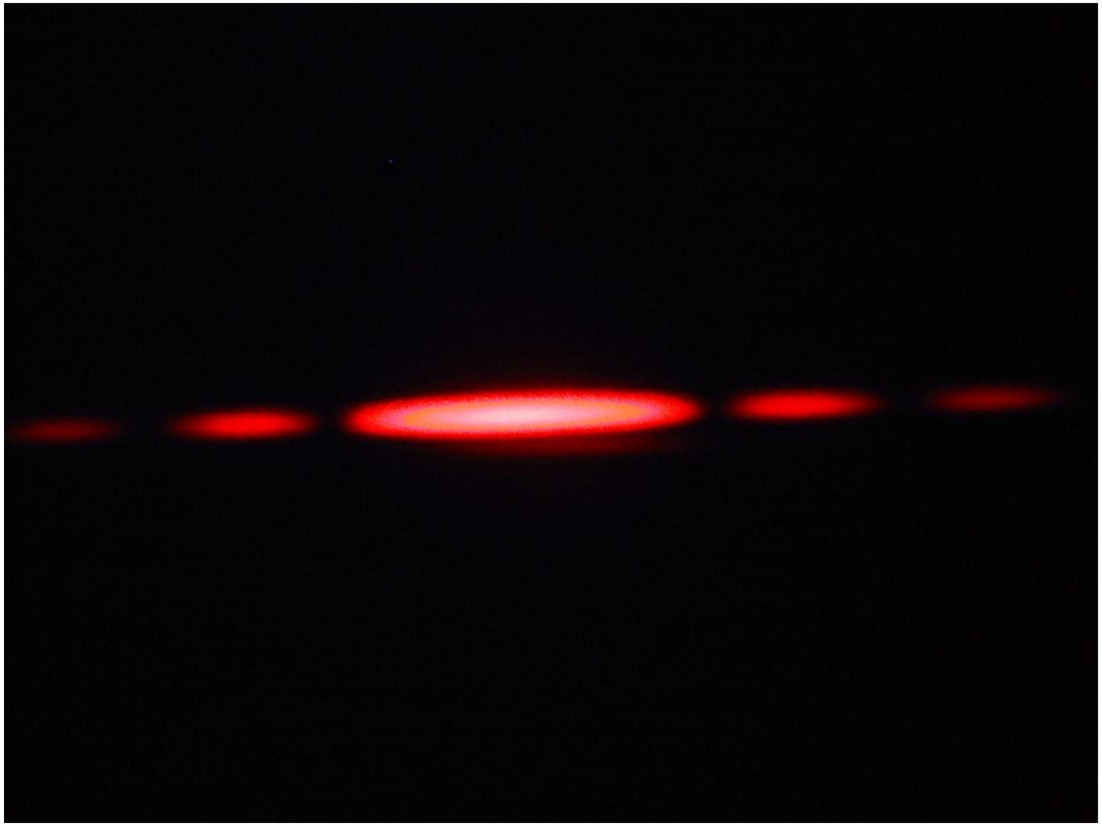
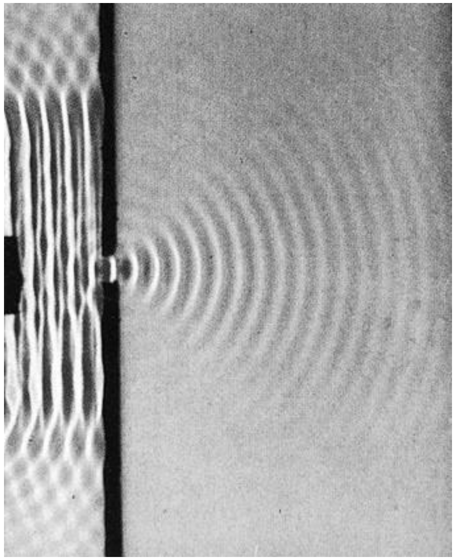
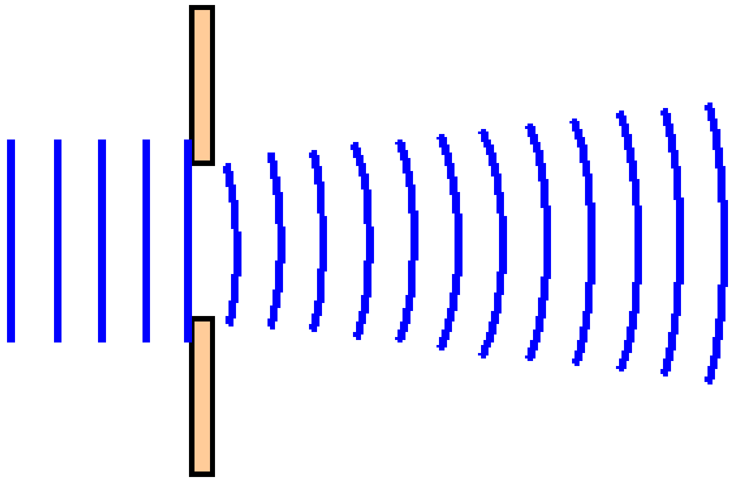
Single Slit Diffraction



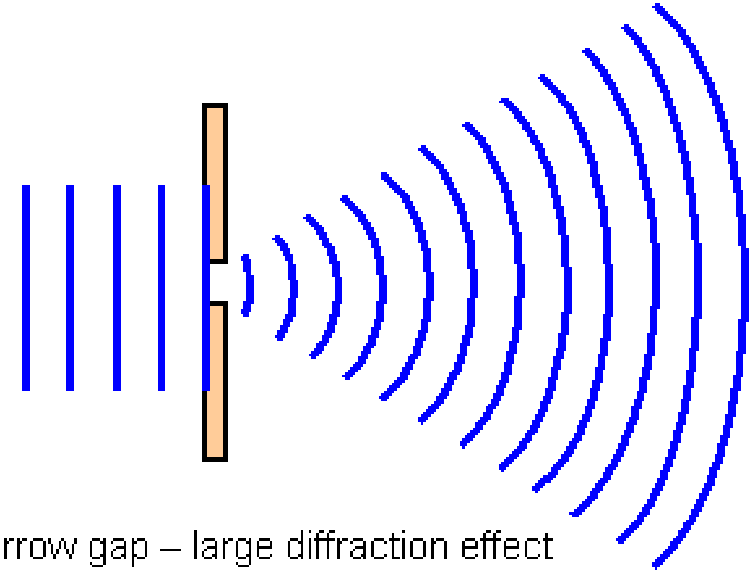
Reminder: What is Diffraction?

- Bending and spreading of a wave into a region behind an obstruction
- Examples: waves passing through openings or around corners
- Effects depend on how wide the opening is relative to wave length
  - Wide opening: little wave spreading
  - Narrow opening: wave fans out, changes shape
    - (Wide: opening > wave length;
    - Narrow: opening ~ wavelength

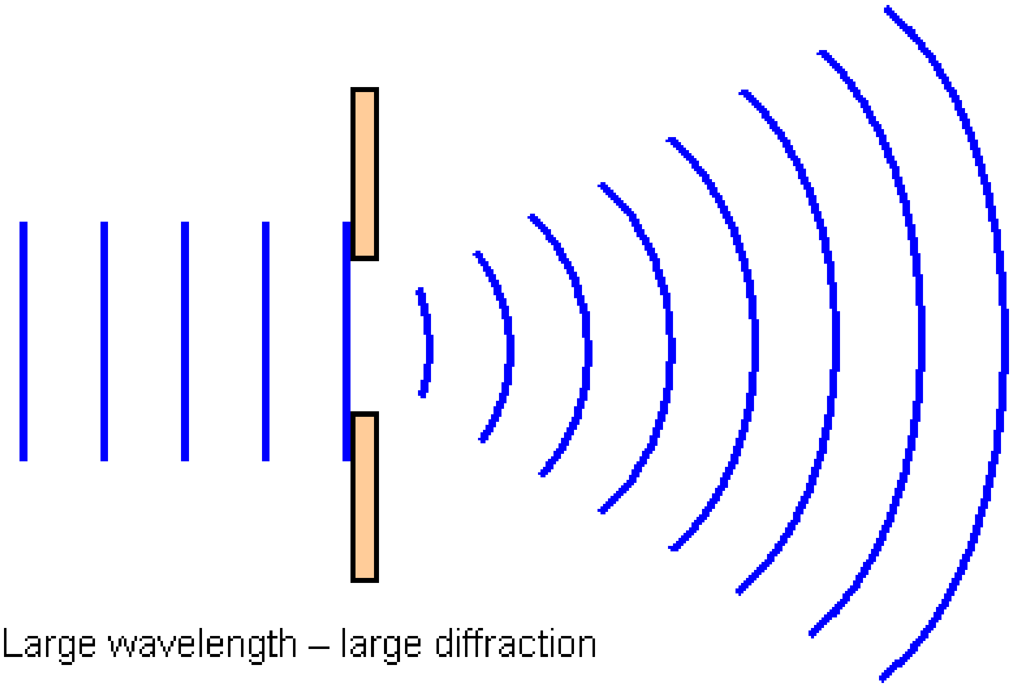




Wide gap – small diffraction effect



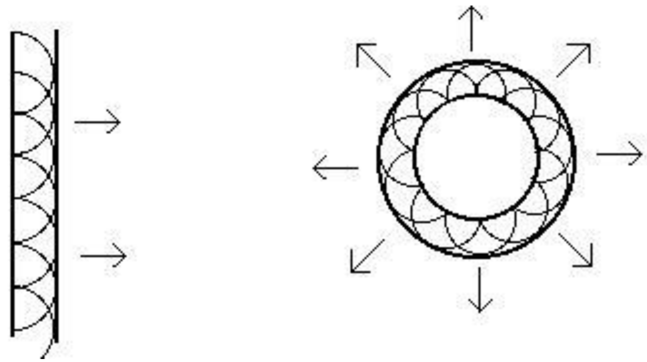
Narrow gap – large diffraction effect



Large wavelength – large diffraction

Diffraction: Why does it occur?

- According to **Huygens' principle**, each point on a wavefront serves as a source of the next wavefront



- After passing through an aperture, there will be locations where the wavelets interfere constructively and destructively
- http://www.acoustics.salford.ac.uk/feschools/waves/flash/huygens.swf
- With light, this will result in bright and dark **fringes**

Interference Reminders

- **Constructive interference** (bright fringes):
  - difference in path length = $n\lambda$
  - phase difference = $2n\pi$ radians

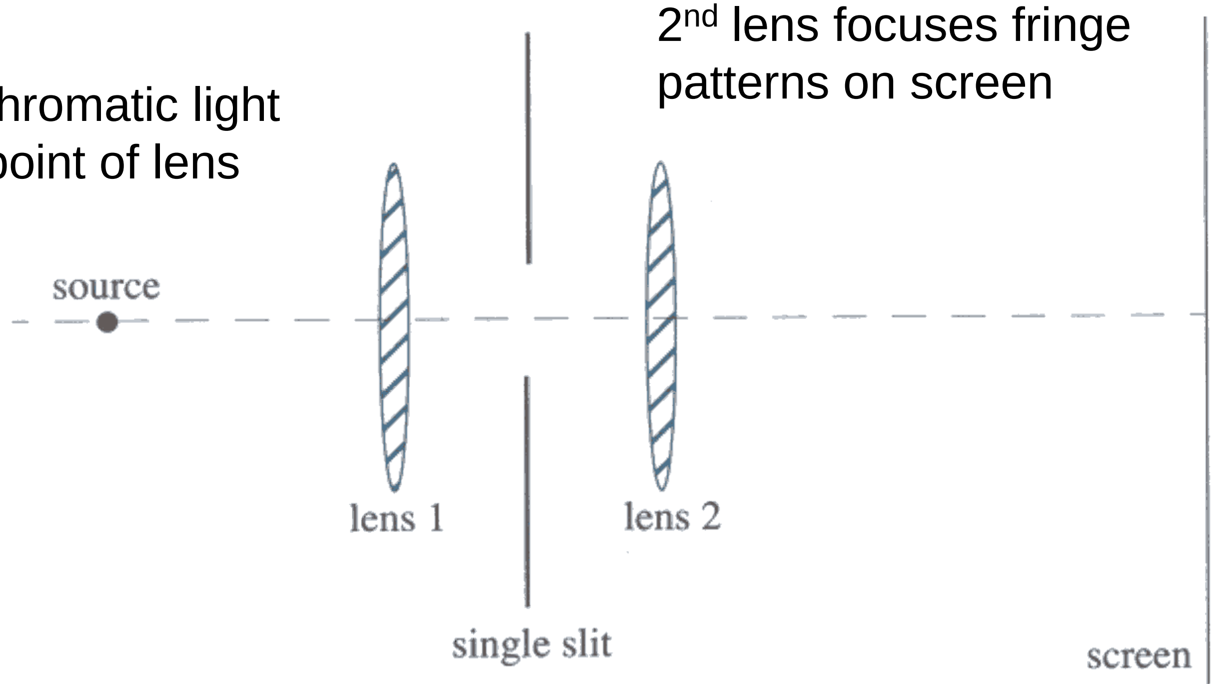
- **Destructive interference** (dark fringes):
  - difference in path length = $(n + \frac{1}{2})\lambda$
  - phase difference = $(n + \frac{1}{2})\pi$ radians

How single slit pattern is achieved

1. Use monochromatic, uncollimated, incoherent light:
   - Lens 1 produces parallel wave fronts passing through slit (collimated)
   - Lens 2 focuses pattern on screen
2. Use a **laser** as the source: produces collimated coherent light

Source of monochromatic light located at focal point of lens

2nd lens focuses fringe patterns on screen

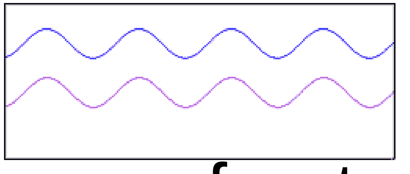


Definitions

- Monochromatic light: light waves all have same wavelength (or frequency)
- Collimated light: all waves are parallel to each other

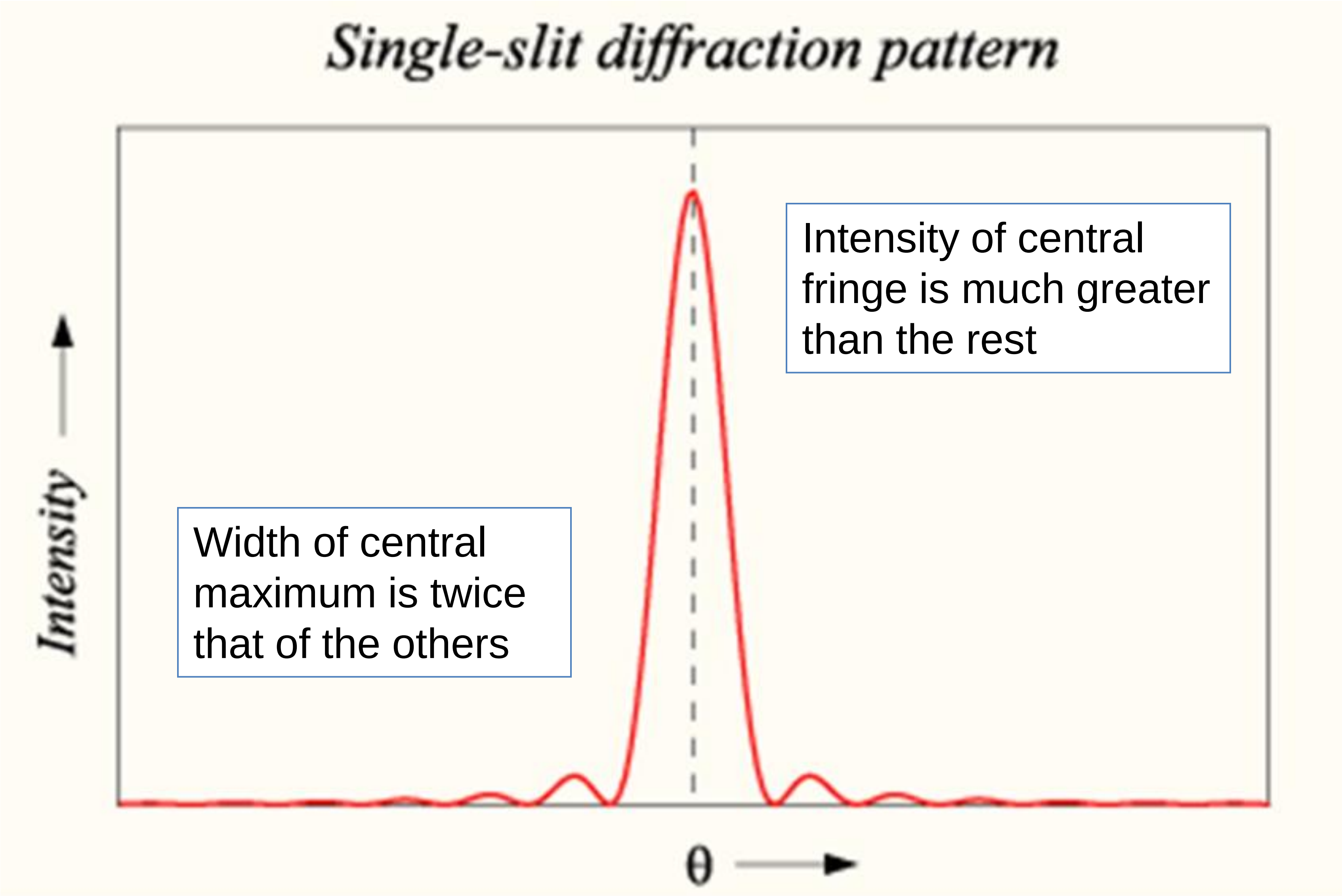
http://en.wikipedia.org/wiki/Collimated_light

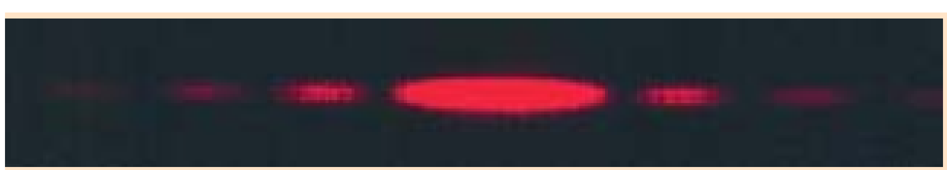
- Coherent light: constant phase difference between sources of individual waves
  - Laser light is coherent



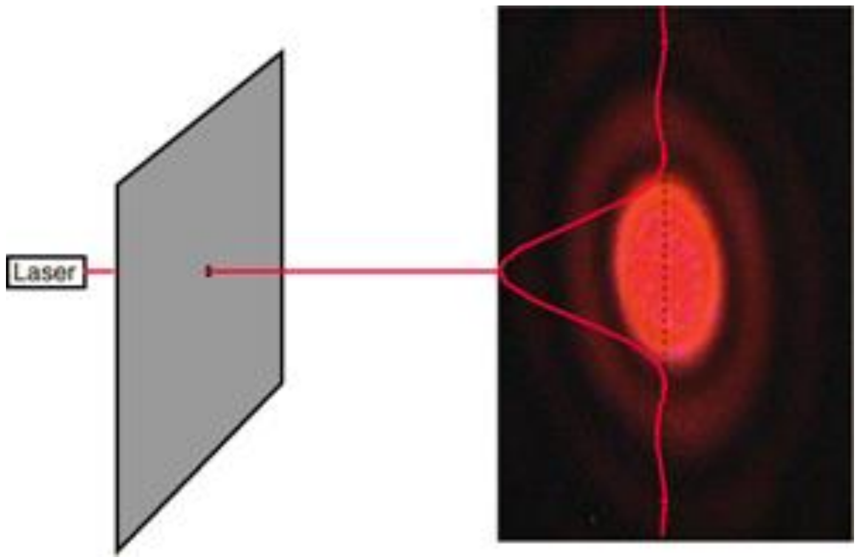
  - All wavelets on a given wavefront are, by definition, coherent

http://schools.matter.org.uk/content/Interference/coherent.html


Single-slit diffraction pattern
Intensity
θ
Intensity of central fringe is much greater than the rest
Width of central maximum is twice that of the others



Circular Aperture Diffraction Pattern



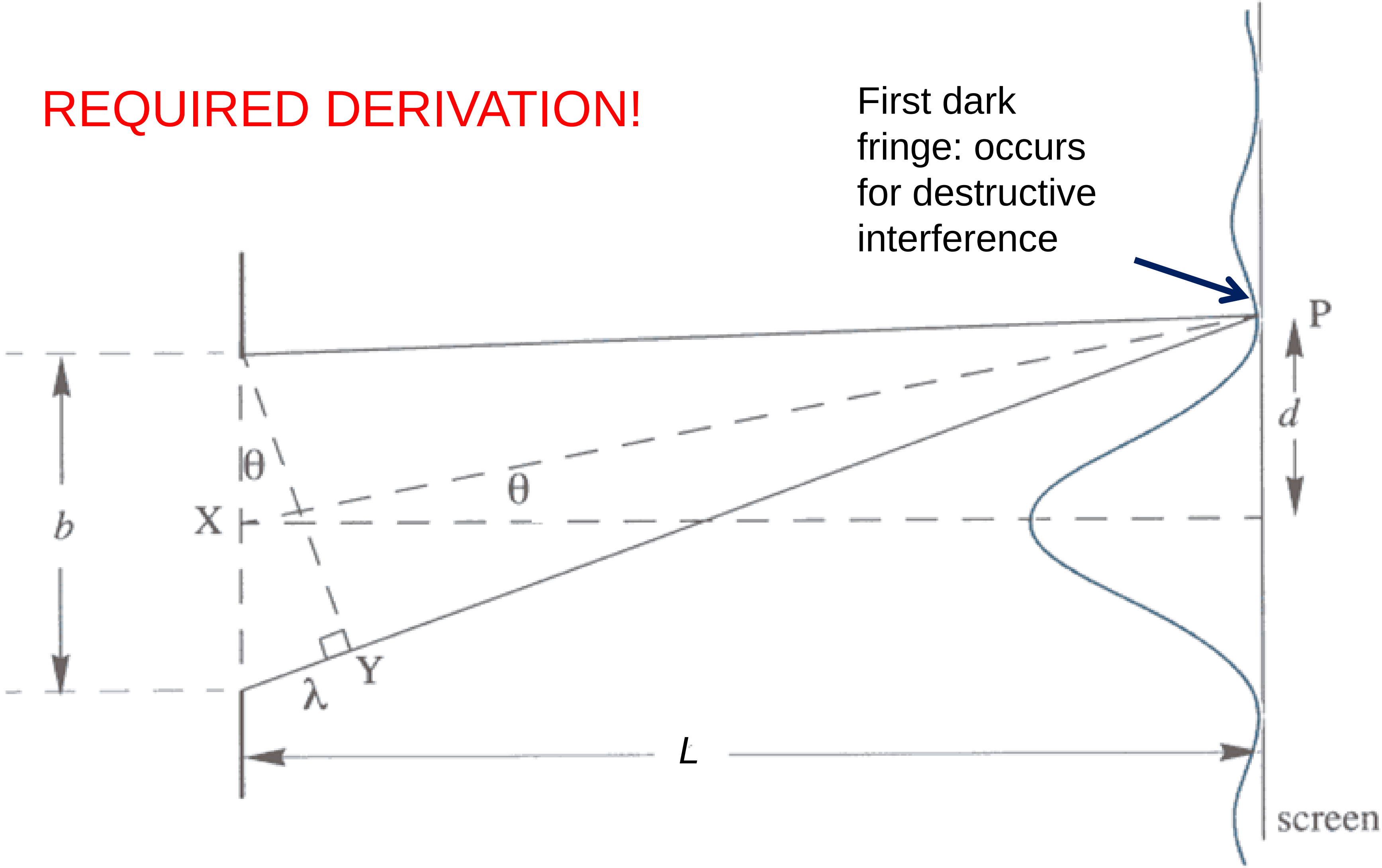
- Central maximum is much brighter and wider than the rest
- This pattern is called an Airy disk

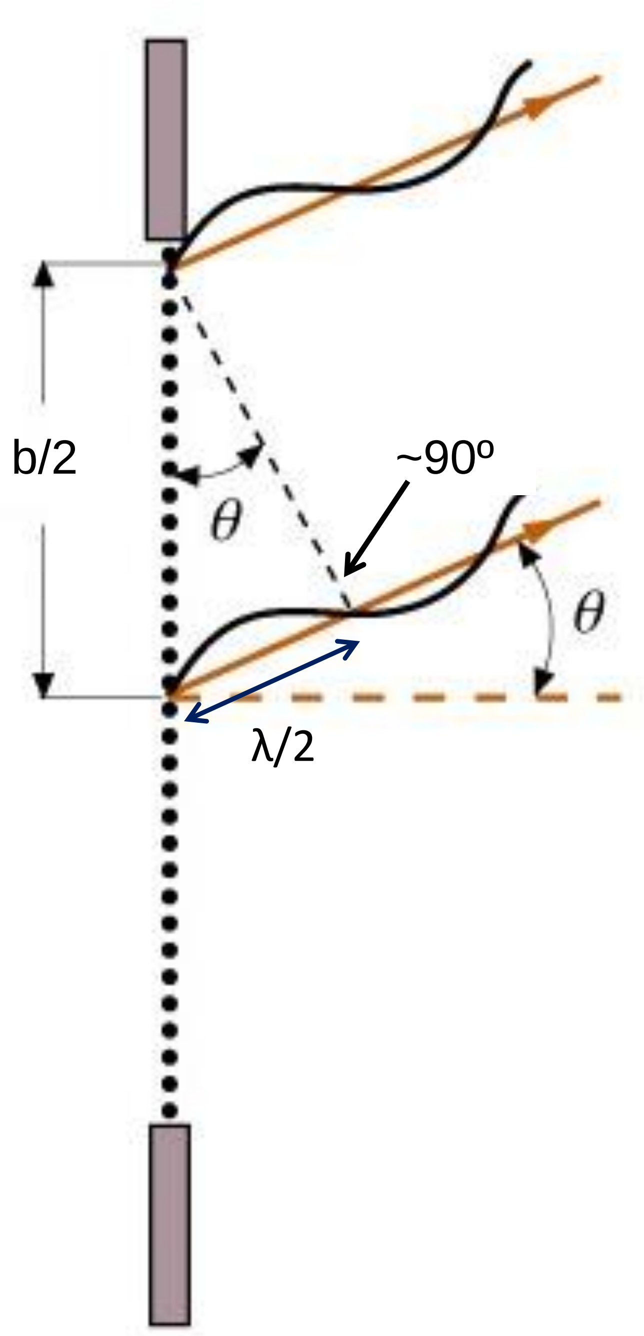
http://www.youtube.com/watch?v=Wy3oR6fY6W8

Derivation of Single Slit Diffraction Equation: the Setup

REQUIRED DERIVATION!

First dark fringe: occurs for destructive interference





• Consider wavelets arising from the top of the slit and from the center of the slit

• If the difference in path length between the two is λ/2, there will be **destructive interference**, resulting in a **dark fringe**

• From the geometry

$$\sin\theta = \frac{\lambda/2}{b/2}$$

• We also consider all of the other symmetrically placed wavelets along entire slit

Derivation (cont.)

- Assuming θ is small: $\sin\theta \approx \theta$

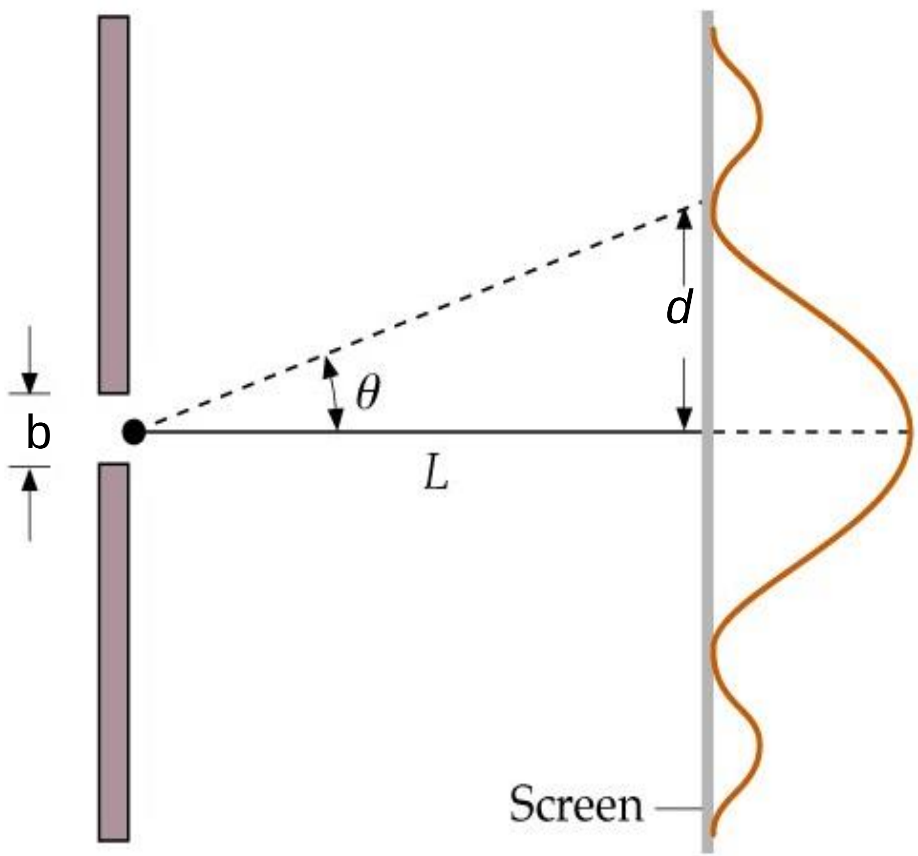
- Final result:

$$\theta = \lambda/b$$

Remember: this angle has units of radians!

- This the angular distance from the central maximum to first dark fringe
- It is also **half** of central maximum angle:
- To find the total angular displacement of the central maximum, **multiply by 2**

Distance on screen from middle of central maximum to first dark fringe:



$$\tan\theta = d/L$$

d = <u>half-width</u> of the central maximum projected on the screen

L = distance from slit to screen

If θ is small (as it will be if L >> d), then

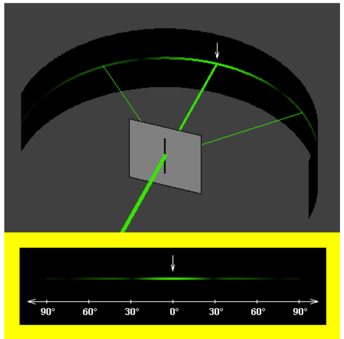
$$\theta \approx d/L$$

$$\text{Then } \lambda b \approx d/L$$

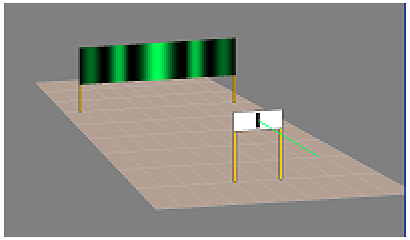
Single Slit Diffraction Applets

What happens to fringe width
when you change wavelength,
slit width, and distance to screen?

- http://www.walter-fendt.de/ph14e/singleslit.htm



- http://surendranath.tripod.com/Applets/Optics/Slits/SingleSlit/SS.html



- http://lectureonline.cl.msu.edu/~mmp/kap27/Gary-Diffraction/app.htm

Example Problem #1

- Light from a laser is used to form a single slit diffraction pattern. The width of the slit is 0.10 mm and the screen is placed 3.0 m from the slit. The width of the central maximum is measured as 2.6 cm. (Hint: this is 2*d*)

  What is the wavelength of the laser light?

Answer

- Since the screen is far from the slit we can use the small angle approximation such that

$$d/L = \lambda/b, \text{ so } \lambda = db/L$$

- *d*, the half width of the center maximum is 1.3 cm so we have

$$\lambda = (1.3 \times 10^{-2}\ \text{m}) \times (0.10 \times 10^{-3}\ \text{m}) / 3.0\ \text{m}$$

$$\lambda = 430 \times 10^{-9}\ \text{m or } 430\ \text{nm}$$

Example Problem #2

- Light of λ = 500 nm is diffracted by a single slit 0.05 mm wide. Find the angular position of the 1^{st} minimum. If a screen is placed 2 meters from the slit, find the half-width of the central bright fringe.

- Answer: 0.01 rad; 2 cm

Example Problem #3

- When light of λ = 440 nm is diffracted through a single slit, the angular position of the 1^{st} minimum is at 0.02 rad. Find the slit width.

- Answer: 0.022 mm