Electric circuits

This chapter explains how simple electric circuits can be solved: that is, how the current through and potential difference across resistors can be determined. The chapter begins with the concept of emf (which stands for electromotive force). The name is unfortunate, as emf is a potential difference and not a force. Hence, we always use the initials emf and never the full name. The chapter ends with a look at the potential divider circuit and sensors that use it.

Objectives

By the end of this chapter you should be able to:

- define *emf* and explain the role of *internal resistance* – the potential difference across a battery is $V = \mathcal{E} - Ir$;
- find the total resistance in *series* and *parallel* connections using $R_{\text{total}} = R_1 + R_2 + \cdots$ and $\frac{1}{R_{\text{total}}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots$;
- find the current through, and potential difference across, resistors in *simple circuits*;
- find the *power dissipated* by a resistor in a circuit using $P = VI$;
- describe the *potential divider*;
- explain the use of *sensors* in potential divider circuits.

Emf

Charges will not drift in the same direction inside a conductor unless a potential difference is established at the ends of the conductor. There are many ways of providing *a source of potential difference* to the circuit. The most common is the connection of a *battery* in the circuit. Others include a *generator*, a *thermocouple* and a *photosurface*. To understand the function of the battery, we can use the standard analogy in which the battery is likened to a pump that forces water through pipes up to a certain height and down again (see Figure 5.1). The gravitational force does work equal to $-mgh$ in lifting a mass m of water up to the height h, and work equal to $+mgh$ on the way down. The net work done by the gravitational force is thus zero. Because frictional forces are present, work must be done by the pump to compensate for the work done by these forces. In the absence of the pump, the water flow would stop.

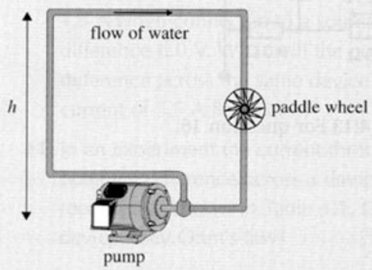


Figure 5.1 In the absence of the pump, the water flow would stop. The work done by the pump equals the work done to overcome frictional forces plus work done to operate devices, such as, for example, a paddle wheel.

If, in addition, the water drives a machine to perform useful work (for example, by turning a paddle wheel), then the pump would have to do work to allow for that as well.

In an electric circuit a battery performs a role similar to the pump's. A battery converts the energy it stores (chemical energy) into electrical energy. The work done by the electrical forces on a charge that moves in the circuit is zero, just as the net work done by gravitational force in the pump and water system described above is zero. Similarly, if a generator is used, the energy that gets converted into electrical energy is the mechanical energy that turns the coils of the generator. In the case of the thermocouple, it is thermal energy. In the case of the photosurface, it is solar energy.

In a battery, the electrons must be pushed from the positive to the negative terminal, which means work must be done *on the electrons* (see Figure 5.2).

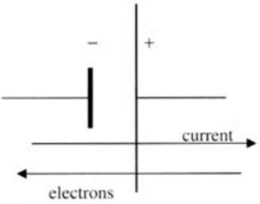


Figure 5.2.

This is what we mean by **emf**:

▶ In the case of the battery, the ratio of work done by non-electrical forces, W, to a quantity of charge q that moves from one terminal of the battery to the other is called the emf of the battery. Emf thus has units of electric potential, i.e. volts:

$$\mathcal{E} = \text{emf} = \frac{W}{q}$$

Suppose we connect a voltmeter to the ends of a battery. We may assign the value of 0 V to the negative terminal of the battery. Then the positive terminal has a potential equal to the emf, $\mathcal{E}$. The chemicals inside the battery create a small resistance r, called the internal resistance of the cell. We cannot isolate this resistance – it is inside the battery and we may assume that it is connected in series to the cell. If the current that leaves the battery is I, then the potential difference across the internal resistance is Ir. In other words, the internal resistance reduces the voltage from a value of $\mathcal{E}$ on its left side to the value $\mathcal{E}-Ir$ on the right side. The potential difference across the battery is therefore

$$V = \mathcal{E} - Ir$$

We see that $V = \mathcal{E}$ when $I = 0$. This gives an alternative and less precise definition of the emf: the emf is the potential difference across the battery when the battery sends out zero current. (See Figure 5.3.)

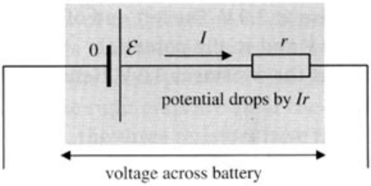


Figure 5.3 The potential difference across the battery terminals is less than the emf of the battery.

Example question

Q1

A battery of emf 12 V and internal resistance $r = 1.5\ \Omega$ produces a current of 3.0 A. What is the potential difference across the battery terminals?

Answer

We find

$$\begin{aligned} V &= \mathcal{E} - Ir \\ &= 12 - 3 \times 1.5 \\ &= 7.5\ \text{V} \end{aligned}$$

In Figure 5.4, a battery forces a current I into a circuit that contains a resistor of resistance R. The connecting wires are assumed to have zero resistance.

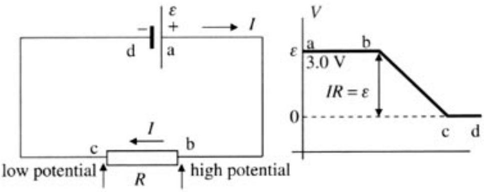


Figure 5.4 A battery connected to a circuit. The current flows into the circuit away from the positive terminal. This is the conventional definition of current. The electrons actually move in the opposite way.

If the emf of the battery in the circuit is 3.0 V (neglecting its internal resistance) and the resistance of the circuit is 1.5 Ω, the current can easily be determined. The positive terminal of the battery may be taken to be 3.0 V and so the negative terminal must be taken to be at 0 V (to give an emf of 3 V). Thus, the right end of the resistor is also at 3.0 V. The left end of the resistor is at 0.0 V and so the potential difference across the resistor is 3.0 V. Hence, the current is 2.0 A.

Simple electric circuits

A simple circuit will consist of a single battery and a number of resistors. When we talk about solving a circuit, we mean finding the current through and voltage across every resistor in the circuit. Here we will develop the methods to do just that. Table 5.1 shows the circuit symbols that you need.

Series circuits

First, let us consider a part of a circuit consisting of a number of resistors connected in *series*. This means that the resistors have the same current through them. An example with three resistors is shown in Figure 5.5. Let I be the common current in the three resistors.

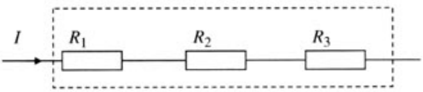


Figure 5.5 Three resistors in series.

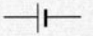
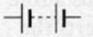
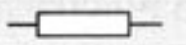
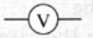
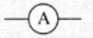
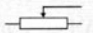
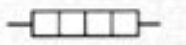
Symbols	Component name
	connection lead
	cell
	battery of cells
	resistor
	power supply
	junction of conductors
	crossing conductors (no connection)
	filament lamp
	voltmeter
	ammeter
	switch
	ac supply
	galvanometer
	potentiometer
	heating element

Table 5.1 Names of electrical components and their circuit symbols.

The potential difference across the resistors is

$$V_1 = I R_1, \quad V_2 = I R_2 \quad \text{and} \quad V_3 = IR_3$$

The sum of the potential differences is thus

$$V = IR_1 + IR_2 + IR_3 = I(R_1 + R_2 + R_3)$$

If we were to replace the three resistors by a *single* resistor of value $R_1 + R_2 + R_3$ (in other words, if we were to replace the contents of the dotted box in Figure 5.5 with a single resistor, as in the circuit shown in Figure 5.6), we would not be able to tell the difference. The *same current* flows into the dotted box and the *same potential difference* exists across its ends. We thus define the equivalent or total resistance of the three resistors of Figure 5.5 by

$$R_{\text{total}} = R_1 + R_2 + R_3$$

(If more than three were present, we would simply add all of them. The formula shows that the total resistance is larger than the individual ones being added.)

In a circuit, the combination of resistors of Figure 5.5 is equivalent to the single total or equivalent resistor. Suppose we now connect the three resistors to a battery of negligible internal resistance and emf equal to 24 V. Suppose that $R_1 = 2\,\Omega$, $R_2 = 6\,\Omega$ and $R_3 = 4\,\Omega$. The circuit is shown in the top diagram of Figure 5.6. Note that we know that the potential at point A is 24 V and at point B it is 0 V. (We *do not* know the potential difference across any of the three resistors individually.) In the bottom diagram, we have replaced the three resistors by the equivalent resistor of $R_{\text{total}} = 2 + 6 + 4 = 12\,\Omega$. We now observe that the potential difference across the equivalent resistor is known. It is simply 24 V and hence the current through the equivalent resistor is found as follows:

$$R = \frac{V}{I}$$
$$\Rightarrow I = \frac{V}{R} = \frac{24}{12} = 2\,\text{A}$$

This current, therefore, is also the current that enters the dotted box: that is, it is the current in each of the three resistors of the original circuit. We may thus deduce that the potential differences across the three resistors are

$$V_1 = IR_1 = 4\,\text{V}$$
$$V_2 = IR_2 = 12\,\text{V}$$
$$V_3 = IR_3 = 8\,\text{V}$$

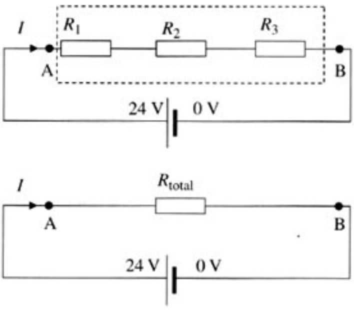


Figure 5.6 The top circuit is replaced by the equivalent circuit containing just one resistor.

Suppose now that we cannot neglect the internal resistance of the battery. The internal resistance is connected in series to the other resistances and so, if its value is $r = 1.0\,\Omega$, the total circuit resistance is $1 + 2 + 6 + 4 = 13\,\Omega$. The current leaving the battery is thus $\frac{24}{13} = 1.85$ A. The potential difference across the battery terminals is

$$V = \mathcal{E} - Ir$$
$$= 24 - 1.85 \times 1$$
$$= 22.15\,\text{V}$$

which is less than the emf, as we expected.

Parallel circuits

Consider now part of another circuit, in which the current splits into three other currents that flow in three resistors, as shown in Figure 5.7. The current that enters the junction at A must equal the current that leaves the junction, by the law of conservation of charge. Furthermore, we note that the left ends of the three resistors are at the same potential (the potential at A) and the right ends are all at the potential of B. Hence, the three resistors have the same potential difference across them. This is called a *parallel* connection.

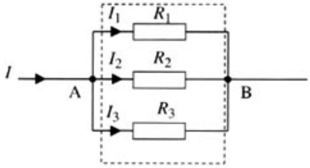


Figure 5.7 Three resistors connected in parallel.

We must then have that

$$I = I_1 + I_2 + I_3$$

Let V be the common potential difference across the resistors. Then

$$I_1 = \frac{V}{R_1}, \quad I_2 = \frac{V}{R_2} \quad \text{and} \quad I_3 = \frac{V}{R_3}$$

and so

$$I = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$
$$= V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

If we replace the three resistors in the dotted box with a single resistor, the potential difference across it would be V and the current through it would be I. Thus

$$I = \frac{V}{R_{\text{total}}}$$

Comparing with the last equation, we find

$$\frac{1}{R_{\text{total}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

The formula shows that the total resistance is *smaller* than any of the individual resistances being added.

▶ We have thus learned how to replace resistors that are connected in series (same current) or parallel (same potential difference across) by a single resistor in each case, thus greatly simplifying the circuit.

A typical circuit will contain both series and parallel connections. In Figure 5.8, the two top resistors are in series. They are equivalent to a single resistor of 12 Ω. This resistor and the 6 Ω resistor are in parallel, so together they are equivalent to a single resistor of

$$\frac{1}{R_{\text{total}}} = \frac{1}{12} + \frac{1}{6} = \frac{1}{4}$$

$$\Rightarrow R_{\text{total}} = 4\ \Omega$$

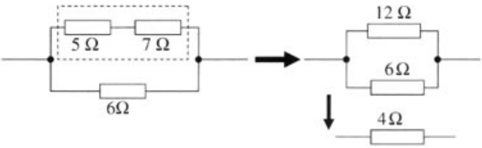


Figure 5.8 Part of a circuit with both series and parallel connections.

Consider now Figure 5.9. The two top 6 Ω resistors are in series, so they are equivalent to a 12 Ω resistor. This, in turn, is in parallel with the other 6 Ω resistor, so the left block is equivalent to

$$\frac{1}{R_{\text{total}}} = \frac{1}{12} + \frac{1}{6} = \frac{1}{4}$$

$$\Rightarrow R_{\text{total}} = 4\ \Omega$$

Let us go to the right block. The 12 Ω and the 24 Ω resistors are in series, so they are equivalent to 36 Ω. This is in parallel with the top 12 Ω, so the equivalent resistor of the right block is

$$\frac{1}{R_{\text{total}}} = \frac{1}{36} + \frac{1}{12} = \frac{1}{9}$$

$$\Rightarrow R_{\text{total}} = 9\ \Omega$$

The overall resistance is thus

$$4 + 9 = 13\ \Omega$$

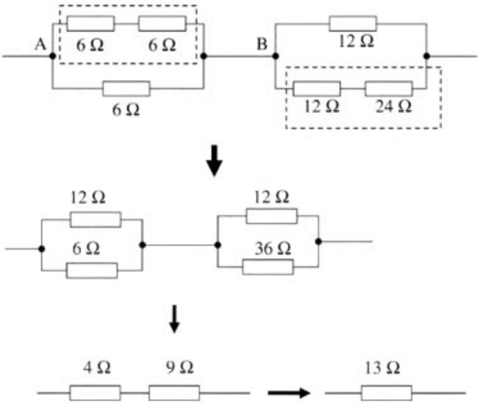


Figure 5.9 A complicated part of a circuit containing many parallel and series connections.

Suppose now that this part of the circuit is connected to a source of emf 156 V (and negligible internal resistance). The current that leaves the source is $I = \frac{156}{13} = 12$ A. When it arrives at point A, it will split into two parts. Let the current in the top part be I_1 and that in the bottom part I_2. We have $I_1 + I_2 = 12$ A. We also have that $12I_1 = 6I_2$, since the top and bottom resistors of the block beginning at point A are in parallel and so have the same potential difference across them. Thus, $I_1 = 4$ A and $I_2 = 8$ A. Similarly, in the block beginning at point B the top current is 9 A and the bottom current is 3 A.

Example questions

Q2

Find the total resistance in each of the circuits shown in Figure 5.10.

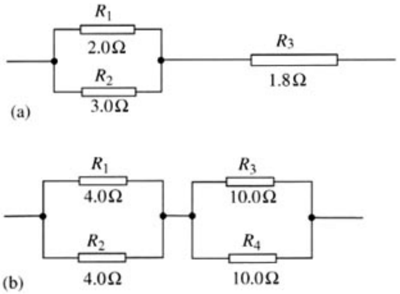


Figure 5.10.

Answer

(a) R_1 and R_2 are in parallel, so together they are equivalent to a resistor R where

$$\frac{1}{R} = \frac{1}{2.0} + \frac{1}{3.0}$$
$$= \frac{5.0}{6.0}$$
$$\Rightarrow R = \frac{6.0}{5.0}$$
$$= 1.2\,\Omega$$

Now, this R is in series with R_3, so together they are equivalent to

$$R_{total} = (1.2 + 1.8)\,\Omega$$
$$= 3.0\,\Omega$$

(b) R_1 and R_2 are in parallel, so together they are equivalent to a resistor R where

$$\frac{1}{R} = \frac{1}{4.0} + \frac{1}{4.0}$$
$$= \frac{1.0}{2.0}$$
$$\Rightarrow R = \frac{2.0}{1.0}$$
$$= 2.0\,\Omega$$

Similarly, R_3 and R_4 are in parallel so they are equivalent to a resistor of 5.0 Ω. The 2.0 Ω and 5.0 Ω are in series, so the overall total is 7.0 Ω.

Q3

What is the total current in the circuit in Figure 5.11?

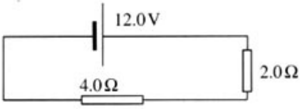


Figure 5.11.

Answer

The emf of the battery is 12 V. The total resistance of the circuit is 2.0 + 4.0 = 6.0 Ω. Thus, the total current is

$$I = \frac{12.0}{6.0}\,\text{A}$$
$$= 2.0\ \text{A}$$

Q4

What is the potential difference across each resistor in Example question 3?

Answer

The current through the 2.0 Ω resistor is 2.0 A, so the potential difference across it is $RI = 4.0$ V. Across the other resistor it is $RI = 4.0 \times 2.0$ V = 8.0 V. Note that the sum of the potential differences across each resistor adds up to the emf of the battery.

Q5

Find the current in each of the resistors in the circuit shown in Figure 5.12.

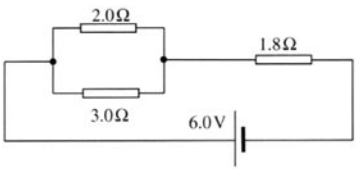


Figure 5.12.

Answer

The resistors of 2.0 Ω and 3.0 Ω are connected in parallel and are equivalent to a single resistor of resistance R found from

$$\frac{1}{R} = \frac{1}{2} + \frac{1}{3}$$
$$= \frac{5}{6}$$
$$\Rightarrow R = \frac{6}{5}$$
$$= 1.2\ \Omega$$

In turn, this is in series with the resistance of 1.8 Ω, so the total equivalent circuit resistance is $1.8 + 1.2 = 3.0\ \Omega$. The current that leaves the battery is thus

$$I = \frac{6.0}{3.0}$$
$$= 2.0\ \text{A}$$

The potential difference across the 1.8 Ω resistor is thus $V = 1.8 \times 2.0 = 3.6$ V, leading to a potential difference across the two parallel resistors of $V = 6.0 - 3.6 = 2.4$ V. Thus the current in the 2 Ω resistor is

$$I = \frac{2.4}{2.0}$$
$$= 1.2\ \text{A}$$

and in the 3 Ω resistor is

$$I = \frac{2.4}{3.0}$$
$$= 0.80\ \text{A}$$

As a check, we see that $1.2 + 0.80 = 2.0$ A, as it should be.

Q6

Find the current in each resistor in the circuit in Figure 5.13.

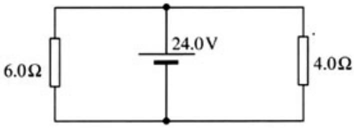


Figure 5.13.

Answer

The voltage across the 4.0 Ω resistor is 24.0 V and thus the current is 6.0 A. The voltage is 24.0 V across the other resistor as well, and so the current through it is 4.0 A. The current leaving the battery is 10.0 A.

Q7

Look at Figure 5.14. What is the potential difference between A and B? What is the current leaving the battery?

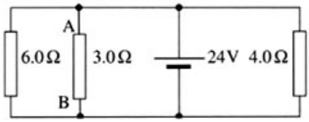


Figure 5.14.

Answer

The potential difference is 24 V for all resistors. The currents in the resistors are 8 A, 6 A and 4 A, respectively. The total current is thus 18 A.

Q8

Look at Figure 5.15. What is the current in the 2.0 Ω resistor when the switch is open and when the switch is closed? What is the potential difference across the two marked points, A and B, when the switch is open and when the switch is closed?

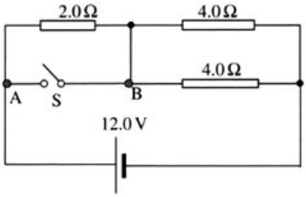


Figure 5.15.

Answer

When the switch is open, the total resistance is 4.0 Ω and thus the total current is 3.0 A. This is the current through the 2.0 Ω resistor. The potential at A is 12 V. The potential difference across the 2.0 Ω resistor is $2 \times 3 = 6$ V and so the potential at its right end, and hence at B, is 6 V. The potential difference across points A and B is thus 6 V.

When the switch is closed, no current flows through the 2.0 Ω resistor, since all the current takes the path through the switch, which offers no resistance. (The 2.0 Ω resistor has been *shorted out*.) The resistance of the circuit is then 2.0 Ω and the current leaving the battery is 6 A. The potential difference across points A and B is now zero. There is current flowing from A to B, but the resistance from A to B is zero. Hence the potential difference is $6 \times 0 = 0$ V.

Q9

Four light bulbs each of constant resistance 60 Ω are connected as shown in Figure 5.16. Find the power in each light bulb. If light bulb A burns out, find the power in each light bulb and the potential difference across the burned-out light bulb.

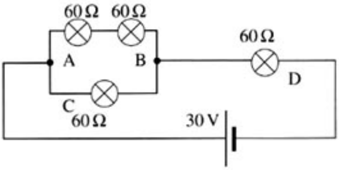


Figure 5.16.

Answer

A and B are connected in series so they are equivalent to one resistor of value $R = 60 + 60 = 120$ Ω. This is connected in parallel to C, giving a total resistance of

$$\frac{1}{R} = \frac{1}{120} + \frac{1}{60} = \frac{1}{40}$$

$$\Rightarrow R = 40\ \Omega$$

Finally, this is in series with D, giving a total circuit resistance of $R = 40 + 60 = 100$ Ω. The current leaving the battery is thus $I = \frac{30}{100} = 0.3$ A. The current through A and B is 0.1 A and that through C is 0.2 A. The current through D is 0.3 A. Hence the power in each light bulb is

$$P_A = P_B = 60 \times (0.1)^2 = 0.6\ \text{W}$$

$$P_C = 60 \times (0.2)^2 = 2.4\ \text{W}$$

$$P_D = 60 \times (0.3)^2 = 5.4\ \text{W}$$

With light bulb A burnt out, the circuit is as shown in Figure 5.17.

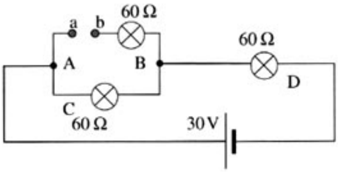


Figure 5.17.

Light bulb B gets no current, so we are left with only C and D connected in series, giving a total resistance of $R = 60 + 60 = 120$ Ω. The current is thus $I = 0.25$ A. The power in C and D is thus

$$P_C = P_D = 60 \times (0.25)^2 = 3.75\ \text{W}$$

We see that D becomes dimmer and C brighter. The potential at point a is 30 V. The potential difference across light bulb C is

$$V = IR = 0.25 \times 60 = 15\ \text{V}$$

and so the potential at the right end of C is 15 V. Light bulb B takes no current, so the potential difference across it is zero. Thus, the potential at point b is also 15 V. The potential difference across points a and b is therefore 15 V.

Ammeters and voltmeters

The current through a resistor is measured by an instrument called an ammeter, which is connected in series to the resistor as shown in Figure 5.18.

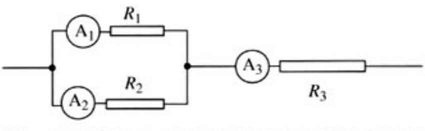


Figure 5.18 An ammeter measures the current in the resistor connected in series to it.

The ammeter itself has a small electric resistance. However, an *ideal ammeter has zero resistance* and throughout this book we are assuming that we are dealing with ideal ammeters.

Example question

Q10

How are the readings of the ammeters of Figure 5.18 related?

Answer

$I_3 = I_1 + I_2$

The potential difference across the ends of a resistor is measured by a voltmeter, which is connected in parallel to the resistor, as shown in Figure 5.19.

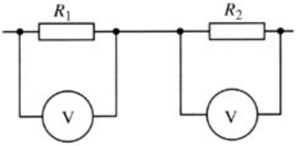


Figure 5.19 A voltmeter measures the potential difference across a resistor it is connected in parallel to.

Thus, to measure the potential difference across and current through a resistor, the arrangement shown in Figure 5.20 is used.

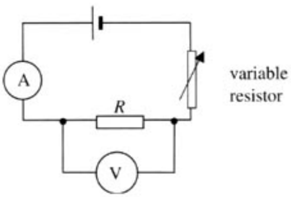


Figure 5.20 The correct arrangement for measuring the current through and potential difference across a resistor. The variable resistor allows the current in the resistor R to be varied.

An ideal voltmeter has infinite resistance (in practice about 50 000 Ω), which means that it takes no current when it is connected to a resistor.

Supplementary material

Voltmeters and ammeters are both based on a current sensor called a galvanometer. An ammeter has a small resistance connected in parallel to the galvanometer and a voltmeter is a galvanometer connected to a large resistance in series.

Example question

Q11

In the circuit in Figure 5.21, the emf of the battery is 9.00 V and the internal resistance is assumed negligible. A voltmeter whose resistance is 500 Ω is connected in parallel to a resistor of 500 Ω. What is the reading of the ammeter? If we assume that the current registered by the ammeter actually flows into the resistor, what value of the resistance would we measure? Repeat this calculation, this time assuming that the voltmeter's resistance is 5000 Ω.

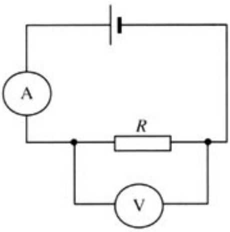


Figure 5.21.

Answer

The total resistance of the circuit is 250 Ω and so the current that leaves the battery is 36.0 mA. If this current is assumed to flow in the resistor, the resistance would be measured as $\frac{9.0\text{ V}}{36\text{ mA}} = 250\ \Omega$. With the higher voltmeter resistance, the total circuit resistance is 454.5 Ω. The current flowing is then $\frac{9.0\text{ V}}{454.5\ \Omega} = 19.8\text{ mA}$. If we assume all of this current goes into the resistance, the resistance would be measured as 454.5 Ω. In other words, what the experimental arrangement actually measures is not the resistance of the resistor R but the total resistance of R and the voltmeter's resistance. The higher the voltmeter resistance, the closer the total is to R.

Sensors based on the potential divider

The potential divider

The circuit in Figure 5.22(a) shows a potential divider. It can be used to investigate, for example, the current–voltage characteristic of some device denoted by resistance R. This complicated-looking circuit is simply equivalent to the circuit in Figure 5.22(b). In this circuit, the resistance R_1 is the resistance of the variable resistor XY from end X to the slider S, and R_2 is the resistance of the variable resistor from S to end Y. The current that leaves the battery splits at point M. Part of the current goes from M to N, and the rest goes into the device with resistance R. The right end of the resistance R can be connected to a point S on the variable resistor XY.

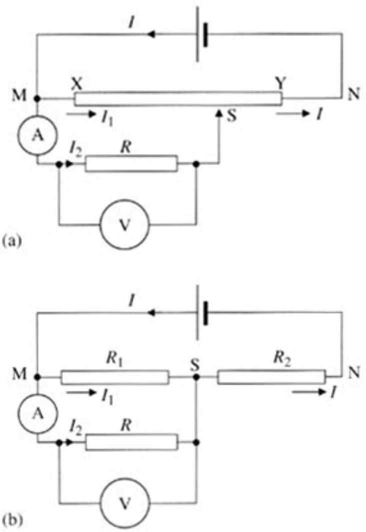


Figure 5.22 (a) This circuit uses a potential divider. The voltage and current in the device with resistance R can be varied by varying the point where the slider S is attached to the variable resistor. (b) The potential divider circuit is equivalent to this simpler-looking circuit.

By varying where the slider S connects to XY, different potential differences and currents are obtained for the device R. The variable resistor XY could also be just a wire of uniform diameter. One advantage of the potential divider over the conventional circuit arrangement (Figure 5.20) is that now the potential difference across the resistor can be varied from a minimum of zero volts, when the slider S is placed at X, to a maximum of $\mathcal{E}$, the emf of the battery (assuming zero internal resistance), by connecting the slider S to point Y. In the conventional arrangement of Figure 5.20, the voltage can be varied from zero volts up to some maximum value *less than* the emf.

Example question

Q12

In the circuit in Figure 5.23, the battery has emf $\mathcal{E}$ and negligible internal resistance. Derive an expression for the potential difference V across resistor R_1.

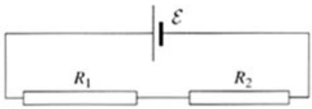


Figure 5.23.

Answer

Since $V = IR_1$ and $I = \frac{\mathcal{E}}{R_1+R_2}$,

we have that

$$V = \frac{R_1}{R_1 + R_2}\mathcal{E}$$

Using sensors

This section includes a use of a particular sensor, a light-dependent resistor in a circuit. Other examples using the potential divider circuit discussed earlier can also be used with various other types of sensor, for example strain gauges and temperature-dependent resistors. A few examples are given in the questions at the end of the chapter.

Consider the circuit in Figure 5.24 that contains a light-dependent resistor (LDR). An LDR is a resistor whose resistance decreases as the light falling on the resistor increases. Typically, the resistance is 100 Ω in bright light and more than 1.0 MΩ in the dark. A voltmeter is connected across the LDR. Because the resistance of the LDR

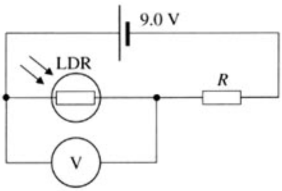


Figure 5.24 A light-dependent resistor in a potential divider circuit.

changes with varying intensity of incident light, the reading of the voltmeter across the LDR also changes as in a potential divider circuit.

The reading of the voltmeter across the LDR is

$$V = \frac{R_{LDR}}{R_{LDR} + R} \times 9.0 \text{ volts}$$

Assume that the LDR has a resistance of 900 kΩ when dark and 100 Ω when bright. With a fixed resistor of resistance $R = 500$ kΩ, the reading of the voltmeter is then:

- Dark

$$V = \frac{900 \times 10^3}{900 \times 10^3 + 500 \times 10^3} \times 9.0 \text{ volts}$$
$$= 5.8 \text{ volts}$$

- Bright

$$V = \frac{100}{100 + 500 \times 10^3} \times 9.0 \text{ volts}$$
$$= 1.8 \times 10^{-3} \text{volts}$$

The reading of the voltmeter is a measure of the illumination of the LDR and can therefore be used as a light sensor. A high value means the LDR is dark, and a very small value means the LDR is bright.

To have a sensitive sensor, we would like to have as large a difference as possible in the readings of the voltmeter for a dark and a bright LDR. This depends on the particular value of the fixed resistor chosen in relation to the dark and bright resistances of the LDR. Using your graphics calculator, you should be able to show that, with the numbers used here, the value of R resulting in the largest difference in the dark and bright readings of the voltmeter is about 9.5 kΩ.

Supplementary material

The mathematically inclined should be able to show that the value of R resulting in the largest possible difference in the dark and bright readings of the voltmeter equals $R = \sqrt{R_D R_B}$, where R_D and R_B are the resistances of the LDR in the dark and bright.

Questions

1 Find the total resistance for each of the circuit parts in Figure 5.25.

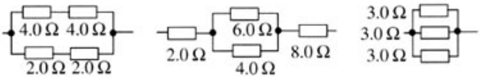


Figure 5.25 For question 1.

2 What is the resistance between A and B in Figure 5.26?

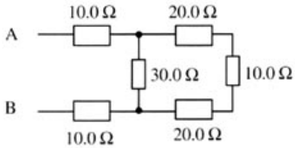


Figure 5.26 For question 2.

3 Each resistor in Figure 5.27 has a value of 6.0 Ω. Calculate the resistance of the combination.

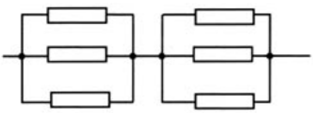


Figure 5.27 For question 3.

4 You are given one hundred 1 Ω resistors. What is the smallest and largest resistance you can make in a circuit using these?

5 A wire that has resistance R is cut into two equal pieces. The two parts are joined in parallel. What is the resistance of the combination?

6 Find the current in, and potential difference across, each resistor in the circuits shown in Figure 5.28.

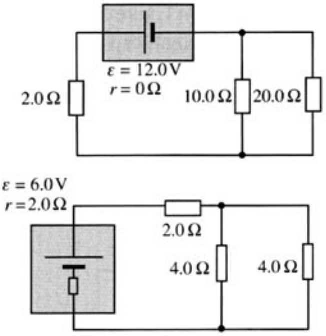


Figure 5.28 For question 6.

7 A battery has emf = 10.0 V and internal resistance 2.0 Ω. The battery is connected in series to a resistance R. Make a table of the power dissipated in R for various values of R and then use your table to plot the power as a function of R. For what value of R is the power dissipated maximum?

8 Six light bulbs, each of constant resistance 3.0 Ω, are connected in parallel to a battery of emf = 9.0 V and negligible internal resistance. The brightness of a light bulb is proportional to the power dissipated in it. Compare the brightness of one light bulb when all six are on, to that when only five are on, the sixth having burned out.

9 A toaster is rated as 1200 W and a mixer as 500 W, both at 220 V.

(a) If both appliances are connected (in parallel) to a 220 V source, what current does each appliance draw?

(b) How much energy do these appliances use if both work for one hour?

10 Find the current in each of the resistors in the circuit shown in Figure 5.29. What is the total power dissipated in the circuit?

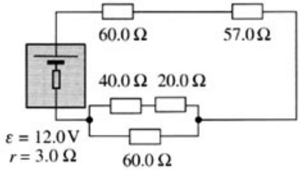


Figure 5.29 For question 10.

11 An electric kettle rated as 2000 W at 220 V is used to warm 2.0 L of water from 15 °C to 90 °C.

(a) How much current flows in the kettle?

(b) What is the resistance of the kettle?

(c) How long does it take to warm the water? (Specific heat capacity of water = 4200 J kg^{-1} K^{-1}.)

(d) How much does this cost if the power company charges \$0.10 per kW h?

12 One light bulb is rated as 60 W at 220 V and another as 75 W at 220 V.

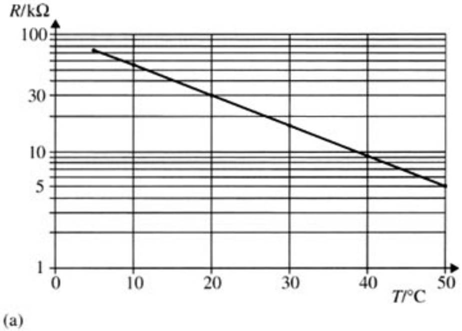
(a) If both of these are connected in parallel to a 110 V source, find the current in each light bulb. (Assume that the resistances of the light bulbs are constant.)

(b) Would it cost more or less (and by how much) to run these two light bulbs connected in parallel to a 110 V or a 220 V source?

13 Three appliances are connected (in parallel) to the same outlet, which provides a voltage of 220 V. A fuse connected to the outlet will blow if the current drawn from the outlet exceeds 10 A. If the three appliances are rated as 60 W, 500 W and 1200 W at 220 V, will the fuse blow?

14 An electric kettle rated as 1200 W at 220 V and a toaster rated at 1000 W at 220 V are both connected in parallel to a source of 220 V. If the fuse connected to the source blows when the current exceeds 9.0 A, can both appliances be used at the same time?

15 The graph in Figure 5.30a shows the temperature dependence of a special resistor *R*. The resistance drops with increasing temperature.



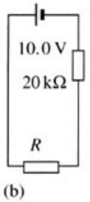


Figure 5.30 For question 15.

(a) Estimate the resistance of this resistor at 20 °C.

(b) If this resistor is connected in a circuit as shown in Figure 5.30b, find the current in the resistor when the temperature is 20 °C.

16 The temperature-dependent resistor of question 15 is connected in a circuit to a lamp of resistance 10 kΩ as shown in Figure 5.31. What will happen to the brightness of the lamp if the temperature of the room increases from 20 °C to 30 °C?

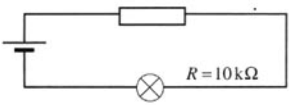


Figure 5.31 For question 16.

17 A clothes dryer operates at 220 V and draws a current of 20.0 A.

(a) What is the power of the machine?

(b) If the dryer is filled with wet clothes that contain 2.0 kg of water at 40 °C, how long will it take to dry them? (The specific heat capacity of water is 4200 J kg^{-1} K^{-1} and the specific latent heat of vaporization of water is 2257 kJ kg^{-1}.) Ignore any heat absorbed by the clothes themselves.

18 In the *potentiometer* in Figure 5.32 wire AB is uniform and has a length of 1.00 m. When contact is made at C with BC = 54.0 cm, the galvanometer G shows zero current. What is the emf of the second cell?

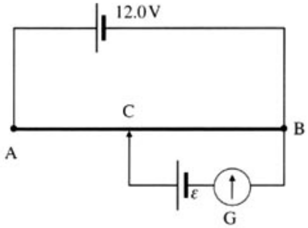


Figure 5.32 For question 18.

19 Two light bulbs are rated as 60 W and 75 W at 220 V. If these are connected in series to a source of 220 V, what will the power in each be? Assume a constant resistance for the light bulbs.

20 At a given time a home is supplied with 100.0 A at 220 V. How many 75 W (rated at 220 V) light bulbs could be on in the house at that time, assuming they are all connected in parallel?

21 (a) What is the reading of the voltmeter in the circuit shown in Figure 5.33 if both resistances are 200 Ω and the voltmeter also has a resistance of 200 Ω?

(b) What is the reading of the ammeter?

(c) If the voltmeter was ideal, what would the readings of the voltmeter and ammeter be?

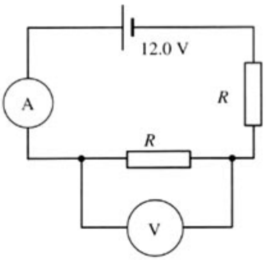


Figure 5.33 For question 21.

22 For the circuit shown in Figure 5.34, find the current taken from the supply.

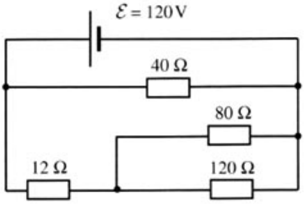


Figure 5.34 For question 22.

23 A direct current supply of constant emf 12.0 V and internal resistance 0.50 Ω is connected to a load of constant resistance 8.0 Ω. Find (a) the power dissipated in the load resistance and (b) the energy lost in the internal resistance in 10 min.

24 Consider the circuit in Figure 5.35, where A, B and C are three identical light bulbs of constant resistance. The battery has negligible internal resistance.

(a) Order the light bulbs in order of increasing brightness.

(b) If C burns out, what will be the brightness of A now compared with before?

(c) If B burns out instead, what will be the brightness of A and C compared with before?

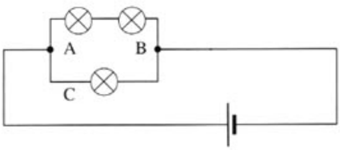


Figure 5.35 For question 24.

25 (a) Determine the potential difference across each resistor in the circuit in Figure 5.36.

(b) A voltmeter of resistance 2 kΩ is connected in parallel across the 3 kΩ resistor. What is the reading of the voltmeter?

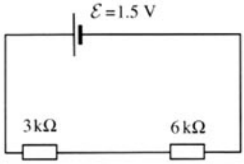


Figure 5.36 For question 25.

26 A battery of emf $\mathcal{E}$ and internal resistance r sends a current I into a circuit.

(a) Sketch the potential difference across the battery as a function of the current.

(b) What is the significance of (i) the slope and (ii) the vertical intercept of the graph?

HL only

27 Each resistor in the circuit shown in Figure 5.37 has value R and the circuit extends to the right forever. Find the total resistance between A and B.

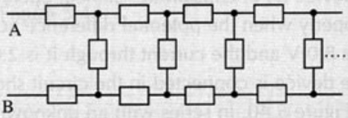


Figure 5.37 For question 27.

28 Twelve 1.0 Ω resistors are placed on the edges of a cube and connected to a 5.0 V battery, as shown in Figure 5.38. What is the current leaving the battery?

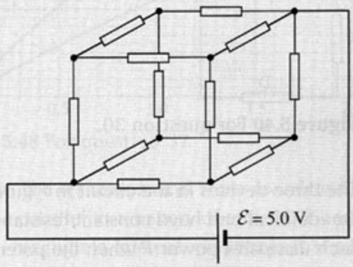


Figure 5.38 For question 28.

29 Two identical lamps, each of constant resistance *R*, are connected as shown in the circuit on the left in Figure 5.39. A third identical lamp is connected in parallel to the other two.

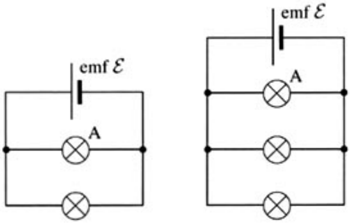


Figure 5.39 For question 29.

Compare the brightness of lamp A in the original circuit (left) with its brightness in the circuit with three lamps (right), when

(a) the battery has no internal resistance, and

(b) the battery has an internal resistance equal to *R*.

30 A device D, of constant resistance, operates properly when the potential difference across it is 8.0 V and the current through it is 2.0 A. The device is connected in the circuit shown in Figure 5.40, in series with an unknown resistance *R*. Calculate the value of the resistance *R*. (The battery has negligible internal resistance.)

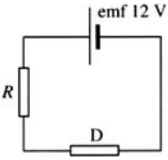


Figure 5.40 For question 30.

31 The three devices in the circuit in Figure 5.41 are identical and have constant resistance. Each dissipates power *P* when the potential difference across it is $\mathcal{E}$. (The battery has negligible internal resistance.)

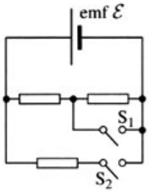


Figure 5.41 For question 31.

Calculate the total power dissipated in the circuit when

(a) S_1 is closed and S_2 is open;

(b) S_1 is closed and S_2 is closed;

(c) S_1 is open and S_2 is open;

(d) S_1 is open and S_2 is closed.

32 Two identical lamps are connected to a battery of emf 12 V and negligible internal resistance, as shown in Figure 5.42. Calculate the reading of the (ideal) voltmeter when lamp B burns out.

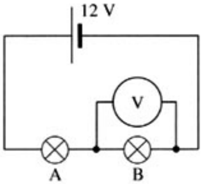


Figure 5.42 For question 32.

33 State the reading of the ideal voltmeter in the circuit in Figure 5.43.

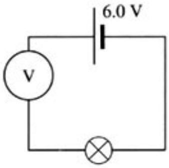


Figure 5.43 For question 33.

34 In an experiment, a voltmeter was connected across the terminals of a battery as shown in Figure 5.44.

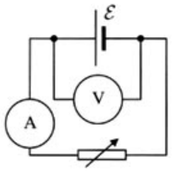


Figure 5.44 For question 34.

The current in the circuit is varied using the variable resistor. The graph in Figure 5.45 shows the variation with current of the reading of the voltmeter.

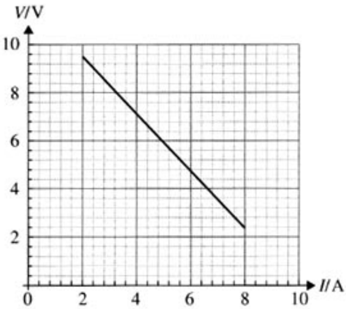


Figure 5.45 For question 34.

(a) Calculate the internal resistance of the battery.
(b) Calculate the emf of the battery.

35 Two resistors are connected in series as shown in Figure 5.46. The battery has negligible internal resistance. Resistor R has a constant resistance of 1.5 Ω.

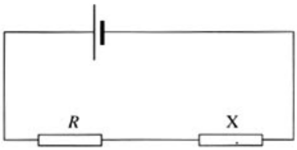


Figure 5.46 For question 35.

The current–voltage (I–V) characteristic of resistance X is shown in Figure 5.47.

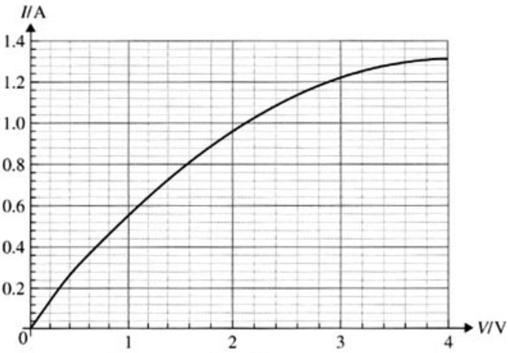


Figure 5.47 For question 35.

The potential difference across resistor R is 1.2 V. Calculate the emf of the battery.

36 When two resistors, each of resistance 4.0 Ω, are connected in parallel with a battery, the current leaving the battery is 3.0 A. When the same two resistors are connected in series with the battery, the total current in the circuit is 1.4 A. Calculate
(a) the emf of the battery;
(b) the internal resistance of the battery.

37 Two resistors, X and Y, have I–V characteristics given by the graph in Figure 5.48.

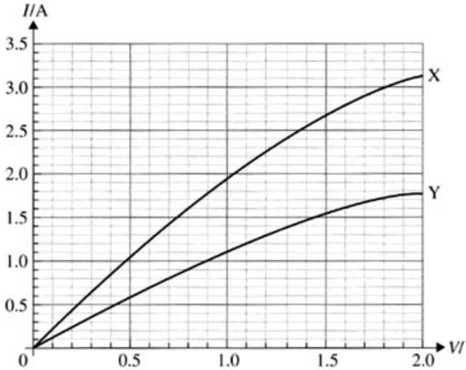


Figure 5.48 For question 37.

(a) The resistors X and Y are connected in parallel to a battery of emf 1.5 V and negligible internal resistance, as shown in Figure 5.49(a). Calculate the total current leaving the battery.

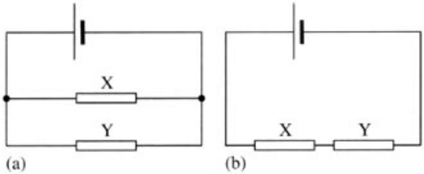


Figure 5.49 For question 37.

(b) In Figure 5.49(b) the resistors X and Y are connected in series. Estimate the total current leaving the battery in this circuit.

38 The circuit in Figure 5.50 contains a positive temperature coefficient (PTC) resistor whose resistance increases with increasing temperature, and a negative temperature coefficient (NTC) resistor whose resistance decreases with increasing temperature.

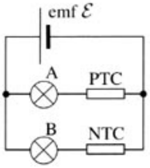


Figure 5.50 For question 38.

At room temperature the lamps (which are identical) are equally bright. Determine the changes, if any, in the brightness of lamps A and B when the temperature is increased. (The battery has negligible internal resistance.)

39 Figure 5.51 shows an NTC resistor (the resistance decreases with increasing temperature) in a circuit.

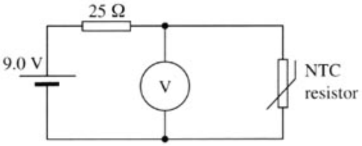


Figure 5.51 For question 39.

Figure 5.52 shows the variation with temperature T of the resistance of the NTC resistor.

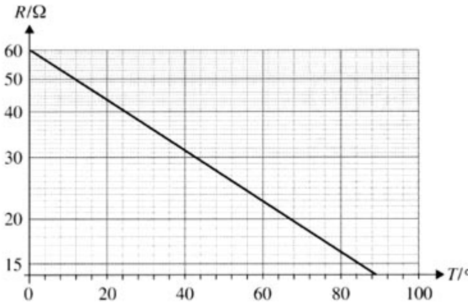


Figure 5.52 For question 39.

(a) State the resistance of the NTC resistor at a temperature of 25 °C.

(b) Deduce that the reading of the voltmeter, in volts, is given by

$$V = \frac{9.0 \times R_{NTC}}{R_{NTC} + 25}$$

where R_{NTC} is the resistance of the NTC resistor in ohms.

(c) Calculate the reading of the (ideal) voltmeter at 25 °C.

(d) The NTC resistor may be used as a temperature sensor. Describe how this circuit may be used to measure the temperature to which the NTC resistor is exposed.

40 (a) Calculate the potential difference between points A and B in the circuit in Figure 5.53. (The battery has negligible internal resistance.)

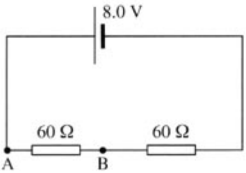


Figure 5.53 For question 40.

A lamp of constant resistance operates at normal brightness when the potential difference across it is 4.0 V and the current through it is 0.20 A. To light up the lamp, a student uses the circuit shown in Figure 5.54.

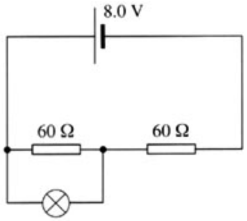


Figure 5.54 For question 40.

(b) Calculate the resistance of the light bulb at normal brightness.

(c) Calculate the potential difference across the light bulb in the circuit in Figure 5.54.

(d) Calculate the current through the light bulb.

(e) Hence explain why the light bulb will not light.

41 The circuit in Figure 5.55 contains a strain gauge, S. The resistance of S when it is not under stress is 100 Ω. The emf of the battery is 6.00 V. (The battery has negligible internal resistance.)

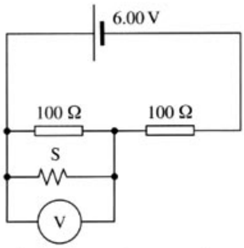


Figure 5.55 For question 41.

(a) Calculate the reading of the voltmeter when the strain gauge S is not under stress.

(b) When the strain gauge is under a certain load, its resistance increases to 110 Ω. Calculate the reading of the voltmeter now.