The realm of physics

Physics is an experimental science in which measurements made must be expressed in units. In the International System of units used throughout this book, the SI system, there are seven fundamental units, which are defined in this chapter. All quantities are expressed in terms of these units directly or as a combination of them.

Objectives

By the end of this chapter you should be able to:

- appreciate the order of magnitude of various quantities;
- perform simple order-of-magnitude calculations *mentally*;
- state the *fundamental units* of the SI system.

Orders of magnitude and units

How many molecules are there in the sun? This may sound like a very difficult question with which to start a physics textbook, but very basic physics can give us the answer. Before we try to work out the answer, guess what you think the answer is by giving a power of 10. The number of molecules in the sun is 10 to the power . . . ?

To answer the question we must first have an idea of the mass of the sun. You may know this, or you can easily look it up (to save you doing this for this example, we can tell you that it is about 10^{30} kg). Next, you will need to know what the chemical composition of the sun is. It is made up of 75% hydrogen and 25% helium, but as we are only making a rough estimate, we may assume that it is made out of hydrogen entirely. The molar mass of hydrogen is 2 g mol^{-1} and so the sun contains $10^{33}/2$ mol $= 5 \times 10^{32}$ mol. The number of molecules in one mole of any substance is given by the Avogadro constant, which is about 6×10^{23}, so the sun has around $5 \times 10^{32} \times 6 \times 10^{23} = 3 \times 10^{56}$ molecules. How close was your guess?

The point of this exercise is that, first, we need units to express the magnitude of physical quantities. We must have a consistent set of units we all agree upon. One such set is the International System (SI system), which has seven basic or fundamental units. The units of all other physical quantities are *combinations* of these seven. These units are presented later in this section. The second point is that we have been able to answer a fairly complicated sounding question without too much detailed knowledge – a few simplifying assumptions and general knowledge have been enough. The third point you may already have experienced. How close was your guess for the number of molecules in the sun? By how much did your exponent differ from 56? Many of you will have guessed a number around 10^{1000} and that is way off. The number 10^{1000} is a huge number – you cannot find anything real to associate with such a number. The mass of the universe is about 10^{53} kg and so repeating the calculation above we find that the number of hydrogen molecules in the entire universe (assuming it is all hydrogen) is about 10^{79} – a big number to be sure but nowhere near 10^{1000}. Part of learning physics is to appreciate the magnitude of things – whether they are masses, times, distances, forces or just pure numbers such as the number of hydrogen molecules in the universe. Hopefully, you will be able to do that after finishing this course.

The SI system

The seven basic SI units are:

1. The *metre* (m). This is the unit of distance. It is the distance travelled by light in a vacuum in a time of 1/299 792 458 seconds.
2. The *kilogram* (kg). This is the unit of mass. It is the mass of a certain quantity of a platinum–iridium alloy kept at the Bureau International des Poids et Mesures in France.
3. The *second* (s). This is the unit of time. A second is the duration of 9 192 631 770 full oscillations of the electromagnetic radiation emitted in a transition between the two hyperfine energy levels in the ground state of a caesium-133 atom.
4. The *ampere* (A). This is the unit of electric current. It is defined as that current which, when flowing in two parallel conductors 1 m apart, produces a force of 2×10^{-7} N on a length of 1 m of the conductors.
5. The *kelvin* (K). This is the unit of temperature. It is $\frac{1}{273.16}$ of the thermodynamic temperature of the triple point of water.
6. The *mole* (mol). One mole of a substance contains as many molecules as there are atoms in 12 g of carbon-12. This special number of molecules is called Avogadro's number and is approximately 6.02×10^{23}.
7. The *candela* (cd). This is a unit of luminous intensity. It is the intensity of a source of frequency 5.40×10^{14} Hz emitting $\frac{1}{683}$ W per steradian.

The details of these definitions should not be memorized.

In this book we will use all of the basic units except the last one. Some of these definitions probably do not make sense right now – but eventually they will.

Physical quantities other than those above have units that are combinations of the seven fundamental units. They have *derived* units. For example, speed has units of distance over time, metres per second (i.e. m/s or, preferably, m s^{-1}). Acceleration has units of metres per second squared (i.e. m/s^2, which we write as m s^{-2}). In other words, we treat the symbols for units as algebraic quantities. Similarly, the unit of force is the newton (N). It equals the combination kg m s^{-2}. Energy, a very important quantity in physics, has the joule (J) as its unit. The joule is the combination N m and so equals (kg m s^{-2} m), or kg m^2 s^{-2}. The quantity power has units of energy per unit of time and so is measured in J s^{-1}. This combination is called a watt. Thus,

$$1\ \text{W} = (1\ \text{N m s}^{-1}) = (1\ \text{kg m s}^{-2}\ \text{m s}^{-1})$$
$$= 1\ \text{kg m}^2\ \text{s}^{-3}.$$

Occasionally, small or large quantities can be expressed in terms of units that are related to the basic ones by powers of 10. Thus, a nanometre (symbol nm) is 10^{-9} m, a microgram (μg) is 10^{-6} g = 10^{-9} kg, a gigaelectron volt (GeV) equals 10^9 eV, etc. The most common prefixes are given in Table 1.1.

Power	Prefix	Symbol	Power	Prefix	Symbol
10^{-18}	atto-	a	10^1	deka-	da*
10^{-15}	femto-	f	10^2	hecto-	h*
10^{-12}	pico-	p	10^3	kilo-	k
10^{-9}	nano-	n	10^6	mega-	M
10^{-6}	micro-	μ	10^9	giga-	G
10^{-3}	milli-	m	10^{12}	tera-	T
10^{-2}	centi-	c	10^{15}	peta-	P*
10^{-1}	deci-	d	10^{18}	exa-	E*

*Rarely used.

Table 1.1 Common prefixes.

When we write an equation in physics, we have to make sure that the units of the quantity on the left-hand side of the equation are the same as the units on the right-hand side. If the units do not match, the equation cannot be right. For example, the period T (a quantity with units of time) of a pendulum is related to the length of the pendulum l (a quantity with units of length) and the acceleration due to gravity g (units of acceleration) through

$$T = 2\pi\sqrt{\frac{l}{g}}$$

The units on the right-hand side must reduce to units of time. Indeed, the right-hand side units are

$$\sqrt{\frac{\text{m}}{\text{m s}^{-2}}} = \sqrt{\text{s}^2} = \text{s}$$

as required (note that 2π is a dimensionless constant). The fact that the units on both sides of an equation must match actually offers a powerful method for guessing equations.

For example, the velocity of a wave on a string is related to the length l and mass m of the string, and the tension force F the string is subjected to. How exactly does the velocity depend on these three variables? One guess is to write

$$v = cF^x l^y m^z$$

where c is a numerical constant (a pure number without units) and x, y and z are numbers to be determined. There could be some confusion here because m stands for mass but we also use the symbol m for the metre. To avoid this we will use the notation $[M]$ to stand for the unit of mass, $[L]$ for the unit of length, $[T]$ for the unit of time, etc. Then, looking at the units of the last equation we have that

$$\frac{[L]}{[T]} = ([M][L][T]^{-2})^x [L]^y [M]^z$$
$$[L][T]^{-1} = [M]^{x+z}[L]^{x+y}[T]^{-2x}$$

The two equations match if the exponents of $[L]$, $[M]$ and $[T]$ match – that is, if

$$x + z = 0$$
$$x + y = 1$$
$$-2x = -1$$

These equations imply that

$$x = \frac{1}{2}, \quad y = \frac{1}{2} \quad \text{and} \quad z = -\frac{1}{2}$$

In other words, the original formula becomes

$$v = cF^{1/2} l^{1/2} m^{-1/2} = c\sqrt{\frac{Fl}{m}}$$

Obviously this method cannot give the value of the dimensionless constant c. To do that we have to learn some physics!

Tables 1.2–1.4 give approximate values for some interesting sizes, masses and time intervals.

Expressing a quantity as a plain power of 10 gives what is called the 'order of magnitude' of that quantity. Thus, the mass of the universe

	Length/m
Distance to edge of observable universe	10^{26}
Distance to the Andromeda galaxy	10^{22}
Diameter of the Milky Way galaxy	10^{21}
Distance to nearest star	10^{16}
Diameter of solar system	10^{13}
Distance to sun	10^{11}
Radius of the earth	10^{7}
Size of a cell	10^{-5}
Size of a hydrogen atom	10^{-10}
Size of a nucleus	10^{-15}
Size of a proton	10^{-17}
Planck length	10^{-35}

Table 1.2 Some interesting sizes.

	Mass/kg
The universe	10^{53}
The Milky Way galaxy	10^{41}
The sun	10^{30}
The earth	10^{24}
Boeing 747 (empty)	10^{5}
An apple	0.25
A raindrop	10^{-6}
A bacterium	10^{-15}
Smallest virus	10^{-21}
A hydrogen atom	10^{-27}
An electron	10^{-30}

Table 1.3 Some interesting masses.

	Time/s
Age of the universe	10^{17}
Age of the earth	10^{17}
Time of travel by light to nearby star	10^{8}
One year	10^{7}
One day	10^{5}
Period of a heartbeat	1
Period of red light	10^{-15}
Time of passage of light across a nucleus	10^{-24}
Planck time	10^{-43}

Table 1.4 Some interesting time intervals.

has an order of magnitude of 10^{53} kg and the mass of the Milky Way galaxy has an order of magnitude of 10^{41} kg. The ratio of the two masses is then simply 10^{12}.

Fundamental interactions

There are four basic or fundamental interactions in physics. However, in 1972, the electromagnetic and weak interactions were unified into one – the electroweak interaction. In this sense, then, we may speak of just three fundamental interactions (see Figure 1.1).

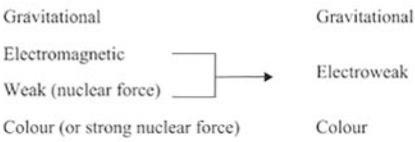

Figure 1.1 The fundamental interactions of physics. Since 1972, the electromagnetic and weak interactions have been shown to be part of a generalized interaction called the electroweak interaction.

Example questions

Let us close this chapter with a few problems similar to the one we started with. These problems are sometimes known as Fermi problems, after the great physicist Enrico Fermi, who was a master in this kind of estimation.

Q1

How many grains of sand are required to fill the earth? (This is a classic problem that goes back to Aristotle.)

Answer

The radius of the earth is about 6400 km, which we may approximate to 10 000 km. The volume of the earth is thus approximately $8 \times (10 \times 10^6)^3\ \mathrm{m}^3 \approx 8 \times 10^{21}\ \mathrm{m}^3$. We are assuming a cubical earth of side equal to twice the radius. This is a simplifying assumption. The true volume is $\frac{4}{3}\pi R^3 = 1.1 \times 10^{21}\ \mathrm{m}^3$, which agrees with our estimate (we are only interested in the power of 10 not the number in front). The diameter of a grain of sand varies of course but we will take 1 mm as a fair estimate. Then the number of grains of sand required to fill the earth is

$$\frac{8 \times 10^{21}\ \mathrm{m}^3}{(1 \times 10^{-3})^3\ \mathrm{m}^3} = 8 \times 10^{30} \approx 10^{31}$$

Q2

Estimate the speed with which human hair grows.

Answer

I cut my hair every 2 months and the barber cuts a length of about 2 cm. The speed is thus

$$\frac{2 \times 10^{-2}}{2 \times 30 \times 24 \times 60 \times 60}\ \mathrm{m\,s^{-1}} \approx \frac{10^{-2}}{3 \times 2 \times 36 \times 10^4} \approx \frac{10^{-6}}{6 \times 40} = \frac{10^{-6}}{240} \approx 4 \times 10^{-9}\ \mathrm{m\,s^{-1}}$$

Q3

If all the people on earth were to hold hands in a straight line, how long would the line be? How many times would it wrap around the earth?

Answer

Assume that each person has his or her hands stretched out to a distance of 1.5 m and that the population of earth is 6×10^9 people. Then the length would be $6 \times 10^9 \times 1.5\ \mathrm{m} = 9 \times 10^9\ \mathrm{m}$. The circumference of the earth is $2\pi R \approx 6 \times 6 \times 10^6\ \mathrm{m} \approx 4 \times 10^7\ \mathrm{m}$ and so the line would wrap $\frac{9 \times 10^9}{4 \times 10^7} \approx 200$ times around the equator.

Q4

How many revolutions do the wheels of a car make before it is junked?

Answer

We assume that the car runs 250 000 km before it is junked and that the wheels have a radius of 30 cm. Then the number of revolutions is

$$\frac{2.5 \times 10^8}{2\pi \times 0.3} \approx \frac{2.5}{2 \times 1} 10^8 \approx 10^8$$

Q5

What depth of car tyre wears off with each turn? (This is another classic problem.)

Answer

We assume that a depth of 5 mm wears off every 60 000 km. (These numbers are 'standard' for people who own cars.) Then, for a wheel of radius 30 cm the number of revolutions is (see previous problem) $\frac{6 \times 10^7}{2\pi \times 0.3} \approx \frac{6}{2 \times 1} 10^7 \approx 3 \times 10^7$ and so the wear per revolution is $\frac{5}{3 \times 10^7}$ mm/rev $\approx$ 10^{-7} mm/rev.

Questions

Have a look through these questions and answer any that you can. However, don't worry about any you can't answer; leave them for now and come back to them when you reach the end of the course.

1 How long does light take to travel across a proton?

2 How many hydrogen atoms does it take to make up the mass of the earth?

3 What is the age of the universe expressed in units of the Planck time?

4 What is the radius of the earth (6380 km) expressed in units of the Planck length?

5 How many heartbeats are there in the lifetime of a person (75 years)?

6 What is the mass of our galaxy in terms of a solar mass?

7 What is the diameter of our galaxy in terms of the astronomical unit, i.e. the distance between the earth and the sun?

8 The molar mass of water is 18 g mol^{-1}. How many molecules of water are there in a glass of water (of volume 0.3 L)?

9 Assuming that the mass of a person is made up entirely of water, how many molecules are there in a human body (of mass 60 kg)?

10 Assuming the entire universe to be made up of hydrogen gas, how many molecules of hydrogen are there?

11 Give an order-of-magnitude estimate of the density of a proton.

12 How long does light from the sun take to arrive on earth?

13 How many apples do you need to make up the mass of an average elephant?

14 How many bricks are used to build an average two-storey family house?

15 (a) How many metres are there in 5.356 nm?
(b) How many in 1.2 fm?
(c) How many in 3.4 mm?

16 (a) How many joules of energy are there in 4.834 MJ?
(b) How many in 2.23 pJ?
(c) How many in 364 GJ?

17 (a) How many seconds are there in 4.76 ns?
(b) How many in 24.0 ms?
(c) How many in 8.5 as?

18 What is the velocity of an electron that covers a distance of 15.68 mm in 87.50 ns?

19 An electron volt (eV) is a unit of energy equal to 1.6×10^{-19} J. An electron has a kinetic energy of 2.5 eV.
(a) How many joules is that?
(b) What is the energy in eV of an electron that has an energy of 8.6×10^{-18} J?

20 What is the volume in cubic metres of a cube of side 2.8 cm?

21 What is the side in metres of a cube that has a volume of 588 cubic millimetres?

22 One inch is 2.54 cm and one foot has 12 inches. The acceleration due to gravity is about 9.8 m s^{-2}. What is it in feet per square second?

23 One fluid ounce is a volume of about 2.96×10^{-5} m^3. What is the side, in inches, of a cube whose volume is 125 fluid ounces? (One inch is 2.54 cm.)

24 A horsepower (hp) is a unit of power equal to about 746 W. What is the power in hp of a 224 kW car engine?

25 Give an order-of-magnitude estimate for the mass of:
(a) an apple;
(b) this physics book;
(c) a soccer ball.

26 Give an order-of-magnitude estimate for the time taken by light to travel across the diameter of the Milky Way galaxy.

27 A white dwarf star has a mass about that of the sun and a radius about that of the earth. Give an order-of-magnitude estimate of the density of a white dwarf.

28 A sports car accelerates from rest to 100 km per hour in 4.0 s. What fraction of the acceleration due to gravity is the car's acceleration?

29 Give an order-of-magnitude estimate for the number of electrons in your body.

30 Give an order-of-magnitude estimate for the gravitational force of attraction between two people 1 m apart.

31 Give an order-of-magnitude estimate for the ratio of the electric force between two electrons 1 m apart to the gravitational force between the electrons.

32 The frequency f of oscillation (a quantity with units of inverse seconds) of a mass m attached to a spring of spring constant k (a quantity with units of force per length) is related to m and k. By writing $f = cm^x k^y$ and matching units on both sides show that $f = c\sqrt{\frac{k}{m}}$, where c is a dimensionless constant.

33 Without using a calculator *estimate* the value of the following expressions and then compare with the exact value using a calculator:

(a) $\frac{243}{43}$;

(b) 2.80×1.90;

(c) $\frac{312 \times 480}{160}$;

(d) $\frac{8.99 \times 10^{9} \times 7 \times 10^{-6} \times 7 \times 10^{-6}}{(8 \times 10^{2})^{2}}$;

(e) $\frac{6.6 \times 10^{-11} \times 6 \times 10^{24}}{(6.4 \times 10^{6})^{2}}$.