Kinematic concepts

Motion is a fundamental part of physics and this chapter introduces the basic quantities used in the description of motion. Even the simplest of motions, such as a leaf falling from a tree, can be a fairly complicated thing to analyse. To learn how to do that requires that we sharpen our definitions of everyday concepts such as speed, distance and time. As we will see, once we master motion in a straight line, more complicated types of motion such as circular and parabolic motion will follow easily.

Objectives

By the end of this chapter you should be able to:

- describe the difference between *distance* and *displacement*;
- state the definitions of *velocity, average velocity, speed* and *average speed*;
- solve problems of *motion in a straight line with constant velocity*, $x = x_0 + vt$;
- appreciate that different observers belonging to different *frames of reference* can give differing but equally valid descriptions of motion;
- use graphs in describing motion;
- understand that the slope of a displacement–time graph is the velocity and that the area under a velocity–time graph is the change in displacement.

Displacement and velocity

Consider the motion of a point particle that is constrained to move in a straight line, such as the one in Figure 1.1. Our first task is to choose a point on this line from which to measure distances. This point can be chosen arbitrarily and we denote it by O.

When we say that the distance of a point P from O is 3 m, we mean that the point in question could be 3 m to the left or right of O. To distinguish the two points we introduce the

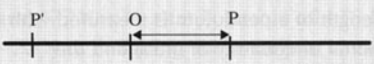

Figure 1.1 To measure distance we need an origin to measure distances from.

concept of *displacement*. The displacement of a point from O will be a quantity whose numerical value will be the distance and its sign will tell us if the point is to the right or left of O. Thus a displacement of −4 m means the point is at a distance of 4 m to the left of O, whereas a displacement of 5 m means a distance of 5 m to the right of O. Displacement is a vector; for the case of motion in a straight line, the displacement vector is very simple. It can be determined just by giving its magnitude and its sign. We will use the convention that positive displacements correspond to the right of O, negative to the left. (This is entirely arbitrary and we may choose any side of the origin as the positive displacement; this takes care of cases where it is not obvious what 'right' means.) We will use the symbol x for displacement in a straight line (we reserve the symbol $\vec{r}$ for displacements in more than one dimension) and s for distance (from the Latin *spatium*). Displacement, being a vector, is represented graphically by an arrow that begins at O and ends at the point of interest. (See Figure 1.2.)

Figure 1.2 Displacement can be positive (point A) or negative (point B).

We will use the capital letter S to stand for the total distance travelled, and Δx for the change in displacement. The change in displacement is defined by

$$\Delta x = \text{final displacement} - \text{initial displacement}$$

If the motion consists of many parts, then the change in displacement is the sum of the displacements in each part of the motion. Thus, if an object starts at the origin, say, and changes its displacement first by 12 m, then by -4 m and then by 3 m, the change in displacement is $12 - 4 + 3 = 11$ m. The final displacement is thus $11 + 0 = 11$ m.

Example questions

Q1

A mass initially at O moves 10 m to the right and then 2 m to the left. What is the final displacement of the mass?

Answer

$\Delta x = +10 \text{ m} - 2 \text{ m} = 8 \text{ m}$. Hence the final displacement is $0 \text{ m} + 8 \text{ m} = 8 \text{ m}$.

Q2

A mass initially at O, first moves 5 m to the right and then 12 m to the left. What is the total distance covered by the mass and what is its change in displacement?

Answer

The total distance is $5 \text{ m} + 12 \text{ m} = 17 \text{ m}$. The change in displacement is $+5 \text{ m} - 12 \text{ m} = -7 \text{ m}$. The mass now finds itself at a distance of 7 m to the left of the starting point.

Q3

An object has a displacement of -5 m. It moves a distance to the right equal to 15 m and then a distance of 10 m to the left. What is the total distance travelled and final displacement of the object? What is the change in displacement of the object?

Answer

The distance travelled is $15 \text{ m} + 10 \text{ m} = 25 \text{ m}$. The object now finds itself at a distance of 0 m from O and thus its displacement is zero. The original displacement was $x = -5$ m and thus the change in displacement is $\Delta x = 0 \text{ m} - (-5 \text{ m}) = +5 \text{ m}$.

Speed

If an object covers a total distance S in a total time T, the average speed of the object is defined by

$$\bar{v} = \frac{S}{T}$$

Suppose that you drive your car for a given amount of time, say 50 minutes. The odometer of the car shows that in those 50 minutes a distance of 30 km was covered. The average speed for this motion is

$$\bar{v} = \frac{30\,\text{km}}{50\,\text{min}} = 0.60\,\frac{\text{km}}{\text{min}}$$
$$= 0.60\,\frac{1000\,\text{m}}{60\,\text{s}} = 10\,\text{m}\,\text{s}^{-1}$$

Using the concept of the average speed is only a crude way of describing motion. In the example above, the car could, at various times, have gone faster or slower than the average speed of 10 m s^{-1}. Cars are equipped with an instrument called a speedometer, which shows the speed of the car at a particular instant in time. We call the speedometer reading the instantaneous speed or just speed. Speed is defined by measuring the distance the car (or whatever it is that is moving) covers in a very short interval of time (see Figure 1.3). If this distance is δs and the time interval δt then the *instantaneous speed* is

$$v = \frac{\delta s}{\delta t}$$

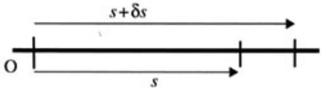

Figure 1.3 Speed at a given time t is defined in terms of the small distance δs travelled in a small interval of time δt right after t.

Note that, by definition, speed is always a positive number.

A word on notation

If Q is any physical quantity, we will use ΔQ to denote the change in Q:

$$\Delta Q = Q_{\text{final}} - Q_{\text{initial}}$$

The symbol Δ will thus represent a *finite* change in a quantity. The symbol δQ represents an *infinitesimal* change in Q. Thus, δQ plays roughly the same role as the calculus quantity dQ. So the definition of instantaneous speed $\frac{\delta s}{\delta t}$ is to be understood as the *calculus quantity* $\frac{ds}{dt}$: that is, the derivative of distance with respect to time. Equivalently, we may understand it as

$$\frac{\delta s}{\delta t} = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t}$$

If a quantity Q depends on, say, time in a linear way, then the graph showing the variation of Q with t will be a straight line. In that case (and only in that case)

$$\frac{\delta Q}{\delta t} = \frac{\Delta Q}{\Delta t}$$

and each of these quantities represent the (constant) gradient of the graph.

In order to avoid a proliferation of deltas, we will mostly use the capital delta; when infinitesimal quantities are involved, we will simply state it explicitly.

Example question

Q4

A car of length 4.2 m travelling in a straight line takes 0.56 s to go past a mark on the road. What is the speed of the car?

Answer

From $v = \frac{\Delta s}{\Delta t}$, we find $v = 7.5\ \text{m s}^{-1}$. This is taken as the speed of the car the instant the middle point of the car goes past the mark on the road.

Velocity

Average speed and instantaneous speed are positive quantities that do not take into account the direction in which the object moves. To do that we introduce the concept of velocity. The average velocity for a motion is defined as the change in displacement of the object divided by the total time taken. (Recall that the change in displacement, Δx, means final minus initial displacement.)

$$\bar{v} = \frac{\Delta x}{\Delta t}$$

Similarly, the **instantaneous velocity** at some time t, or just velocity, is defined by the ratio of the change in displacement, δx, divided by the time taken, δt.

$$v = \frac{\delta x}{\delta t}$$

(See Figure 1.4.)

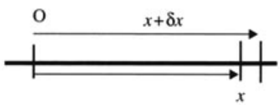

Figure 1.4 The definition of velocity at time t involves the small displacement change δx in the small time interval δt right after t.

We are using the same symbol for speed and velocity. It will always be clear which of the two we are talking about.

Unlike speed, which is always a positive number, velocity can be positive or negative. Positive velocity means the object is *increasing its displacement* – that is, it moves toward the 'right' by our convention. Negative velocity signifies motion in which the displacement is *decreasing* – that is, toward the 'left'. Thus, it is important to realize that the quantity δx can be positive or negative. (It is worthwhile to note that the magnitude of velocity is speed but the magnitude of average velocity is not related to average speed.)

Example questions

Q5

A car starts out from O in a straight line and moves a distance of 20 km towards the right, and then returns to its starting position 1 h later. What is the average speed and the average velocity for this trip?

Answer

The total distance covered is 40 km. Thus, the average speed is 40 km h^{-1}. The change in displacement for this trip is 0 m because

$$\begin{aligned}\text{displacement} &= \text{final} - \text{initial}\\ &= 0\text{ m} - 0\text{ m}\\ &= 0\text{ m}\end{aligned}$$

So the average velocity is zero.

Q6

A car moves in exactly the same way as in example question 1, but this time it starts out not at O but a point 100 km to the right of O. What is the average speed and the average velocity for this trip?

Answer

The distance travelled is still 40 km and hence the average speed is the same, 40 km h^{-1}. The change in displacement is given by

$$\begin{aligned}\text{displacement} &= \text{final} - \text{initial}\\ &= 100\text{ km} - 100\text{ km}\\ &= 0\text{ km}\end{aligned}$$

Hence the average velocity is zero as before. This example shows that the starting point is irrelevant. We have the freedom to choose the origin so that it is always at the point where the motion starts.

Q7

A car 4.0 m long is moving to the left. It is observed that it takes 0.10 s for the car to pass a given point on the road. What is the speed and velocity of the car at this instant of time?

Answer

We can safely take 0.10 s as a small enough interval of time. We are told that in this interval of time the distance travelled is 4.0 m and so the speed is 40 m s^{-1}. The velocity is simply -40 m s^{-1}, since the car is moving to the left.

Motion with uniform velocity (or just *uniform motion*) means motion in which the velocity is constant. This implies that the displacement changes by equal amounts in equal intervals of time (no matter how small or large). Let us take the interval of time from $t = 0$ to time t.

▶ If the displacement at $t = 0$ is x_0 and the displacement at time t is x, then it follows that

$$\begin{aligned}v &= \frac{\Delta x}{\Delta t}\\ &= \frac{x - x_0}{t - 0}\\ \Rightarrow x &= x_0 + vt\end{aligned}$$

This formula gives the displacement x at time t in terms of the constant velocity v and the initial displacement x_0. Note that t in this formula stands for the time for which the object has been moving.

Example questions

Q8

The initial displacement of a body moving with a constant velocity 5 m s^{-1} is -10 m. When does the body reach the point with displacement 10 m? What distance does the body cover in this time?

Answer

From

$x = x_0 + vt$
$\Rightarrow 10 = -10 + 5t$
$\Rightarrow t = 4\text{ s}$

So the distance travelled is 20 m.

Q9

Bicyclist A starts with initial displacement zero and moves with velocity 3 m s^{-1}. At the same time, bicyclist B starts from a point with displacement 200 m and moves with velocity -2 m s^{-1}. When does A meet B and where are they when this happens?

Answer

The formula giving the displacement of A is

$x_A = 0 + 3t$

and that for B is

$x_B = 200 + (-2)t$
$= 200 - 2t$

When they meet they have the same displacement, so

$x_A = x_B$
$\Rightarrow 3t = 200 - 2t$
$\Rightarrow 5t = 200$
$\Rightarrow t = 40\text{ s}$

Their common displacement is then 120 m.

Q10

Object A starts from the origin with velocity 3 m s^{-1} and object B starts from the same place with velocity 5 m s^{-1}, 6 seconds later. When will B catch up with A?

Answer

We take object A to start its motion when the clock shows zero. The displacement of A is then given by

$x_A = 3t$

and that of B by

$x_B = 5(t - 6)$

The formula for B is understood as follows. When the clock shows that t seconds have gone by, object B has only been moving for $(t - 6)$ seconds. When B catches up with A, they will have the same displacement and so

$3t = 5(t - 6)$
$\Rightarrow 2t = 30$
$\Rightarrow t = 15\text{ s}$

The displacement then is 45 m.

Frames of reference

We are used to measuring velocities with respect to observers who are 'at rest'. Thus, velocities of cars, aeroplanes, clouds and falling leaves are all measured by observers who are at rest on the surface of the earth. However, other observers are also entitled to observe and record a given motion and they may reach different results from the observer fixed on the surface of the earth. These other observers, who may themselves be moving with respect to the fixed observer on earth, are just as entitled to claim that they are 'at rest'. There is in fact no absolute meaning to the statement 'being at rest' – a fact that is the starting point of Einstein's theory of special relativity. No experiment can be performed the result of which will be to let observers know that they are moving with constant velocity and that they are not at rest. Consider two observers: observer A is fixed on the earth; observer B moves past A in a box without windows. B cannot, by performing experiments within his box (he cannot look outside) determine that he is moving, let alone determine his velocity with respect to A.

An observer who uses measuring tapes and stopwatches to observe and record motion is called a *frame of reference*. Consider the following three frames of reference: the first consists of observer A on the ground; the second consists of observer B, who is a passenger in a train sitting in her seat; the third consists of observer C, a passenger on the train who walks in the direction of the motion of the train at 2 m s^{-1}, as measured by the passenger sitting in her seat. The train moves in a straight line with constant velocity of 10 m s^{-1}, as measured by the observer on the ground. The three observers describe their situation as follows: A says he is at rest, that B moves forward at 10 m s^{-1} and that C moves forward at 12 m s^{-1}. This is because in 1 s the train moves forward a distance of 10 m but, in this same second, C has walked an additional distance of 2 m making him 12 m away from A. Thus A measures a velocity of 12 m s^{-1} for C. Observer B says that she is at rest. As far as B is concerned, A is moving backwards (the station is being left behind) with a velocity of -10 m s^{-1}, and C is moving forward at a velocity of 2 m s^{-1}. Observer C claims he is at rest. As far as he is concerned, A is moving backwards at -12 m s^{-1} and B at -2 m s^{-1}.

Example question

Q11

A cart moves in a straight line with constant speed. A toy cannon on the cart is pointed vertically up and fires a ball. Ignoring air resistance, where will the ball land?

Answer

The ball will land back into the cannon. For an observer moving along with the cannon, this is obvious. This observer considers herself to be at rest; so the ball will move vertically up and then fall vertically down into the cannon. As far as an observer on the ground is concerned, the cart moves forward with a certain velocity but so does the ball. The horizontal component of velocity of the cannon is the same as that of the ball, which means that the ball is at all times vertically over the cannon.

This introduces the concept of relative velocity. Let two observers P and Q have velocities $\vec{v}_P$ and $\vec{v}_Q$ as measured by the *same* frame of reference. Then the relative velocity of P with respect to Q, denoted by $\vec{v}_{PQ}$, is simply $\vec{v}_{PQ} = \vec{v}_P - \vec{v}_Q$.

(Note that we are subtracting vectors here.) This definition makes use of the fact that by subtracting the vector velocity $\vec{v}_Q$ it is as if we make Q be at rest, so that we can refer to the velocity of P. In the example of the cannon above, the relative velocity in the horizontal direction between the cannon and the ball is zero. This is why it falls back into the cannon.

Example questions

Q12

A car (A) moves to the left with speed 40 km h^{-1} (with respect to the road). Another car (B) moves to the right with speed 60 km h^{-1} (also with respect to the road). Find the relative velocity of B with respect to A.

Answer

The relative velocity of B with respect to A is given by the difference

$60 \text{ km h}^{-1} - (-40 \text{ km h}^{-1}) = 100 \text{ km h}^{-1}$.

Note that we must put in the negative sign for the velocity of A.

Q13

Rain comes vertically down and the water has a velocity vector given in Figure 1.5a (as measured by an observer fixed on the surface of the earth). A girl runs towards the right with a velocity vector as shown. (Again as measured by the observer fixed on the earth.) Find the velocity of the rain relative to the running girl.

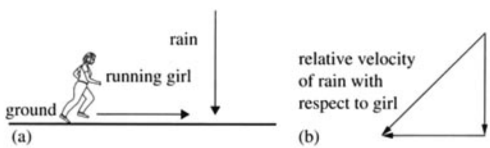

Figure 1.5.

Answer

We are asked to find the difference in the vector velocities of rain minus girl and this vector is given by Figure 1.5b. The rain thus hits the girl from the front.

Q14

This is the same as example question 9, which we did in the last section. We will do it again using the concept of relative velocity. Bicyclist A starts with initial displacement zero and moves with velocity 3 m s^{-1}. At the same time, bicyclist B starts from a point with displacement 200 m and moves with velocity -2 m s^{-1}. When does A meet B and where are they when this happens?

Answer

The velocity of B relative to A is

$$
\begin{aligned}
v_{BA} &= v_B - v_A \\
&= -3 - 2 \\
&= -5 \text{ m s}^{-1}
\end{aligned}
$$

When B meets A, the displacement of B becomes zero, since A thinks of herself sitting at the origin. Thus

$$0 = 200 - 5t$$
$$\Rightarrow t = 40\text{s}$$

Graphs for uniform motion

In uniform motion, if we make a graph of velocity versus time we must get a horizontal straight line. Figure 1.6 shows the v–t graph for motion with constant velocity v.

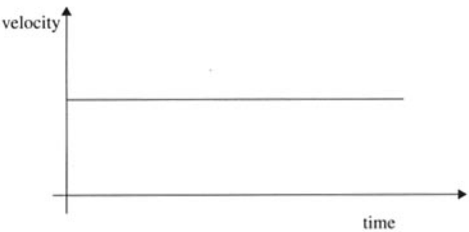

Figure 1.6 Uniform velocity means that the velocity–time graph is a horizontal straight line.

If we wanted to find the displacement from $t = 0$ to time t, the answer would be given by the formula $x = x_0 + vt$. The same answer can, however, also be obtained directly from the graph: vt is simply the area under the graph, as shown in Figure 1.7.

This means that the area under the graph gives the change in displacement. This area added to the initial displacement of the mass gives the final displacement at time t.

Figure 1.7 The area under the curve in a velocity–time graph gives the displacement change.

A graph of displacement versus time for *uniform motion* also gives a straight line (Figure 1.8).

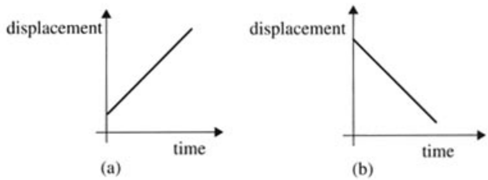

Figure 1.8 The displacement–time graph for uniform motion is a straight line: (a) motion to the right, (b) motion to the left.

This is the graph of the equation $x = x_0 + vt$. Comparing this with the standard equation of a straight line, $y = mx + c$, we see that the slope of this graph gives the velocity. We can also deduce this from the definition of velocity, $\frac{\Delta x}{\Delta t}$. But $\frac{\Delta x}{\Delta t}$ is also the definition of the slope of the straight-line x–t graph, hence the slope is the velocity. In Figure 1.8a the slope is positive, which means, therefore, that the velocity is positive, and the mass is moving to the right. In Figure 1.8b the mass is moving at constant velocity to the left.

▶ The slope of a displacement–time graph gives the velocity.

The time when the graph intersects the time axis is the time the moving object goes past point O, the point from which distances and displacements are measured.

The corresponding velocity–time graph for negative velocity is shown in Figure 1.9.

Figure 1.9 The velocity–time graph for uniform motion towards the left.

The area 'under' the curve is below the time axis and is counted as negative. This is consistent with the fact that negative velocity takes the moving object towards the left and thus towards negative displacements.

Consider now the graph of displacement versus time in Figure 1.10. We may extract the following information from it. The initial displacement is −10 m. The object moves with a positive velocity of 2 m s^{-1} for the first 10 s of the motion and with a negative velocity of 2 m s^{-1} in the next 5 s. The object is at the origin at 5 s and 15 s. The change in displacement is +10 m and the total distance travelled is 30 m. The average speed is thus 2 m s^{-1} and the average velocity is 0.67 m s^{-1}. Make sure you can verify these statements.

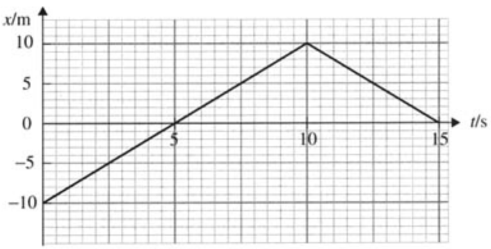

Figure 1.10.

▶ We can thus summarize our findings for uniform motion in a straight line: The graph of displacement versus time is a straight line whose slope is the velocity. The graph of velocity versus time is a horizontal straight line and the area under the graph gives the change in displacement. The displacement after time t is given by the formula $x = x_0 + vt$.

Example question

Q15

A mass starts out from O with velocity 10 m s^{-1} and continues moving at this velocity for 5 s. The velocity is then abruptly reversed to −5 m s^{-1} and the object moves at this velocity for 10 s. For this motion find:

(a) the change in displacement;
(b) the total distance travelled;
(c) the average speed;
(d) the average velocity.

Answer

The problem is best solved through the velocity–time graph, which is shown in Figure 1.11.

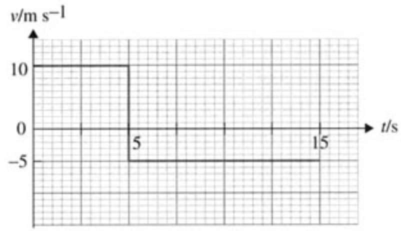

Figure 1.11.

The initial displacement is zero. Thus, after 5 s the displacement is 10×5 m = 50 m (area under first part of the curve). In the next 10 s the displacement changes by $-5 \times 10 = -50$ m. The change in displacement is thus 0 m. The object moved toward the right, stopped and returned to its starting position. The distance travelled was 50 m in moving to the right and 50 m coming back giving a total of 100 m. The average velocity is zero, since the change in displacement is zero. The average speed is 100 m/15 s = 6.7 m s^{-1}.

Questions

1 A plane flies 3000.0 km in 5.00 h. What is its average speed in metres per second?

2 A car must be driven a distance of 120.0 km in 2.5 h. During the first 1.5 h the average speed was 70 km h^{-1}. What must the average speed for the remainder of the journey be?

3 A person walks a distance of 3.0 km due south and then a distance of 2.0 km due east. If the walk lasts for 3.0 h find

(a) the average speed for the motion;
(b) the average velocity.

4 Find the displacement–time graph for an object moving in a straight line whose velocity–time graph is given in Figure 1.12. The displacement is zero initially. You do not have to put any numbers on the axes.

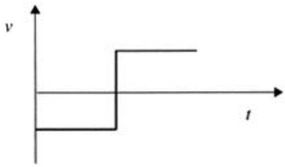

Figure 1.12 For question 4.

5 An object moving in a straight line according to the velocity–time graph shown in Figure 1.13 has an initial displacement of 8.00 m.
(a) What is the displacement after 8.00 s?
(b) What is the displacement after 12.0 s?
(c) What is the average speed and average velocity for this motion?

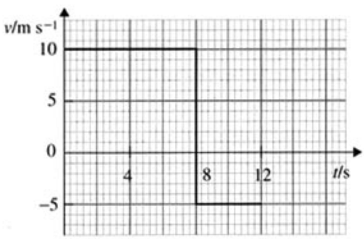

Figure 1.13 For question 5.

6 Two cyclists, A and B, have displacements 0 km and 70 km, respectively. At $t = 0$ they begin to cycle towards each other with velocities 15 km h^{-1} and 20 km h^{-1}, respectively. At the same time, a fly that was sitting on A starts flying towards B with a velocity of 30 km h^{-1}. As soon as the fly reaches B it immediately turns around and flies towards A, and so on until A and B meet.
(a) What will the displacement of the two cyclists and the fly be when all three meet?
(b) What will be the distance travelled by the fly?

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7 A particle of dust is bombarded by air molecules and follows a zigzag path at constant speed v.
(a) Assuming each step has a length d, find the distance travelled by the dust particle in time t.
(b) What is the length of the displacement vector after N steps where N is large? Assume that each step is taken in a random direction on the plane. (This problem assumes you are familiar with the scalar product of two vectors.)

8 Two cars are moving on the same straight-line road. Car A moves to the right at velocity 80 km h^{-1} and car B moves at 50 km h^{-1} to the left. Both velocities are measured by an observer at rest on the road.
(a) Find the relative velocity of car B with respect to car A.
(b) Find the relative velocity of car A with respect to car B.

9 A cyclist A moves with speed 3.0 m s^{-1} to the left (with respect to the road). A second cyclist, B, moves on the same straight-line path as A with a relative velocity of 1.0 m s^{-1} with respect to A.
(a) What is the velocity of B with respect to the road?
(b) A third cyclist has a relative velocity with respect to A of -2.0 m s^{-1}. What is the velocity of C with respect to the road?

10 Two objects A and B move at a constant speed of 4 m s^{-1} along a circular path. What is the relative velocity of B (magnitude and direction) with respect to A when the objects are in the positions shown in Figure 1.14?

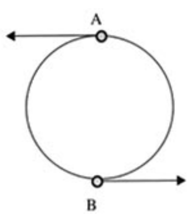
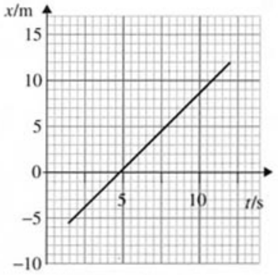
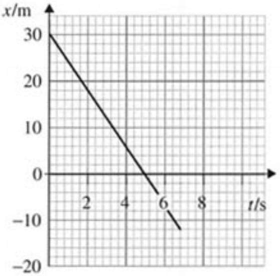


Figure 1.14 For question 10.

11 Find the velocity of the two objects whose displacement–time graphs are shown in Figure 1.15.



(a)



(b)

Figure 1.15 For question 11.

12 An object moving in a straight line has a displacement–time graph as shown in Figure 1.16.

(a) Find the average speed for the trip.

(b) Find the average velocity for the trip.

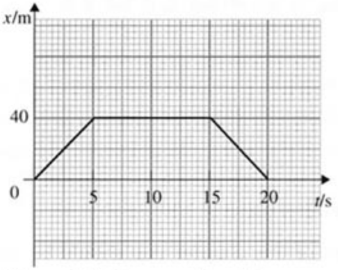


Figure 1.16 For question 12.