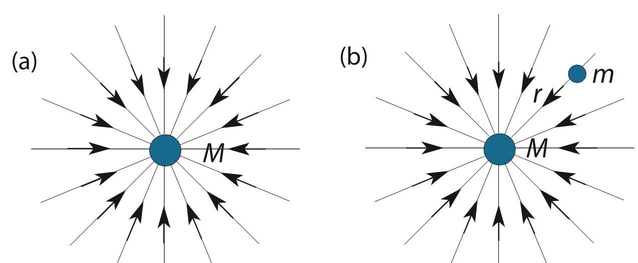

Figure 603 (a) and (b)

Clearly this is only a representation since the field due to M acts at all points in space.

Suppose we now place a particle of mass m a distance r from M as shown in Figure 603 (b).

The gravitational law gives the magnitude of the force that M exerts on m

$$F = \frac{GMm}{r^2}$$

So the magnitude of the gravitational field strength $I = \frac{F}{m}$ is given by

$$I = \frac{GM}{r^2}$$

If we wish to find the field strength at a point due to two or more point masses, then we use vector addition. (This is another example of the general principle of superposition - see 4.5.5). In Figure 604 the magnitude of the field strength produced by the point mass M_1 at point P is I_1 and that of point mass M_2 is I_2.

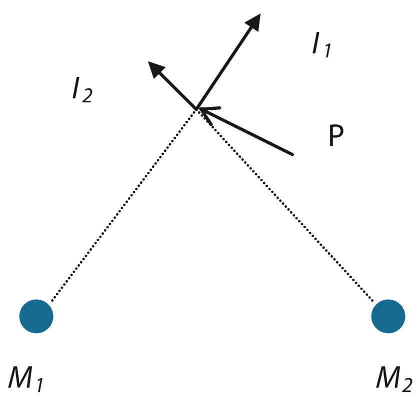

Figure 604 Vector addition

If I_1 and I_2 are at right angles to each other, the resultant magnitude of the field strength I at P is given by:

$$I = \sqrt{I_1^{\,2} + I_2^{\,2}}$$

If the particle of mass M is replaced with a sphere of mass M and radius R, as shown in Figure 605, then rely on the fact that the sphere behaves as a point mass situated at its centre, the field strength at the surface of the sphere will be given by

$$I = \frac{GM}{R^2}$$

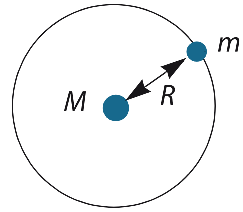

Figure 605 Field strength

If the sphere is the Earth then

$$I = \frac{GM_e}{R_e^2}$$

But the field strength is equal to the acceleration that is produced on a mass hence the acceleration of free fall at the surface of the Earth, g_0 , is given by

$$g_0 = \frac{GM_e}{R_e^2}$$

This actually means that whenever you determine the acceleration of free fall g_0 at any point on the Earth you are in fact measuring the gravitational field strength at that point. It can also be seen now why the value of g varies with height above the surface of the Earth. Since at a height of h above the surface of the Earth the field strength, g, is given by

$$g = \frac{GM_e}{(R_e + h)^2}$$

It can be shown that if we have a hollow sphere then the field strength at all points within the sphere is zero. This fact can be used to deduce an expression for the field strength at points inside the Earth.

It is left as an exercise to demonstrate (*if desired since this is not in the syllabus*) that if ρ is the mean density of the Earth then at a point distance r from the centre of the Earth the value of g is given by

$$g = \frac{4}{3}\pi r G\rho$$

6.1.5 SOLVE PROBLEMS INVOLVING GRAVITATIONAL FORCES AND FIELDS

Example

Take the value of $g_0 = 10 \text{ N kg}^{-1}$ and the mean radius of the Earth to be 'm' to estimate a value for the mass of the Earth.

Solution

We have $g_0 = \dfrac{GM_e}{R_e^2}$, therefore

$$M_e = \frac{g_0 R_e^2}{G} = \frac{10 \times (6.4 \times 10^6)^2}{6.7 \times 10^{-11}}$$

$\approx 6 \times 10^{24}$ *kg.*

Example

Assuming the Earth and Moon to be isolated from all other masses, use the following data to estimate the mass of the Moon.

mass of Earth = 6.0×10^{24} kg

distance between centre of Earth and centre of Moon = 3.8×10^8 m

distance from centre of Earth at which gravitational field is zero = 2.8×10^8 m

Answer: 7.4×10^{22} kg

6.2 ELECTRIC FORCE AND FIELD

6.2.1 State that there are two types of electric charge.

6.2.2 State and apply the law of conservation of charge.

6.2.3 Describe and explain the difference in the electrical properties of conductors and insulators.

6.2.4 State Coulomb's law.

6.2.5 Define *electric field strength*.

6.2.6 Determine the electric field strength due to one or more point charges.

6.2.7 Draw the electric field patterns for different charge configurations.

6.2.8 Solve problems involving electric charges, forces and fields.

6.2.1 TYPES OF ELECTRIC CHARGE

There are 2 types of electric charge – positive charge and negative charge.

It was not until the late 1890's through the work of *J.J. Thomson* that the true nature of electrons was discovered through experiments with cathode ray tubes. With this exploration of atoms and quantum mechanics in the 1900s, the electrical properties of matter were understood.

We now know that

- charge is conserved
- charge is quantised
- the force between two point charges varies as the inverse square law of the distance between the two charges.

These properties will be outlined further in a later section of this chapter.