Waves

Topic 11.1

Standing Waves

Standing Waves

- The Formation

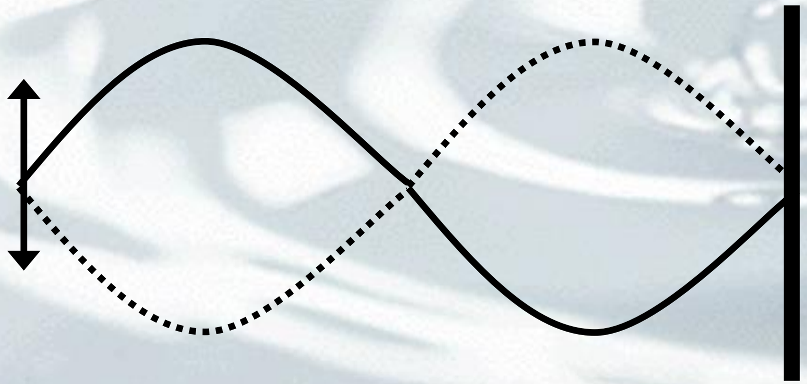
- When 2 waves of
  - the same speed
  - and wavelength
  - and equal or almost equal amplitudes
  - travelling in opposite directions
- meet, a *standing wave* is formed

- The standing wave is the result of the superposition of the two waves travelling in opposite directions

- The main difference between a standing and a travelling wave is that in the case of the standing wave **no energy or momentum is transferred** down the string

- A standing wave is characterised by the fact that there exists a number of points at which the disturbance is always zero
- These points are called Nodes
- And the amplitudes along the waveform is different
- In a travelling wave there are no points where the disturbance is always zero, all the points have the same amplitude.

At the correct frequency a standing wave is formed



The frequency is increased until a different standing wave is formed

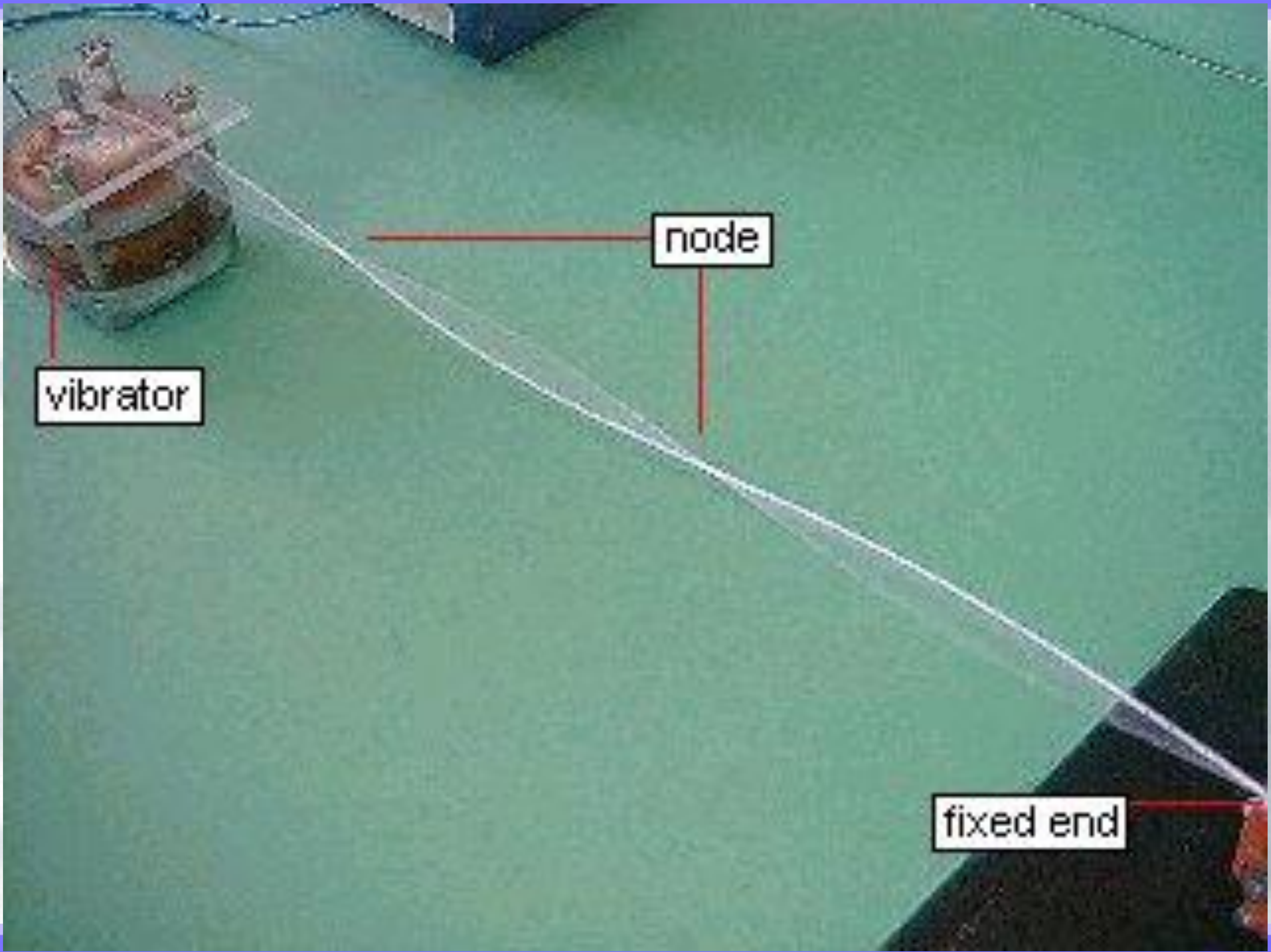
Resonance

- If the frequency of the source of a vibration is exactly equal to the *natural frequency* of the oscillatory system then the system will *resonate*
- When this occurs the *amplitude* will get larger and larger
- Pushing a swing is an example of resonance
- Resonance can be useful and harmful

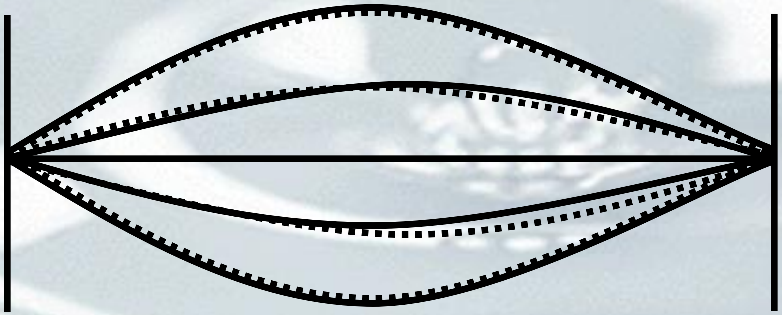
- Airplane wings, engines, bridges, tall buildings are objects that need to be protected against resonance from external vibrations due to wind and other vibrating objects
- Soldiers break step when marching over a bridge in case the force which they exert on the bridge starts uncontrollable oscillations of the bridge

Resonance and Standing Waves

1. Standing Waves on Strings


node
vibrator
fixed end

- If you take a wire and stretch it between two points then you can set up a standing wave
- The travelling waves are reflected to and fro between the two ends of the wire and interfere to produce the standing wave



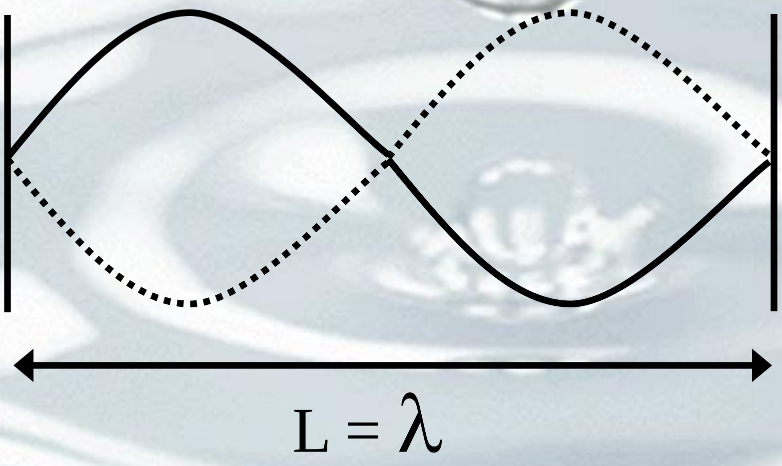
•This has a *node* at both ends

• and an *antinode* in the middle

•– it is called the *fundamental*

- With this wave the length of the string is equal to half the wave length
  - $L = \frac{1}{2} \lambda$
  - $\therefore \lambda = 2L$
  - As $v = f \lambda$
  - Then $f = v / \lambda$
  - $\therefore f = v / 2L$
- This is the *fundamental frequency* of the string (the 1st harmonic)

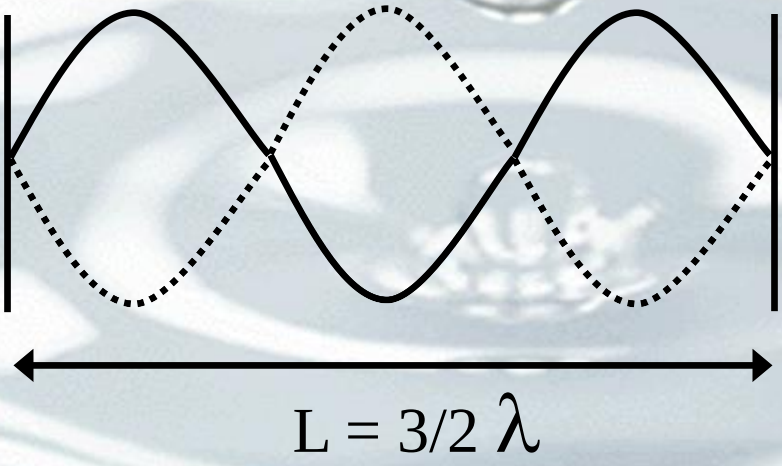
- This is not the only standing wave that can exist on the string
- The next standing wave is



This is called the 2nd *Harmonic*

- With this wave the length of the string is equal to the wave length
  - $L = \lambda$
  - $\therefore \lambda = L$
  - As $v = f \lambda$
  - Then $f = v / \lambda$
  - $\therefore f = v / L$
- This is the *2nd Harmonic frequency* of the string
- Notice it is twice the *fundamental frequency*

- The next standing wave is



This is called the *3rd Harmonic*

- With this wave the length of the string is equal to 3/2 of the wave length
    - $L = 3/2\ \lambda$
    - $\therefore\ \lambda = 2/3L$
    - As $v = f\ \lambda$
    - Then $f = v / \lambda$
    - $\therefore\ f = v / 2/3L$
    - $\therefore\ f = 3v / 2L$
- This is the 3rd *Harmonic frequency* of the string
- Notice it is three times the *fundamental frequency*

- Notice that the only constraint is that the ends of the string are *nodes.*
- In general we find that the wavelengths satisfy

$$\lambda = \frac{2L}{n}$$

Where n = 1,2,3,4……

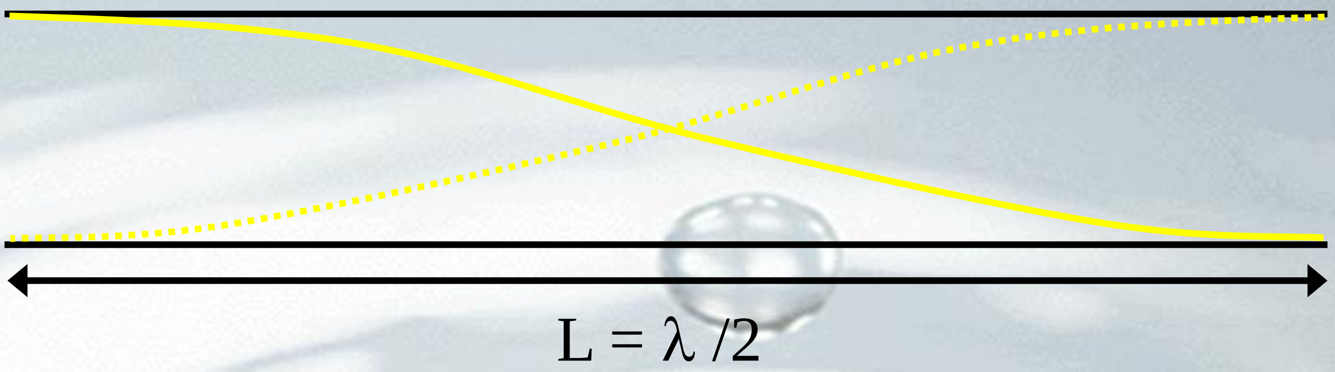
- This is the *harmonic series*
- The *fundamental* is the dominant vibration and will be the one that the ear will hear above all the others
- The *harmonics* effect the quality of the note
- It is for this reason that different musical instruments sounding a note of the same frequency sound different
- (it is not the only way though)

2. Standing Waves in Pipes

- Sound standing waves are also formed in pipes
- Exactly the same results apply
- There are two types of pipes
  - 1. Open ended
  - 2. Closed at one end
- Nodes exist at closed ends
- Antinodes exists at open ends

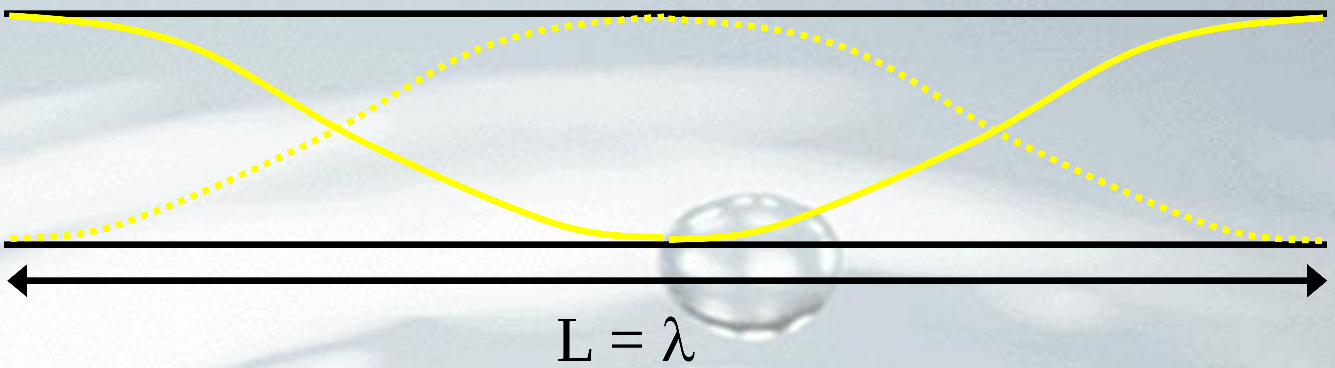
a) Open Ended

- Fundamental Frequency (1st Harmonic)



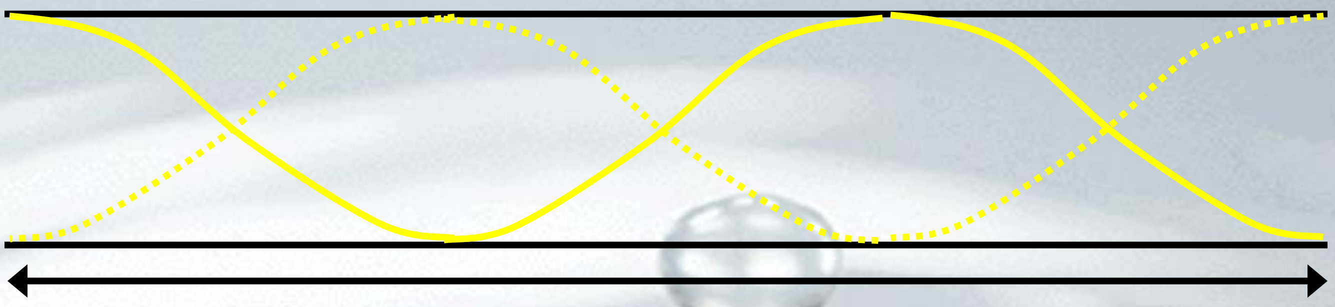
- $\therefore\ \lambda = 2L$
- As $v = f\lambda$
- Then $f = v / \lambda$
- $\therefore\ f = v / 2L$

- 2nd Harmonic



- $\therefore\ \lambda = L$
- As $v = f\lambda$
- Then $f = v / \lambda$
- $\therefore\ f = v / L$

- 3^{rd} Harmonic

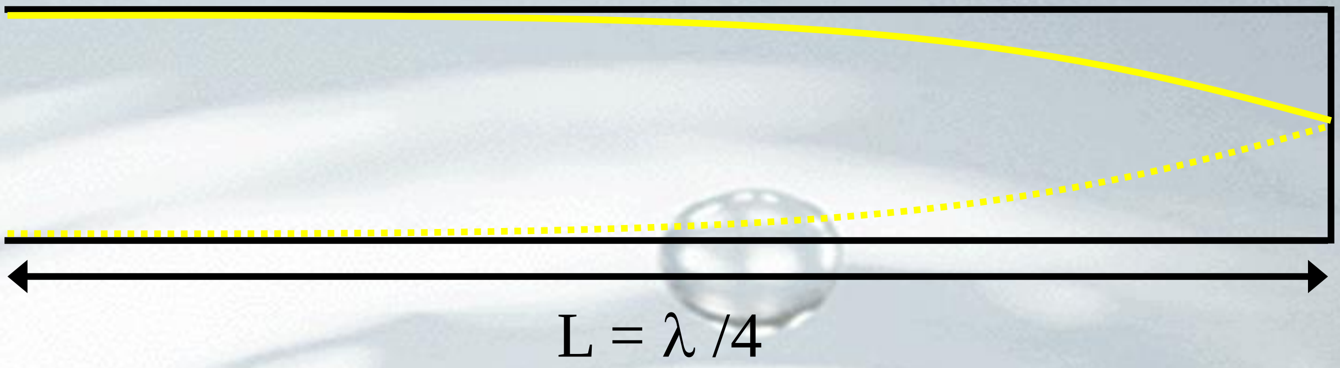


- ∴ $\lambda = 2/3L$
- As $v = f\lambda$
- Then $f = v / \lambda$
- ∴ $f = v / 2/3L$
- ∴ $f = 3v / 2L$

- The harmonics are in the same series as the string series
- If the fundamental frequency = f
- Then the 2nd harmonic is 2f, 3rd is 3f and the 4th is 4f… etc

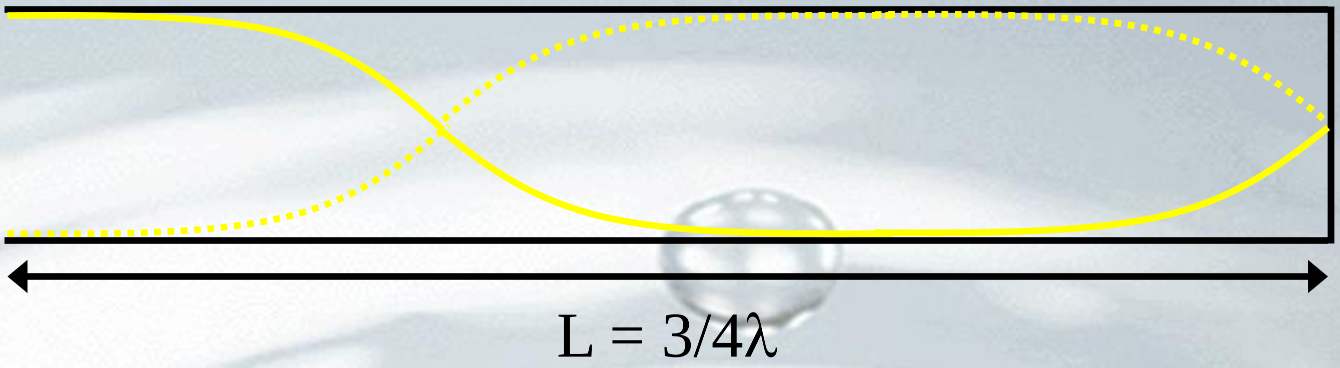
b) Closed at one End

- Fundamental Frequency (1st Harmonic)



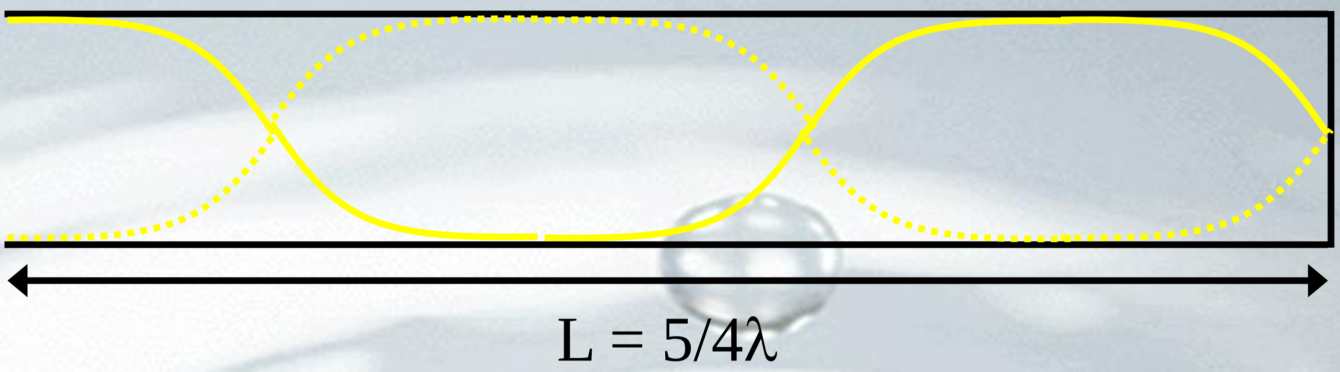
- $\therefore\ \lambda = 4L$
- As $v = f\,\lambda$
- Then $f = v / \lambda$
- $\therefore\ f = v / 4L$

- Next Harmonic



- ∴ $\lambda = 4/3L$
- As $v = f\lambda$
- Then $f = v / \lambda$
- ∴ $f = v / 4/3L$
- ∴ $f = 3v / 4L$

- And the next harmonic



- $\therefore\ \lambda = 4/5L$
- As $v = f\,\lambda$
- Then $f = v / \lambda$
- $\therefore\ f = v / 4/5L$
- $\therefore\ f = 5v / 4L$

- The harmonics are DIFFERENT to the string and open pipe series
- If the fundamental frequency = f
- Then there is no 2nd harmonic
- The 3rd is 3f
- There is no 4th harmonic
- The 5th is 5f