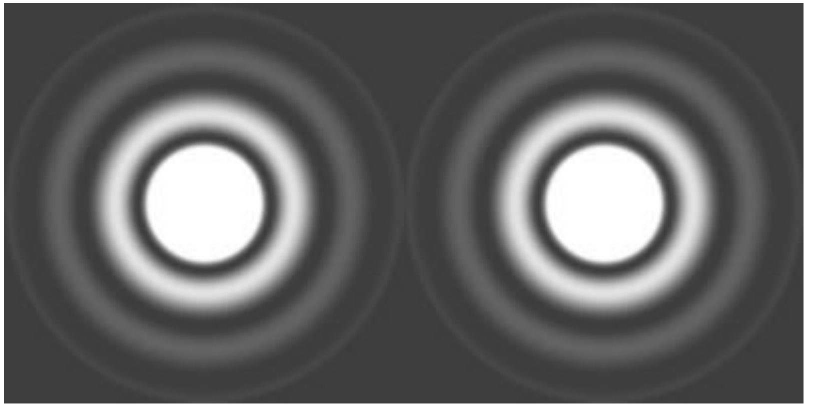
Resolution

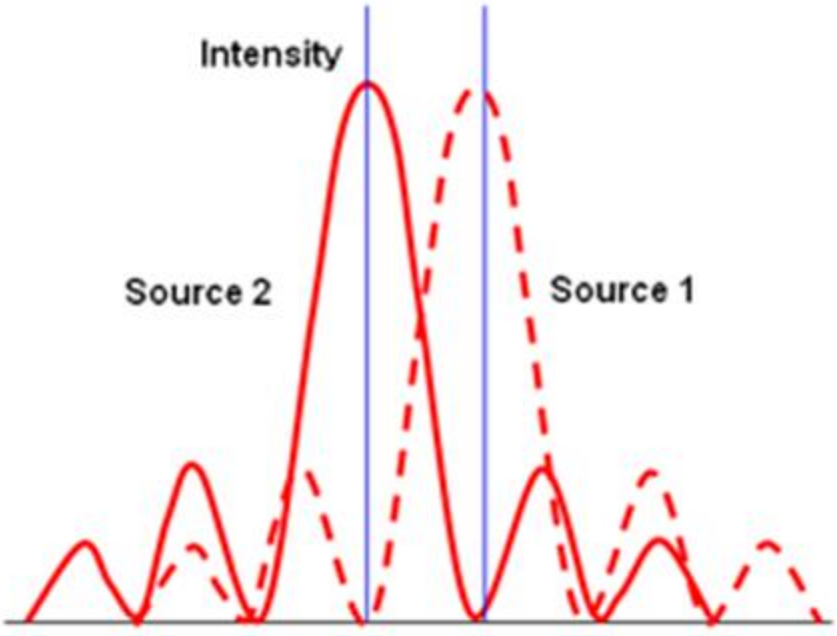


Resolution of images of two objects that are close to each other

- Practical application of single slit diffraction of waves through slits or circular apertures
- Light from two images, when passing through a lens, will diffract around the circular aperture of the lens
- The diffraction patterns may merge with each other, making it difficult to distinguish how many objects there are

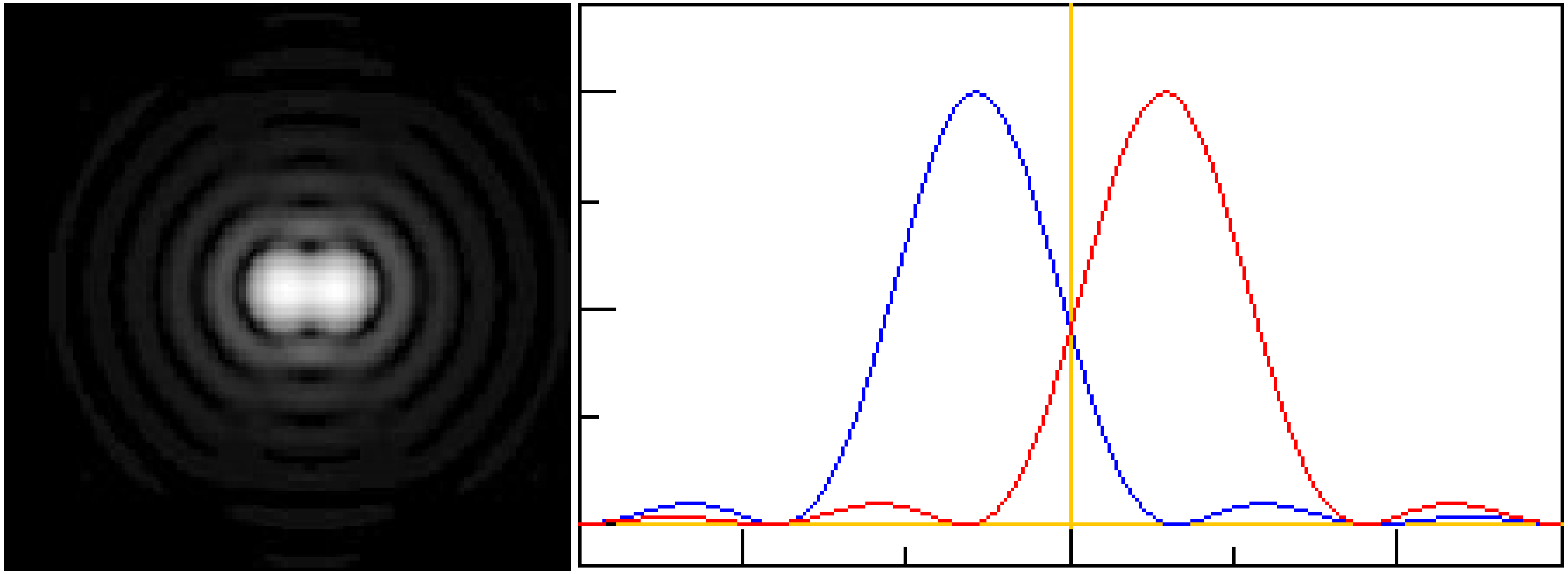
Rayleigh Criterion

Two sources are just resolved if **central maximum** of diffraction pattern of one source falls on **first minimum** of the other

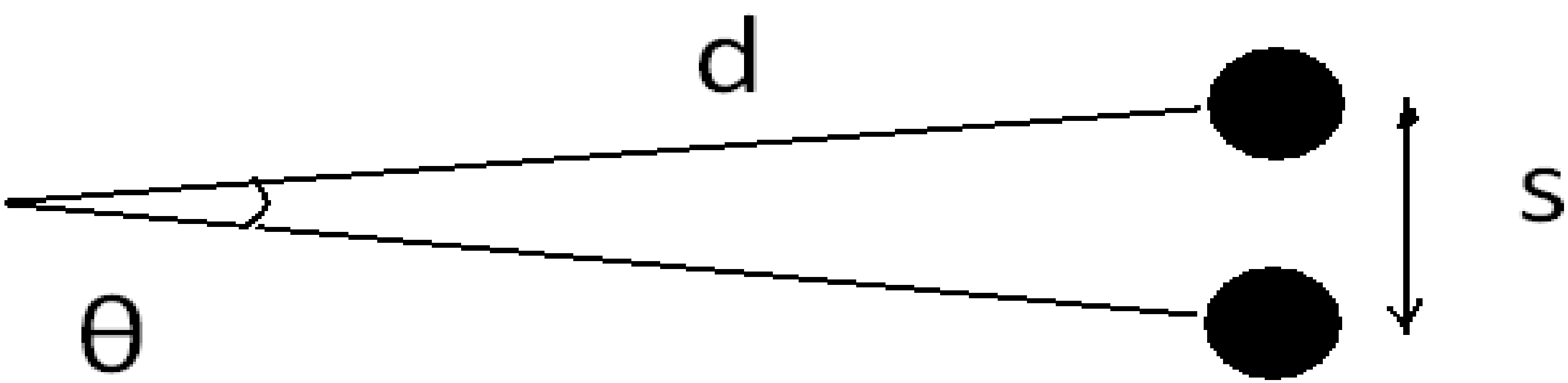


- For square aperture (slit) of side b: $\theta_R \approx \lambda/b$
- For circular aperture of diameter b: $\theta_R \approx 1.22\ \lambda/b$
- Note: θ_R is expressed in radians

Here's what the image will look like when Rayleigh criterion is met



Actual angular separation depends on actual distance between the objects and the distance from the observer:



θ_{actual} = s/d radians

The two sources will be resolved if the actual angle is greater than or equal to the Rayleigh angle: $\theta_{actual} \geq \theta_R$

Resolution Applied to Telescopes

What design parameters could help to be able to resolve far away binary stars?

- Since angular separation between the stars will be very small, the Rayleigh angle must be even smaller: $\theta_c = 1.22\ \lambda/b$
- To make the Rayleigh angle smaller, what can you do?

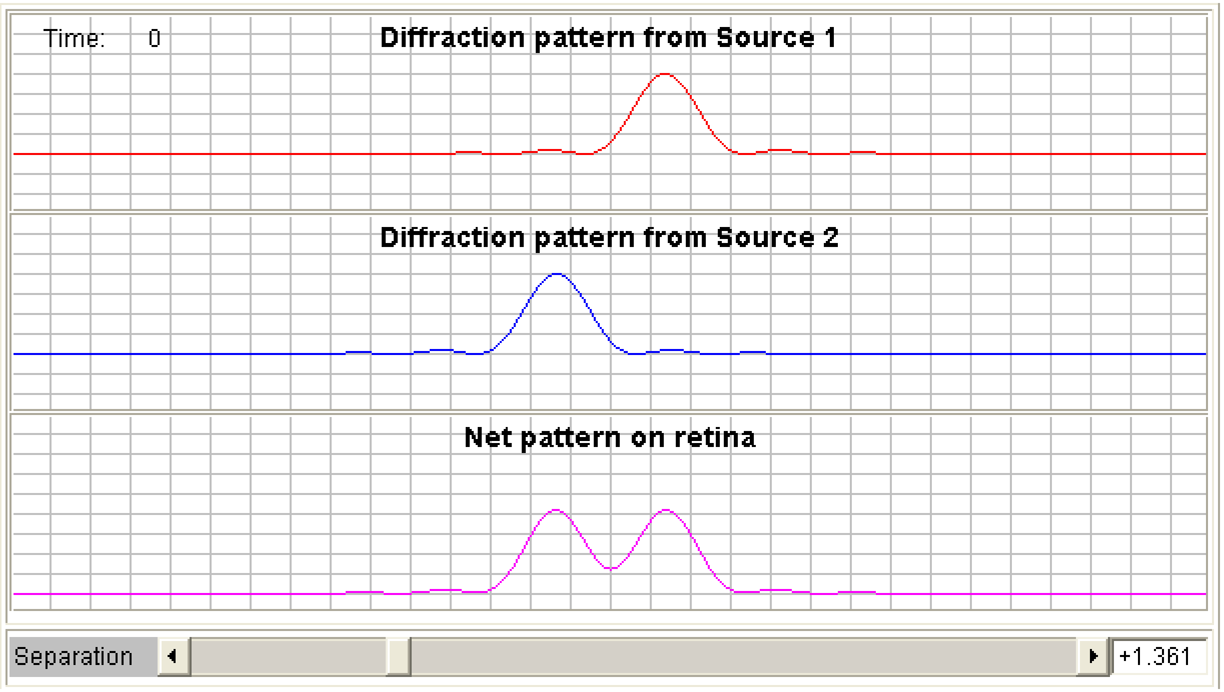
Answer: use small light wavelength, large diameter lens/mirror

Resolution applied to microscopy

- To resolve a small object of size *s*, the wavelength of light used must be of the same order of magnitude as the object size or smaller
- Electron microscopes operate based on the wave nature of electrons. Their wavelength (deBroglie wavelength) is related to their momentum: $\lambda = h/p$
  - (h is Planck's constant = 6.63×10^{-34} J s)
  - Faster (more energetic) electrons have a smaller wave length

Resolution Applet

- http://webphysics.davidson.edu/physlet_resources/bu_semester2/c27_rayleigh.html



Example Problem #1

- A spy satellite orbits at a distance of 180 km above the earth's surface. The diameter of the satellite's camera lens is 45 cm. Find the smallest distance on the ground that can be resolved by the camera. (Use $\lambda = 500$ nm.)

Answer: 0.24 m; set the actual angular separation equal to the Rayleigh angle

Example Problem #2

- The dish of the Arecibo radio telescope has a diameter of 300 m. Two distant radio sources are 4.0×10^{12} m apart. The sources are 3.0×10^{16} m from Earth and they emit radio waves of wavelength 15 cm.
- Can these sources be resolved by this telescope?

Answer to Example #2

- $\theta_R = 1.22\lambda/b = 1.22(0.15\ m)/300\ m$

  $= 6.1 \times 10^{-4}$ radians
- $\theta_{actual} = s/d = (4.0 \times 10^{12}\ m)/(3.0 \times 10^{16}\ m)$

  $= 1.33 \times 10^{-4}$ radians

- The telescope can't resolve the sources because the actual angular separation is too small ($\theta_{actual} < \theta_R$)