The Doppler effect

This chapter looks at the Doppler effect, the change in frequency of a wave when there is relative motion between the source and the observer. The Doppler effect is a fundamental wave phenomenon with many applications. This chapter discusses this phenomenon quantitatively. The phenomenon applies to all waves but sound waves only are considered here.

Objectives

By the end of this chapter you should be able to:

- understand the *Doppler effect* in a qualitative way and explain it by drawing appropriate diagrams for a moving source or a moving observer;
- derive the *Doppler formula* for a moving source $f_o = \frac{f_s}{1-(v_s/c)}$ and a moving observer $f_o = f_s(1 + \frac{v_o}{c})$, and use these in solving problems;
- qualitatively *explain* the Doppler effect by suitable *wavefront diagrams*.

The Doppler effect

Consider a source of waves and an observer who receives them. If there is relative motion between the observer and the source (i.e. the source or the observer, or both, move) then, in general, the observer will receive the wave at a frequency that is *different* from the emitted frequency. This is a phenomenon of everyday life. For example, if an approaching car creates a high-pitched sound, as it goes past us and recedes the frequency of the sound becomes lower.

▶ The Doppler effect is the change in the frequency of a wave received by an observer, compared with the frequency with which it was emitted. The effect takes place whenever there is motion between the emitter and receiver.

We can understand the Doppler effect in terms of wavefront diagrams. Consider first a stationary source of waves emitting circular wavefronts (Figure 5.1). Suppose for simplicity that the source emits a wave of frequency f that travels with speed c. This means that f wavefronts are emitted per second. An observer who is also stationary will clearly receive f wavefronts every second as well, so there is no Doppler effect.

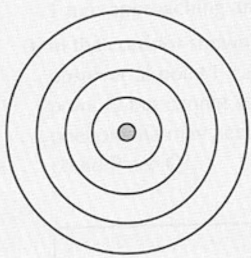

Figure 5.1 The wavefronts emitted by a stationary source are concentric. The common centre is the position of the source.

Now consider a stationary observer and a source of sound that moves with speed v_s ($<c$) towards the observer (Figure 5.2). The source emits sound of a single frequency f_s as measured by an

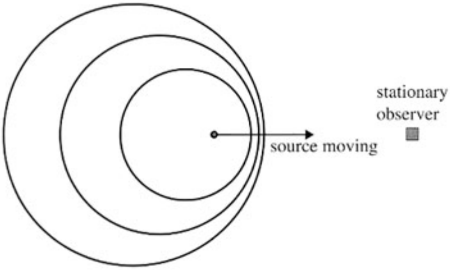

Figure 5.2 A source is approaching the stationary observer with speed v_s.

observer moving along with the source. (In what follows c stands for the speed of the wave, i.e. here for the speed of sound in still air and later on for the speed of light in a vacuum.)

In a time equal to one second, the source will therefore emit f_s wavefronts. In that same time interval, the source will move a distance equal to v_s towards the stationary observer (Figure 5.3).

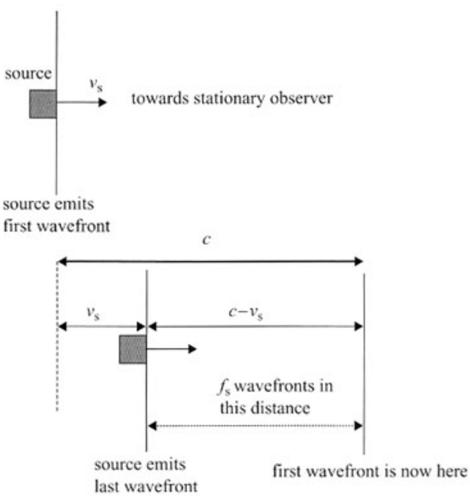

Figure 5.3 Determining the Doppler frequency.

Therefore, these f_s wavefronts are within a distance of $c - v_s$, and so the stationary observer will measure a wavelength (separation of wavefronts) equal to

$$\lambda_o = \frac{c - v_s}{f_s}$$

The frequency measured by the stationary observer is therefore

$$f_o = \frac{c}{\lambda_o} = \frac{c}{(c - v_s)/f_s} = f_s \frac{c}{c - v_s}$$

Dividing through by c gives

$$f_o = \frac{f_s}{1 - \frac{v_s}{c}} \quad \text{source moving towards observer}$$

As the source approaches, the stationary observer thus measures a higher frequency than that emitted by the source.

A similar calculation for the case of the source moving away from the stationary observer gives

$$f_o = \frac{f_s}{1 + \frac{v_s}{c}} \quad \text{source moving away from observer}$$

In the case of a stationary source and a moving observer we may argue as follows. First let us consider the case of the observer moving towards the source. The observer who moves with speed v_o with respect to the source may claim that he is at rest and that it is the source that approaches him with speed v_o. The observer will then measure a higher wave speed, equal to $c + v_o$. We are now back to the case of a moving source, and so the frequency measured by the observer is

$$f_o = \frac{f_s}{1 - \frac{v_o}{c + v_o}} = \frac{f_s(c + v_o)}{c + v_o - v_o} = \frac{f_s(c + v_o)}{c}$$

Dividing through by c gives

$$f_o = f_s\left(1 + \frac{v_o}{c}\right) \quad \text{observer moving towards source}$$

Similarly, if the observer moves away from the source we get

$$f_o = f_s\left(1 - \frac{v_o}{c}\right) \quad \text{observer moving away from source}$$

Notice carefully that, in the case of the moving source and the stationary observer, the wavelength measured by the observer, λ_o, is *different* from that measured by the source, λ_s. Consider the case of a source moving towards the observer:

$$\begin{aligned}\lambda_o &= \frac{c}{f_o}\\ &= \frac{c}{f_s/(1-\frac{v_s}{c})}\\ &= \frac{c}{f_s}\left(1-\frac{v_s}{c}\right)\\ \lambda_o &= \lambda_s\left(1-\frac{v_s}{c}\right)\end{aligned}$$

because

$$\lambda_s = \frac{c}{f_s}$$

However, in the case of the *moving observer* (towards the source for example):

$$\begin{aligned}\lambda_o &= \frac{c+v_o}{f_o}\\ &= \frac{c+v_o}{f_s/(1+\frac{v_o}{c})}\\ &= \frac{c}{f_s}\\ \lambda_o &= \lambda_s\end{aligned}$$

and is the *same* as that measured by the source.

This is why in defining the Doppler effect we refer to the change in *frequency* measured by the observer and not the change in wavelength.

The Doppler effect has many applications. One of the most common is to determine the speed of moving objects from cars on a highway (as the next Example question shows). Another application is to measure the speed of flow of blood cells in an artery.

Example questions

Q1

A sound wave of frequency 300 Hz is emitted towards an approaching car. The wave is reflected from the car and is then received back at the emitter at a frequency of 315 Hz. What is the velocity of the car? (Take the speed of sound to be 340 m s^{-1}.)

Answer

The car is approaching the emitter so the frequency it receives is

$$f_1 = 300 \times \frac{340+u}{340}\ \text{Hz}$$

where u is the unknown car speed. The car now acts as an emitter of a wave of this frequency (f_1), and the original emitter will act as the new receiver. Thus, the frequency received (315 Hz) is (car is approaching)

$$315 = \left(300 \times \frac{340+u}{340}\right) \times \frac{340}{340-u}$$

from which we find $u = 8.29$ m s^{-1}.

Q2

A train with a 500 Hz siren on is moving at a constant speed of 8.0 m s^{-1} in a straight line. An observer is in front of the train and off its line of motion. What frequencies does the observer hear? (Take the speed of sound to be 340 m s^{-1}.)

Answer

What counts is the velocity of the train along the *line of sight* between the train and the observer. When the train is *very far away* (Figure 5.4) it essentially comes straight towards the observer

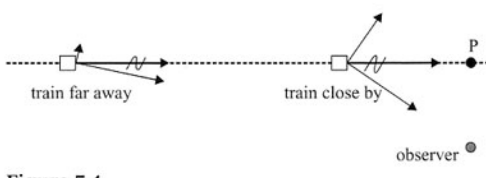

Figure 5.4.

and so the frequency received is

$$f_o = f_s \frac{c}{c - v_s}$$
$$= 500 \times \frac{340}{340 - 8}$$
$$\approx 510 \text{ Hz}$$

When the train is again *very far away* to the right, the train is moving away from the observer and the frequency received will be

$$f_o = f_s \frac{c}{c + v_s}$$
$$= 500 \times \frac{340}{340 + 8}$$
$$\approx 490 \text{ Hz}$$

As the train approaches, we take components of the train's velocity vector in the direction along the line of sight and the direction normal to it (see Figure 5.4).

As is seen from the diagram, the component along the line of sight is decreasing as the train gets closer to the observer. Thus, the observer will measure a *decreasing* frequency. It starts at 510 Hz and falls to 500 Hz when the train is at position P. As the train moves past P to the right, the observer will hear sound of *decreasing* frequency starting at 500 Hz and ending at 490 Hz.

Thus, the observer hears frequencies in the range of 510 Hz to 490 Hz, as shown in Figure 5.5.

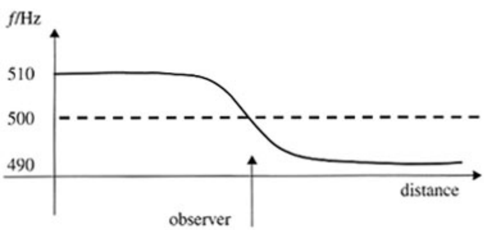

Figure 5.5.

The Doppler effect also applies to light, but the equations giving the frequency observed are more complicated. However, in the case in which the speed of the source or the observer is *small* compared to the speed of light, the equations take a simple form:

$$\Delta f = \frac{v}{c} f \quad \text{for light only}$$

In this formula v is the speed of the source or the observer, c is the speed of light and f and λ stand for the frequency and wavelength emitted. Then Δf gives the change in the observed frequency (and $\Delta\lambda$ the change in the observed wavelength).

Example question

Q3

Hydrogen atoms in a distant galaxy emit light of wavelength 658 nm. The light received on earth is measured to have a wavelength of 689 nm. State whether the galaxy is approaching the earth or moving away, and calculate the speed of the galaxy.

Answer

The received wavelength is longer than that emitted, and so the galaxy is moving away from earth.

The emitted frequency is

$$f = \frac{c}{\lambda}$$
$$= \frac{3.00 \times 10^8}{658 \times 10^{-9}}$$
$$= 4.56 \times 10^{14} \text{ Hz}$$

and the received frequency is

$$f = \frac{c}{\lambda}$$
$$= \frac{3.00 \times 10^8}{689 \times 10^{-9}}$$
$$= 4.35 \times 10^{14} \text{ Hz}$$

giving a shift of $\Delta f = 0.21 \times 10^{14}$ Hz. Hence the speed is found as follows:

$$\Delta f = \frac{v}{c} f \Rightarrow v = \frac{c\Delta f}{f}$$
$$v = \frac{3.00 \times 10^8 \times 0.21 \times 10^{14}}{4.56 \times 10^{14}}$$
$$v = 1.4 \times 10^7 \text{ m s}^{-1}$$

Questions

Take the speed of sound to be 343 m s^{-1} in all the problems that follow.

1 A source of sound is directed at an approaching car. The sound is reflected by the car and is received back at the source. *Carefully* explain what changes in frequency the observer at the source will detect.

2 Light from a nearby galaxy is emitted at a wavelength of 657 nm and is observed on earth at a wavelength of 654 nm. What can we deduce about the motion of this galaxy?

3 Explain, with the help of diagrams, the Doppler effect. Show clearly the cases of a source that (a) moves towards and (b) goes away from a stationary observer as well the case of a moving observer.

4 A source approaches a stationary observer at 40 m s^{-1} emitting sound of frequency 500 Hz. What frequency does the observer measure?

5 A source is moving away from a stationary observer at 32 m s^{-1} emitting sound of frequency 480 Hz. What frequency does the observer measure?

6 A sound wave of frequency 512 Hz is emitted by a stationary source toward an observer who is moving away at 12 m s^{-1}. What frequency does the observer measure?

7 A sound wave of frequency 628 Hz is emitted by a stationary source toward an observer who is approaching at 25 m s^{-1}. What frequency does the observer measure?

8 A sound wave of frequency 500 Hz is emitted by a stationary source toward a receding observer. The signal is reflected by the observer and received by the source, where the frequency is measured and found to be 480 Hz. What is the speed of the observer?

9 A sound wave of frequency 500 Hz is emitted by a moving source toward a stationary observer. The signal is reflected by the observer and received by the source, where the frequency is measured and found to be 512 Hz. What is the speed of the source?

10 A disc rotates about its axis with constant angular velocity. A point on the rim moves with a speed of 7.5 m s^{-1}. Sound of frequency 500.0 Hz is emitted from a source on the circumference of the disc in directions parallel to the source's velocity as shown in Figure 5.6, and is received by an observer very far away from the disc. What frequencies does the observer measure?

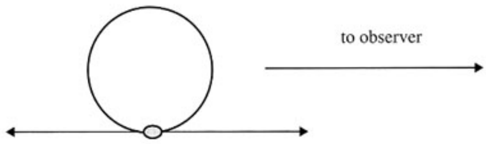

Figure 5.6 For question 10.

11 Consider the general case when both the source and the observer move. Let v_s be the velocity of the source and v_o that of the observer. In the frame of reference in which the observer is at rest, the waves appear to move with velocity $c + v_o$ and the source appears to move with velocity $v_s + v_o$. Thus, show that the frequency received by the observer is

$$f_o = f_s \frac{c + v_o}{c - v_s}$$

12 Consider a source moving away from a stationary observer with speed v. The source emits waves of speed c and wavelength λ_s. Explain why the observer will measure a *longer* wavelength for the waves received and show that the *shift* in wavelength $\Delta\lambda = \lambda_o - \lambda_s$ obeys $\frac{\Delta\lambda}{\lambda_s} = \frac{v}{c}$.

13 A source of sound emits waves of frequency f towards an object moving away from the source. The waves are reflected by the object and are received back at the source. The speed of the object is v.

(a) Deduce that the frequency of the reflected waves as measured by an observer at the source is given by

$$f' = f\frac{1 - \frac{v}{c}}{1 + \frac{v}{c}}$$

(b) If $\frac{v}{c}$ is small, it can be shown in mathematics that

$$\frac{1}{1 + \frac{v}{c}} \approx 1 - \frac{v}{c}$$

Deduce that the magnitude of the frequency shift measured by the observer at the source becomes

$$\Delta f = \frac{2v}{c} f$$

(c) Ultrasound of frequency 5.000 MHz reflected from red blood cells moving in an artery is found to show a frequency shift of 2.4 kHz. The speed of ultrasound in blood is 1500 m s^{-1}.
(i) Estimate the speed of the blood cells.
(ii) In practice, a range of frequency shifts is observed. Explain this observation.

14 The sun rotates about its axis with a period that may be assumed to be constant at 27 days. The radius of the sun is 7.00×10^8 m. Discuss the shifts in frequency of light emitted from the sun's equator and received on earth. Assume that the sun emits monochromatic light of wavelength 5.00×10^{-7} m.

15 The human ear can detect frequencies in the range of about 20 Hz to 20 kHz. A source of sound moves towards and then away from a stationary observer. Describe qualitatively the changes, if any, in the frequency of sound heard by the observer when the source emits
(a) sound of a single frequency 500 Hz;
(b) sound with frequency in the range 500 Hz to 1000 Hz;
(c) all frequencies covering the entire audible range of the observer.

16 In a binary star system, two stars orbit a common point and move so that they are always in diametrically opposite positions. Light from both stars reaches an observer on earth. Assume that both stars emit light of wavelength 6.58×10^{-7} m.

(a) When the stars are in the position shown in Figure 5.7, the observer on earth measures a wavelength of light of 6.58×10^{-7} m from both stars. Explain why there is no Doppler shift in this case.

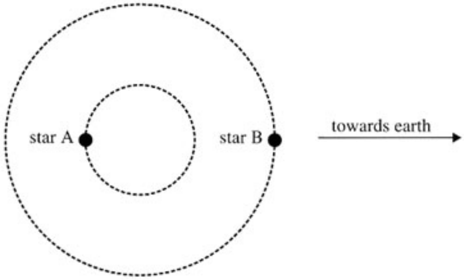

Figure 5.7 For question 16(a).

(b) When the stars are in the position shown in Figure 5.8, the earth observer measures two wavelengths in the received light, 6.50×10^{-7} m and 6.76×10^{-7} m. Determine the speed of each of the stars.

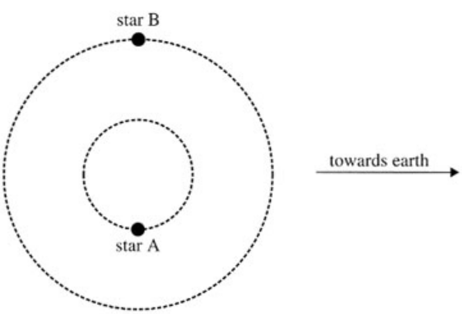

Figure 5.8 For question 16(b).

17 A source of sound emits waves of frequency 850 Hz in all directions as it approaches and then recedes from an observer close to its path. The power of the sound emitted is constant.
(a) Draw a sketch graph (no numbers required) to show the variation with time of the intensity of the sound heard by the observer.

The observer is 4.0 m away from the line of motion of the source. The source moves at a constant speed of 12 m s^{-1}, and its initial position is 24 m away, as shown in Figure 5.9.

(b) Draw a detailed graph to show the variation with time of the frequency of the sound heard by the observer.

(c) How does your graph in (b) change if the source is moving with constant acceleration? (Assume that the acceleration is 2.0 m s^{-2}, the initial position is the same (24 m away) and the initial velocity is 10 m s^{-1}.) Draw a detailed graph and explain the shape you have drawn.

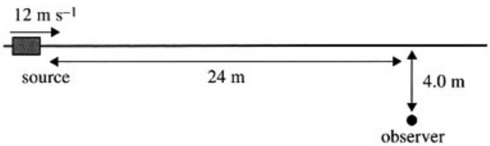

Figure 5.9 For question 17.

18 Sound of frequency 530 Hz is emitted by a stationary source. An observer approaching the source at high speed receives the sound and measures a frequency of 580 Hz.

(a) Determine the speed of the observer.

(b) Calculate the wavelength of the sound as measured by

(i) the source;

(ii) the observer.

Take the speed of sound in still air to be 340 m s^{-1}.

19 (a) The shift in frequency due to a source of light moving at speed v and emitting light of frequency f is given by

$$\Delta f = \frac{v}{c} f$$

Using the approximation (valid if $\frac{v}{c}$ is small)

$$\frac{1}{1 \pm \frac{v}{c}} \approx 1 \mp \frac{v}{c}$$

show that the shift in wavelength is given by

$$\Delta \lambda = \frac{v}{c} \lambda$$

where λ is the emitted wavelength.

(b) Calculate the speed of a galaxy emitting light of wavelength 5.48×10^{-7} m which when received on earth is measured to have a wavelength of 5.65×10^{-7} m.