Motion with constant acceleration

To complete our description of motion we need the concept of acceleration. This concept and its use are introduced here.

Objectives

By the end of this chapter you should be able to:

- recognize situations of accelerated motion and to define *acceleration* as $a = \frac{\Delta v}{\Delta t}$;
- *describe a motion given a graph for that motion*;
- understand that the *slope of a displacement–time graph is the velocity*;
- understand that the *slope of a velocity–time graph is the acceleration* and the *area under a velocity–time graph is the change in displacement*;
- understand that the *area under an acceleration–time graph is the change in velocity*;
- analyse motion from *ticker tape*, *stroboscopic* pictures and *photogate* data;
- solve problems of kinematics for motion in a straight line with constant acceleration using

$$v = v_0 + at$$
$$x = x_0 + v_0 t + \tfrac{1}{2}at^2$$
$$x = x_0 + \left(\frac{v + v_0}{2}\right)t$$
$$v^2 = v_0^2 + 2ax$$

(It must be emphasized that these formulae *only* apply in the case of motion in a straight line with constant acceleration.)

Acceleration

To treat situations in which velocity is not constant we need to define acceleration a. If the velocity changes by Δv in a very short interval of time Δt then

$$a = \frac{\Delta v}{\Delta t}$$

is the definition of the **instantaneous acceleration**. We will mostly be interested in situations where the acceleration is constant, in which case the instantaneous acceleration and the average acceleration are the same thing. Such a motion is called *uniformly accelerated motion*. In this case the intervals Δv and Δt do not have to be infinitesimally small. Then

$$a = \frac{\Delta v}{\Delta t} = \frac{v - v_0}{t - 0}$$

where t is the total time taken for the trip, v the final velocity and v_0 the initial velocity.

▶ In this case, by rewriting the last equation we find

$$v = v_0 + at$$

This is the formula that gives the velocity at a time of t seconds after the start of the motion in terms of the (constant) acceleration a and the initial velocity.

If we put $a = 0$ in this formula, we find that $v = v_0$ at all times. The velocity does not change since there is no acceleration.

Example question

Q1

An object starting with an initial velocity of 2.0 m s^{-1} undergoes constant acceleration. After 5.0 s its velocity is found to be 12.0 m s^{-1}. What is the acceleration?

Answer

From $v = v_0 + at$ we find

$12 = 2 + a \times 5$

$\Rightarrow a = 2.0 \text{ m s}^{-2}$

▶ For motion in a straight line, *positive* acceleration means that the velocity is *increasing* whereas negative acceleration implies a decreasing velocity.

In solving problems it is sometimes confusing to decide whether the acceleration is positive or negative. The only criterion is whether the acceleration increases or decreases the *velocity* (and not speed). In the top part of Figure 2.1 the velocity is increasing and so the acceleration is positive. (The direction of positive velocities is taken to be toward the right.) In the bottom part the velocity is decreasing and thus the acceleration is negative.

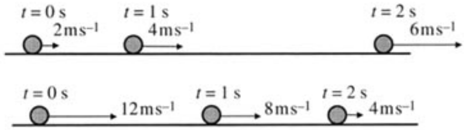

Figure 2.1 In the top part the acceleration is positive. In the bottom part the acceleration is negative.

Similarly, in the top part of Figure 2.2 the velocity is increasing (-15 m s^{-1} is larger than -20 m s^{-1}) and so the acceleration is positive. In the bottom part the velocity is decreasing (-8 m s^{-1} is less than -5 m s^{-1}) and so the acceleration is negative. In the second case note that the *speed* (i.e. the magnitude of velocity) is increasing even though the acceleration is negative.

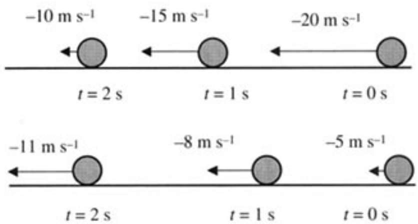

Figure 2.2 In the top diagram the acceleration is positive. In the bottom it is negative.

Acceleration due to gravity

We encounter a very special acceleration when an object is dropped or thrown. This is an acceleration that acts on all objects and has the same magnitude for all bodies independently of their mass. This assumes conditions of free fall – that is, only gravity is acting on the body. Air resistance, friction and other forces are assumed absent. Under these conditions (as will be discussed in detail in later chapters) all objects experience the same acceleration. On earth the magnitude of this acceleration is about 9.8 m s^{-2}, a number we will often approximate to 10 m s^{-2} for convenience. We always use the symbol g for the *magnitude* of the acceleration due to gravity. Consider a body falling freely under gravity. We take, as is customary, the upward direction to be the direction of positive velocities. On the way up the velocity is decreasing, hence we state that the acceleration due to gravity is negative. On the way down the velocity is still decreasing (-12.0 m s^{-1} is less than -2 m s^{-1}) and so the acceleration due to gravity is negative on the way down as well as on the way up (see Figure 2.3). (On the way down the *speed* is increasing.)

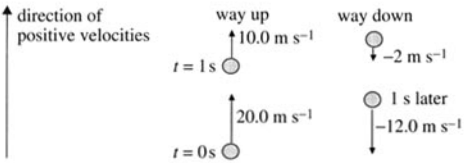

Figure 2.3 Motion in a vertical straight line under gravity. If the upward direction is the positive direction for velocity, then the acceleration due to gravity is negative for both the way up as well as the way down.

If we had decided, instead, to take the *downward* direction as the direction of positive velocities, then the acceleration due to gravity would have been positive for both the way up and the way down. Can you verify yourself that this is the case?

Example question

Q2

An object initially at $x = 12$ m has initial velocity of -8 m s^{-1} and experiences a constant acceleration of 2 m s^{-2}. Find the velocity at $t = 1$ s, 2 s, 3 s, 4 s, 5 s, 6 s and 10 s.

Answer

Applying the equation $v = v_0 + at$ we get the results shown in Table 2.1.

Time/s	1	2	3	4	5	6	10
Velocity/m s^{-1}	−6	−4	−2	0	2	4	12

Table 2.1.

This means that the body stops instantaneously at $t = 4$ s and then continues moving. We do not need to know the initial position of the body to solve this problem. Note also that the acceleration is positive and hence the velocity must be increasing. This is indeed the case as shown in Table 2.1 (e.g. -4 m s^{-1} is larger than -6 m s^{-1}). However, the speed decreases from $t = 0$ s to $t = 4$ s and increases from $t = 4$ s onwards.

In motion with constant positive acceleration the graph showing the variation of velocity with time is one of the three in Figure 2.4.

This represents a mass moving towards the right with increasing velocity. This is the graph of the equation $v = v_0 + at$.

The first graph of Figure 2.5 represents a mass that starts moving to the right (velocity is

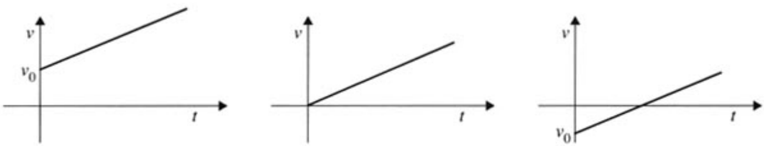

Figure 2.4 Graphs showing the variation of velocity with time when the acceleration is constant and positive. In the graphs above, the only difference is that the initial velocity v_0 is positive, zero and negative, respectively.

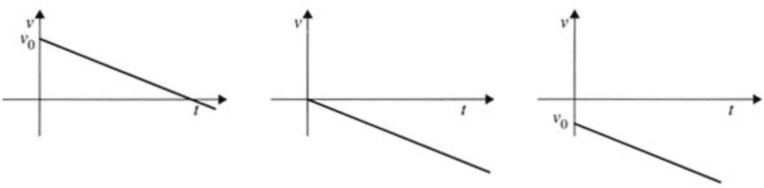

Figure 2.5 Graphs showing the variation of velocity with time when the acceleration is constant and negative. In the graphs above, the only difference is that the initial velocity v_0 is positive, zero and negative, respectively.

positive) but is decelerating (negative acceleration). At a specific time the mass stops instantaneously and begins moving again towards the left (negative velocity).

▶ The acceleration can be found from the velocity–time graph by taking the slope of the graph. We see this directly by comparing $v = v_0 + at$ and the standard equation for a straight line, $y = c + mx$.

If the acceleration is not uniform, the velocity–time graph will not be a straight line. The acceleration at a given point is found by first drawing a tangent to the curve at the point of interest. The slope of the tangent is the acceleration at that point.

In the case of uniform motion (no acceleration) the area under a velocity–time graph gave the change in displacement. We would like to know if a similar result holds in the case of accelerated motion as well.

To do this we will make use of what we learned in uniform motion together with a little trick. Consider the velocity–time graph of an accelerated motion in Figure 2.6. The trick consists of approximating this motion with another motion in four steps. We will assume that during each of the steps the velocity is constant. The velocity then changes abruptly to a new constant value in the next step. The approximation is shown in the figure. Clearly, this is a very crude approximation of the actual motion.

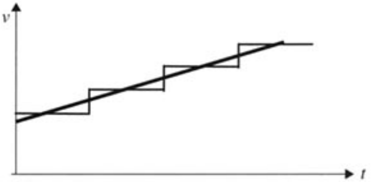

Figure 2.6 The velocity is assumed to increase abruptly and then remain constant for a period of time.

We can improve the approximation tremendously by taking more and thinner steps, as shown in Figure 2.7.

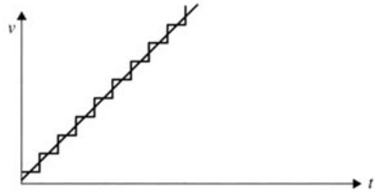

Figure 2.7 The approximation is made better by considering more steps.

Clearly, the approximation can be made as accurate as we like by choosing more and more (and thus thinner and thinner) steps.

The point of the approximation is that during each step the velocity is constant. In each step, the displacement increases by the area under the step, as we showed in the case of uniform motion. To find the total change in displacement for the entire trip we must thus add up the areas under all steps. But this gives the area under the original straight line! So we have managed to show that:

▶ Even in the case of accelerated motion, the change in displacement is the area under the velocity–time graph, just as in uniform motion. (See Figure 2.8.)

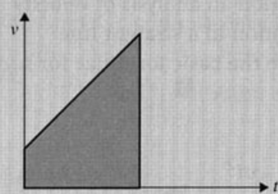

Figure 2.8 The area under the graph is the change in displacement.

Using this result we can now find a formula for the displacement after time t. We are given a velocity–time graph with constant acceleration (a straight-line graph in a v–t diagram).

We want to find the area under the line from $t = 0$ to a time of t s. Since the area we have is the shape of a trapezoid, the area is the sum of two parallel bases times height divided by two:

$$\text{area} = \left(\frac{v + v_0}{2}\right)t$$

This actually gives a useful formula for displacement for motion with constant acceleration. If x_0 is the initial displacement

$$x = x_0\left(\frac{v + v_0}{2}\right)t$$

This is useful when we know the initial and final velocities but not the acceleration.

However, we do know that $v = v_0 + at$ and so the area (i.e. the change in displacement after time t) is

$$\begin{aligned}\text{area} &= \frac{v_0 + at + v_0}{2} \times t \\ &= v_0 t + \tfrac{1}{2}at^2\end{aligned}$$

Thus, the displacement after time t is this area added to the initial displacement, that is

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2$$

(We see that when the acceleration is zero, this formula becomes identical to the one we derived earlier for constant velocity, namely $x = x_0 + v_0 t$.) Note that this formula says that when $t = 0$, $x = x_0$ as it should.

In the previous section an analysis of velocity–time graphs for motion in a straight line allowed us to derive the basic formulae for such motion:

$$\begin{aligned}v &= v_0 + at \\ x &= x_0 + v_0 t + \tfrac{1}{2}at^2\end{aligned}$$

or

$$x = \left(\frac{v + v_0}{2}\right)t$$

All of these involve time. In some cases, it is useful to have a formula that involves velocity and displacement without any reference to time. This can be done by solving the first equation for time

$$t = \frac{v - v_0}{a}$$

and using this value of time in the second equation:

$$x = x_0 + v_0\frac{v - v_0}{a} + \frac{1}{2}a\frac{(v - v_0)^2}{a^2}$$

$$2a(x - x_0) = 2v_0 v - 2v_0^2 + v^2 + v_0^2 - 2vv_0$$

i.e.

$$v^2 = v_0^2 + 2a(x - x_0)$$

If the initial displacement is zero, then this reduces to the simpler

$$v^2 = v_0^2 + 2ax$$

Example questions

Q3

A mass has an initial velocity of 10.0 m s^{-1}. It moves with acceleration -2.00 m s^{-2}. When will it have zero velocity?

Answer

We start with

$v = v_0 + at$

$v = 0$ and so

$0 = v_0 + at$

Putting in the numbers we get

$0 = 10 + (-2.00)t$

so $t = 5.00$ s.

Q4

What is the displacement after 10.0 s of a mass whose initial velocity is 2.00 m s^{-1} and moves with acceleration $a = 4.00$ m s^{-2}?

Answer

We assume that the initial displacement is zero so that $x_0 = 0$.

$x = x_0 + v_0 t + \frac{1}{2}at^2$

so

$$\begin{aligned}x &= 0 + 2 \times 10 + \tfrac{1}{2} \times 4 \times 10^2 \\ &= 220\ \text{m}\end{aligned}$$

Q5

A car has an initial velocity of $v_0 = 5.0 \text{ m s}^{-1}$. When its displacement increases by 20.0 m, its velocity becomes 7.0 m s^{-1}. What is the acceleration?

Answer

Again take $x_0 = 0$ so that

$$v^2 = v_0^2 + 2ax$$

So $7^2 = 5^2 + 2a \times 20$
therefore
$a = 0.60 \text{ m s}^{-2}$

Q6

A body has initial velocity $v_0 = 4.0 \text{ m s}^{-1}$ and a velocity of $v = 12 \text{ m s}^{-1}$ after 6.0 s. What displacement did the body cover in the 6.0 s?

Answer

We may use

$$x = \left(\frac{v + v_0}{2}\right) t$$

to get

$$x = \left(\frac{12 + 4}{2}\right) 6$$
$$= 48 \text{ m}$$

This is faster than using $v = v_0 + at$ in order to find the acceleration as

$$12 = 4 + 6a$$
$$\Rightarrow a = 1.333 \text{ m s}^{-2}$$

and then

$$x = v_0 t + \tfrac{1}{2}at$$
$$= 4 \times 6 + \tfrac{1}{2} \times 1.333 \times 36$$
$$= 48 \text{ m}$$

The two examples that follow involve motions that start at different times.

Q7

Two balls start out moving to the right with constant velocities of 5 m s^{-1} and 4 m s^{-1}. The slow ball starts first and the other 4 s later. How far from the starting position are they when they meet?

Answer

See Figure 2.9.

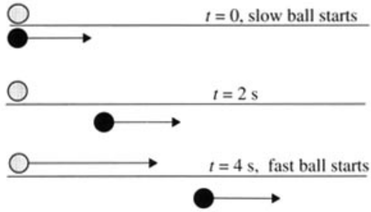

Figure 2.9.

Let the two balls meet t s after the first ball starts moving. The displacement of the slow ball is $x = 4t$ m and that travelled by the fast ball $5(t - 4)$ m. The factor $t - 4$ is there since after t s the fast ball has actually been moving for only $t - 4$ s. These two displacements are equal when the two balls meet and thus $4t = 5t - 20$, or $t = 20$ s. The common displacement is thus 80 m.

Q8

A mass is thrown upwards with an initial velocity of 30 m s^{-1}. A second mass is dropped from directly above, a height of 60 m from the first mass, 0.5 s later. When do the masses meet and how high is the point where they meet?

Answer

See Figure 2.10. We choose the upward direction to be positive for velocities and displacements. The masses experience an acceleration of -10 m s^{-2}, the acceleration due to gravity. Since the motion is along a *vertical* straight line, we use the symbol y for displacement rather than x.

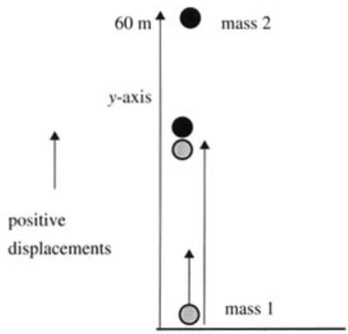

Figure 2.10.

The first mass moves to a displacement given by $y_1 = 30t - 5t^2$. The second moves to a displacement of $y_2 = 60 - 5(t - 0.5)^2$. The displacements are the same when the masses meet. Thus

$$30t - 5t^2 = 60 - 5(t - 0.5)^2$$
$$\Rightarrow t = 2.35 \text{ s}$$

The common displacement at this time is 42.9 m.

Graphs of acceleration versus time

In a graph of acceleration versus time the area under the graph gives the change in velocity. In Figure 2.11 the area from time zero to 4 s is 12 m s^{-1} and thus the velocity after 4 s is 12 m s^{-1} plus whatever initial velocity the object had.

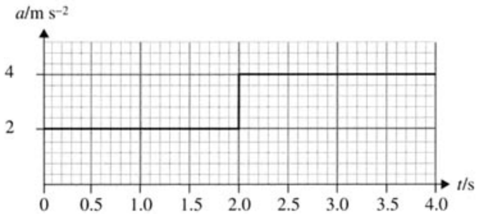

Figure 2.11 The area under an acceleration–time graph is the change in velocity.

Graphs of displacement versus time

In motion with constant acceleration, a graph of displacement versus time is a parabola. Consider a ball that is dropped from rest from a height of 20 m. The graph of displacement versus time is shown in Figure 2.12.

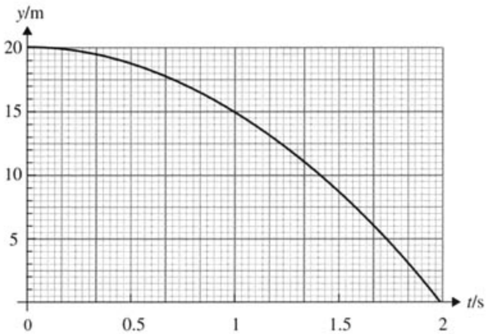

Figure 2.12 Graph of displacement versus time. The object hits the floor at 2 s.

Example question

Q9

An object with initial velocity 20 m s^{-1} and initial displacement of −75 m experiences an acceleration of −2 m s^{-2}. Draw the displacement–time graph for this motion for the first 20 s.

Answer

The displacement is given by $x = -75 + 20t - t^2$ and this is the function we must graph. The result is shown in Figure 2.13.

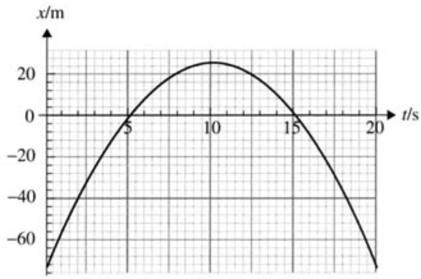

Figure 2.13.

At 5 s the object reaches the origin and overshoots it. It returns to the origin 10 s later ($t = 15$ s). The furthest it gets from the origin on the right side is 25 m. The velocity at 5 s is 10 m s^{-1} and at 15 s it is −10 m s^{-1}. At 10 s the velocity is zero.

In general, if the velocity is not constant, the graph of displacement with time will be a curve. Drawing the tangent at a point on the curve and finding the slope of the tangent gives the velocity at that point.

Measuring speed and acceleration

The speed of an object is determined experimentally by measuring the distance travelled by the object in an interval of time. Dividing the distance by the time taken gives the average speed. To get as close an approximation to the instantaneous speed as possible, we must make the time interval as small as possible. We can measure speed electronically by attaching a piece of cardboard of known length to the object so that a single photogate will record the time taken for that known length to go through the photogate, as in Figure 2.14. The ratio of the cardboard length to time taken is the speed of the object when it is halfway through the photogate.

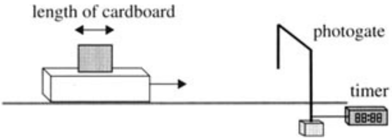

Figure 2.14 Measuring speed with a photogate.

Speed can also be measured with a tickertape, an instrument that makes marks on a paper tape at regular intervals of time (usually 50 marks per second). If one end of the tape is attached to the moving object and the other end goes through the marker, then to find the speed at a particular point we would measure the distance between two consecutive marks (distance travelled by the object) and divide by the time taken (1/50 s). In Figure 2.15 the dotted lines are supposed to be 0.5 cm apart. Then the top tape represents uniform motion with speed

$$v = \frac{0.5}{1/50}\ \text{cm s}^{-1}$$
$$= 25\ \text{cm s}^{-1}$$
$$= 0.25\ \text{m s}^{-1}$$

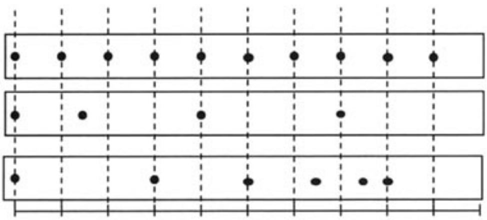

Figure 2.15 Measuring speed with a tickertape.

In the second tape the moving object is accelerating. The distance between the first two dots is about 0.75 cm so the average speed between those two dots is

$$v = \frac{0.75}{1/50}\ \text{cm s}^{-1}$$
$$= 0.375\ \text{m s}^{-1}$$

The distance between dots 2 and 3 is 1.25 cm and so the average speed between those dots is

$$v = \frac{1.25}{1/50}\ \text{cm s}^{-1}$$
$$= 0.625\ \text{m s}^{-1}$$

Between dots 3 and 4 the distance is 1.5 cm and so

$$v = \frac{1.5}{1/50}\ \text{cm s}^{-1}$$
$$= 0.750\ \text{m s}^{-1}$$

We may thus take the average speed between $t = 0$ s and $t = 1/50$ s to be 0.375 m s^{-1}, between $t = 1/50$ s and $t = 2/50$ s to be 0.625 m s^{-1} and between $t = 2/50$ s and $t = 3/50$ s to be 0.750 m s^{-1}. Thus the average acceleration in the first 1/50 s is 12.5 m s^{-2} and in the next 1/50 s it is 6.25 m s^{-2}. The acceleration is thus not constant for this motion.

The third tape shows decelerated motion.

Related to the tickertape method is that of a stroboscopic picture (see Figure 2.16). Here the moving body is photographed in rapid succession with a constant, known interval of time between pictures. The images are then developed on the same photograph, giving a multiple exposure picture of the motion. Measuring the distances covered in the known time interval allows a measurement of speed.

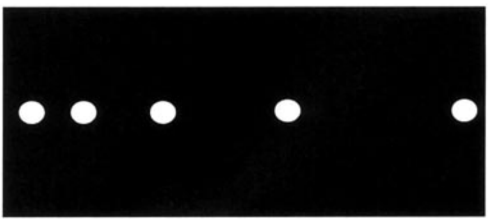

Figure 2.16 Measuring speed with a stroboscope.

Once measurement of speed is possible, acceleration can also be determined. To measure the acceleration at a specific time, t, one must first measure the velocity a short interval of time before t, say $t - T/2$ and again a short time after t, $t + T/2$ (see Figure 2.17). If the values of velocity found are respectively u and v, then

$$a = \frac{v - u}{T}$$

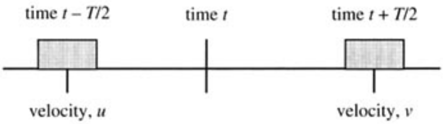

Figure 2.17 Measuring acceleration requires knowing the velocity at two separate points in time.

More on graphs

In kinematics the most useful graph is that of velocity versus time (v–t). The slope of such a graph gives the acceleration and the area under the graph gives the change in displacement. Let's examine this in detail. Consider the following problem, which is hard to solve with equations but is quite easy using a v–t graph. Two masses, A and B, are to follow the paths shown in Figure 2.18. The paths are the same length, but one involves a hill and the other a valley.

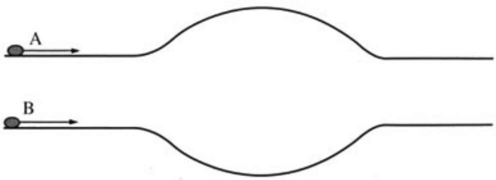

Figure 2.18 Which mass gets to the end first? They both travel the same distance.

Which mass will get to the end first? (Remember, the distance travelled is the same.) We know that the first mass will slow down as it climbs the hill and then speed up on the way down until it reaches its original speed on the level part. The second mass will first speed up on the way down the hill and slow down to its original speed when it reaches the level part. Let us make the v–t graph for each mass. The graphs for A and B must look like Figure 2.19.

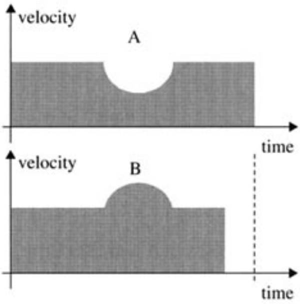

Figure 2.19.

It is then obvious that since the areas under the two curves must be the same (same displacement) the graph for B must stop earlier: that is, B gets to the end first. The same conclusion is reached more quickly if we notice that the average speed in case B is higher and so the time taken is less since the distance is the same.

Consider the following question. The graph of velocity versus time for two objects is given in Figure 2.20. Both have the same initial and final velocity. Which object has the largest average velocity?

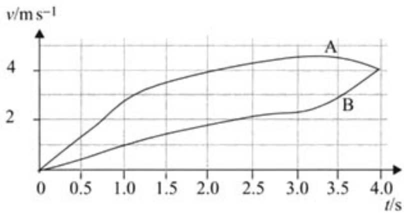

Figure 2.20 Graph showing the variation of velocity with time for two motions that have the same initial and final velocity.

Average velocity is the ratio of total displacement divided by time taken. Clearly, object A has a larger displacement (larger area under curve). Thus, it has a larger average velocity. The point is that you *cannot* say that average velocity is half of the sum of initial and final velocities. (Why? Under what circumstances *can* you say it?)

Consider finally the graph in Figure 2.21, which shows the variation of the displacement of an object with time. We would like to obtain the graph showing the variation of the velocity with time.

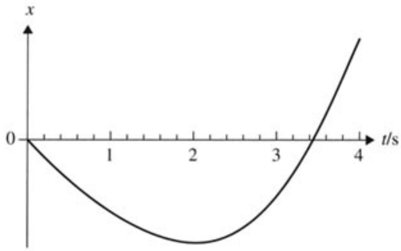

Figure 2.21 Graph showing the variation of displacement with time.

Our starting point is that *velocity is the slope of the displacement–time graph*. We see that initially the slope is negative, it becomes less negative and at $t = 2$ s it is zero. From then on the slope becomes increasingly positive. This leads to the velocity–time graph in Figure 2.22.

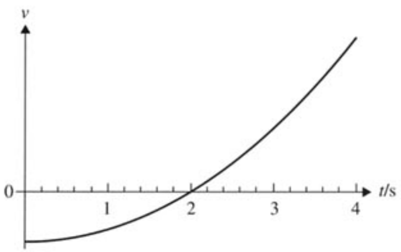

Figure 2.22 Graph showing the variation of velocity with time for the motion in Figure 2.21.

The *slope of the velocity–time graph is acceleration* and from the graph we see that the slope is initially zero but then becomes more and more positive. Hence, the acceleration–time graph must be something like Figure 2.23.

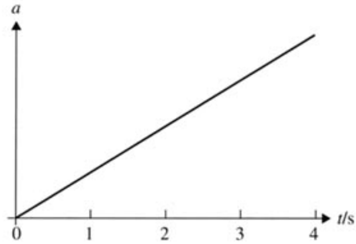

Figure 2.23 Graph showing the variation of acceleration with time for the motion in Figure 2.21.

Questions

In the graphs in this section, the point where the axes cross is the origin unless otherwise indicated.

1 The initial velocity of a car moving on a straight road is 2.0 m s^{-1} and becomes 8.0 m s^{-1} after travelling for 2.0 s under constant acceleration. What is the acceleration?

2 A plane starting from rest takes 15.0 s to take off after speeding over a distance of 450.0 m on the runway with constant acceleration. With what velocity does it take off?

3 The acceleration of a car is assumed constant at 1.5 m s^{-2}. How long will it take the car to accelerate from 5.0 m s^{-1} to 11 m s^{-1}?

4 A car accelerates from rest to 28 m s^{-1} in 9.0 s. What distance does it travel?

5 A body has an initial velocity of 12 m s^{-1} and is brought to rest over a distance of 45 m. What is the acceleration of the body?

6 A body at the origin has an initial velocity of −6.0 m s^{-1} and moves with an acceleration of 2.0 m s^{-2}. When will its displacement become 16 m?

7 A body has an initial velocity of 3.0 m s^{-1} and after travelling 24 m the velocity becomes 13 m s^{-1}. How long did this take?

8 What deceleration does a passenger of a car experience if his car, which is moving at 100.0 km h^{-1}, hits a wall and is brought to rest in 0.100 s? Express the answer in m s^{-2}.

9 A car is travelling at $40.0\ \mathrm{m\ s^{-1}}$. The driver sees an emergency ahead and 0.50 s later slams on the brakes. The acceleration of the car is $-4\ \mathrm{m\ s^{-2}}$.

(a) What distance will the car travel before it stops?

(b) If the driver was able to apply the brakes instantaneously without a reaction time, over what distance would the car stop?

(c) Calculate the difference in your answers to (a) and (b).

(d) Assume now that the car was travelling at $30.0\ \mathrm{m\ s^{-1}}$ instead. Without performing any calculations, would the answer to (c) now be less than, equal to or larger than before? Explain your answer.

10 A ball is thrown upwards with a speed of $24.0\ \mathrm{m\ s^{-1}}$.

(a) When is the velocity of the ball $12.0\ \mathrm{m\ s^{-1}}$?

(b) When is the velocity of the ball $-12.0\ \mathrm{m\ s^{-1}}$?

(c) What is the displacement of the ball at those times?

(d) What is the velocity of the ball 1.50 s after launch?

(e) What is the maximum height reached by the ball?

(Take the acceleration due to gravity to be $10.0\ \mathrm{m\ s^{-2}}$.)

11 A stone is thrown vertically upwards with an initial speed of $10.0\ \mathrm{m\ s^{-1}}$ from a cliff that is 50.0 m high.

(a) When does it reach the bottom of the cliff?

(b) What speed does it have just before hitting the ground?

(c) What is the total distance travelled by the stone?

(Take the acceleration due to gravity to be $10.0\ \mathrm{m\ s^{-2}}$.)

12 A rock is thrown vertically down from the roof of a 25.0 m high building with a speed of $5.0\ \mathrm{m\ s^{-1}}$.

(a) When does the rock hit the ground?

(b) With what speed does it hit the ground?

(Take the acceleration due to gravity to be $10.0\ \mathrm{m\ s^{-2}}$.)

13 A window is 1.50 m high. A stone falling from above passes the top of the window with a speed of $3.00\ \mathrm{m\ s^{-1}}$. When will it pass the bottom of the window? (Take the acceleration due to gravity to be $10.0\ \mathrm{m\ s^{-2}}$.)

14 A ball is dropped from rest from a height of 20.0 m. One second later a second ball is thrown vertically downwards. If the two balls arrive on the ground at the same time, what must have been the initial velocity of the second ball?

15 A ball is dropped from rest from the top of a 40.0 m building. A second ball is thrown downward 1.0 s later.

(a) If they hit the ground at the same time, find the speed with which the second ball was thrown.

(b) What is the ratio of the speed of the thrown ball to the speed of the other as they hit the ground?

(Take the acceleration due to gravity to be $10.0\ \mathrm{m\ s^{-2}}$.)

16 Two balls are dropped from rest from the same height. One of the balls is dropped 1.00 s after the other. What distance separates the two balls 2.00 s after the second ball is dropped?

17 An object moves in a straight line with an acceleration that varies with time as shown in Figure 22.4. Initially the velocity of the object is $2.00\ \mathrm{m\ s^{-1}}$.

(a) Find the maximum velocity reached in the first 6.00 s of this motion.

(b) Draw a graph of the velocity versus time.

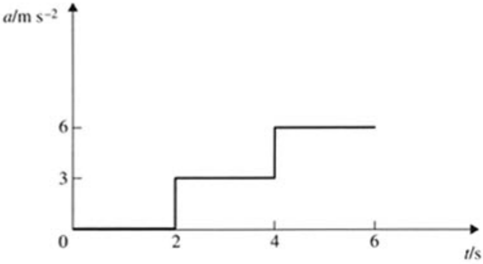

Figure 2.24 For question 17.

18 Figure 2.25 shows the variation of velocity with time of an object. Find the acceleration at 2.0 s.

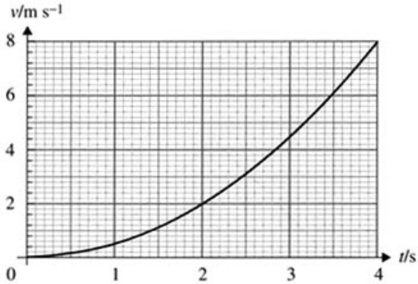

Figure 2.25 For question 18.

19 Figure 2.26 shows the variation of the displacement of a moving object with time. Draw the graph showing the variation of the velocity of the object with time.

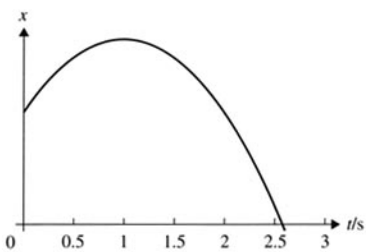

Figure 2.26 For question 19.

20 Figure 2.27 shows the variation of the displacement of a moving object with time. Draw the graph showing the variation of the velocity of the object with time.

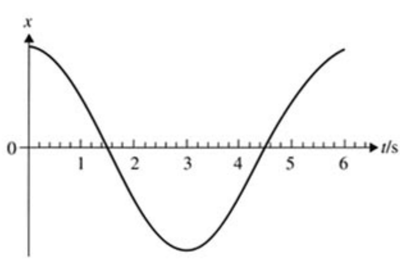

Figure 2.27 For question 20.

21 Figure 2.28 shows the variation of the displacement of a moving object with time. Draw the graph showing the variation of the velocity of the object with time.

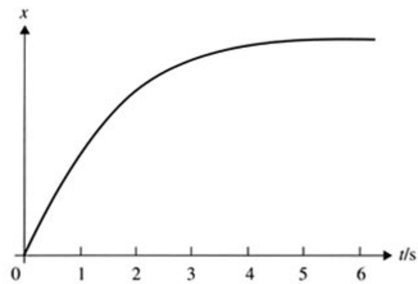

Figure 2.28 For question 21.

22 Figure 2.29 shows the variation of the displacement of a moving object with time. Draw the graph showing the variation of the velocity of the object with time.

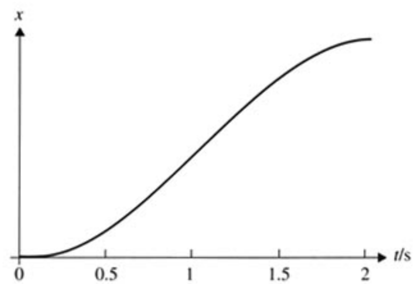

Figure 2.29 For question 22.

23 Figure 2.30 shows the variation of the displacement of a moving object with time. Draw the graph showing the variation of the velocity of the object with time.

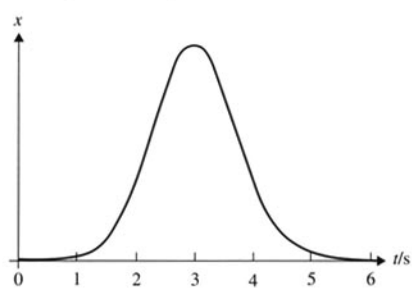

Figure 2.30 For question 23.

24 Figure 2.31 shows the variation of the velocity of a moving object with time. Draw the graph showing the variation of the displacement of the object with time.

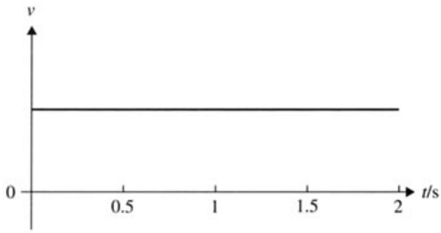

Figure 2.31 For question 24.

25 Figure 2.32 shows the variation of the velocity of a moving object with time. Draw the graph showing the variation of the displacement of the object with time.

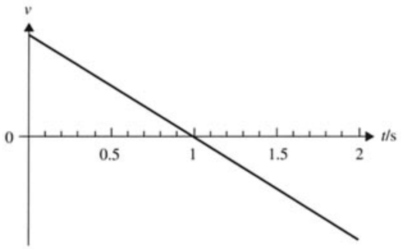

Figure 2.32 For question 25.

26 Figure 2.33 shows the variation of the velocity of a moving object with time. Draw the graph showing the variation of the displacement of the object with time (assuming a zero initial displacement).

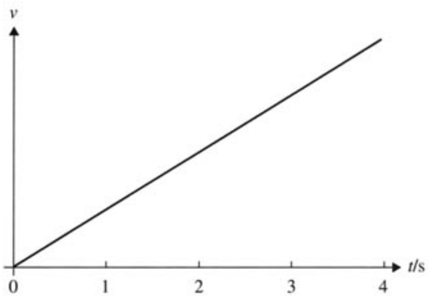

Figure 2.33 For question 26.

27 Figure 2.34 shows the variation of the velocity of a moving object with time. Draw the graph showing the variation of the displacement of the object with time (assuming a zero initial displacement).

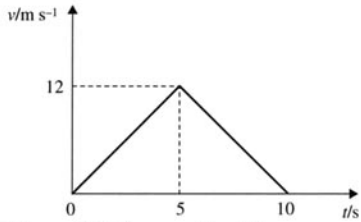

Figure 2.34 For question 27.

28 Figure 2.35 shows the variation of the velocity of a moving object with time. Draw the graph showing the variation of the acceleration of the object with time.

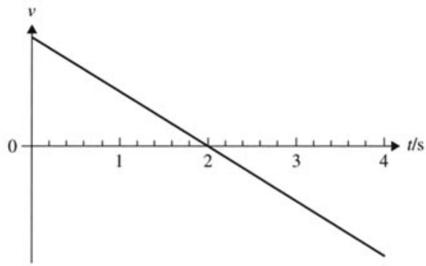

Figure 2.35 For question 28.

29 Figure 2.36 shows the variation of the velocity of a moving object with time. Draw the graph showing the variation of the acceleration of the object with time.

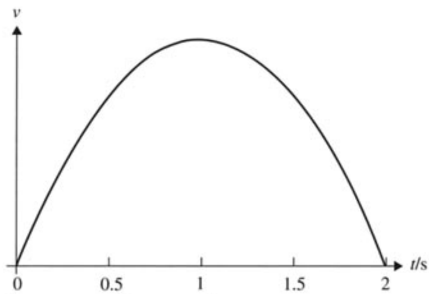

Figure 2.36 For question 29.

30 Your brand new convertible Ferrari is parked 15 m from its garage when it begins to rain. You do not have time to get the keys so you begin to push the car towards the garage. If the maximum acceleration you can give the car is 2.0 m s^{-2} by pushing and 3.0 m s^{-2} by pulling back on the car, find the least time it takes to put the car in the garage. (Assume that the car, as well as the garage, are point objects.)

31 Figure 2.37 shows the displacement versus time of an object moving in a straight line. Four points on this graph have been selected.
(a) Is the velocity between A and B positive, zero or negative?
(b) What can you say about the velocity between B and C?
(c) Is the acceleration between A and B positive, zero or negative?
(d) Is the acceleration between C and D positive, zero or negative?

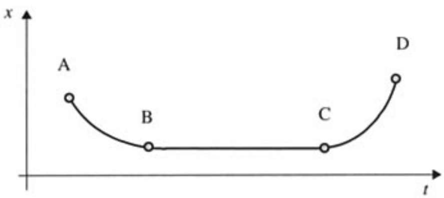

Figure 2.37 For question 31.

32 A hiker starts climbing a mountain at 08:00 in the morning and reaches the top at 12:00 (noon). He spends the night on the mountain and the next day at 08:00 starts on the way down following exactly the same path. He reaches the bottom of the mountain at 12:00. Prove that there must be a time between 08:00 and 12:00 when the hiker was at the same spot along the route on the way up and on the way down.

33 Make velocity–time sketches (no numbers are necessary on the axes) for the following motions.
(a) A ball is dropped from a certain height and bounces off a hard floor. The speed just before each impact with the floor is the same as the speed just after impact. Assume that the time of contact with the floor is negligibly small.
(b) A cart slides with negligible friction along a horizontal air track. When the cart hits the ends of the air track it reverses direction with the same speed it had right before impact. Assume the time of contact of the cart and the ends of the air track is negligibly small.
(c) A person jumps from a hovering helicopter. After a few seconds she opens a parachute. Eventually she will reach a terminal speed and will then land.

34 A cart with a sail on it is given an initial velocity and moves toward the right where, from some distance away, a fan blows air at the sail (see Figure 2.38). The fan is powerful enough to stop the cart before the cart reaches the position of the fan. Make a graph of the velocity of the cart as a function of time that best represents the motion just described. List any assumptions you made in drawing your graph.

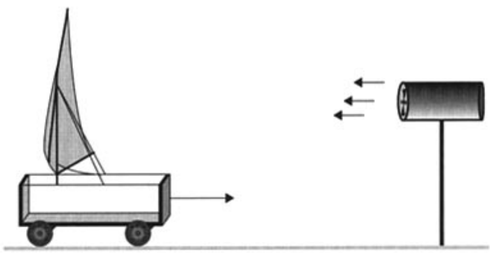

Figure 2.38 For question 34.

35 A stone is thrown vertically up from the edge of a cliff 35.0 m from the ground. The initial velocity of the stone is 8.00 m s^{-1}. (See Figure 2.39.)

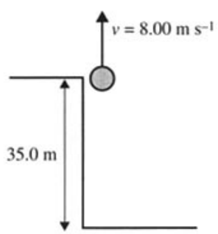

Figure 2.39 For question 35.

(a) How high will the stone get?
(b) When will it hit the ground?
(c) What velocity will it have just before hitting the ground?
(d) What distance will the stone have covered?
(e) What is the average speed and average velocity for this motion?
(f) Make a graph to show the variation of velocity with time.
(g) Make a graph to show the variation of displacement with time.

(Take the acceleration due to gravity to be $10.0\ \text{m s}^{-2}$.)

36 A ball is thrown upward from the edge of a cliff with velocity $20.0\ \text{m s}^{-1}$. It reaches the bottom of the cliff 6.0 s later.
(a) How high is the cliff?
(b) With what speed does the ball hit the ground?

37 A rocket accelerates vertically upwards from rest with a constant acceleration of $4.00\ \text{m s}^{-2}$. The fuel lasts for 5.00 s.
(a) What is the maximum height achieved by this rocket?
(b) When does the rocket reach the ground again?
(c) Sketch a graph to show the variation of the velocity of the rocket with time from the time of launch to the time it falls to the ground.

(Take the acceleration due to gravity to be $10.0\ \text{m s}^{-2}$.)

38 A hot air balloon is rising vertically at constant speed $5.0\ \text{m s}^{-1}$. A sandbag is released and it hits the ground 12.0 s later.
(a) With what speed does the sandbag hit the ground?
(b) How high was the balloon when the sandbag was released?
(c) What is the relative velocity of the sandbag with respect to the balloon 6.0 s after it was dropped?

(Assume that the balloon's velocity increased to $5.5\ \text{m s}^{-1}$ after releasing the sandbag. Take the acceleration due to gravity to be $10.0\ \text{m s}^{-2}$.)