

# Magnetic fields

We have all observed with fascination how magnets attract or repel each other and we are familiar with compasses, which align with the magnetic field of the earth (a fact that has been used for navigation for hundreds of years). But it was only in 1820 that scientists began to understand the cause of magnetism, when the Danish scientist H. C. Ørsted discovered that a wire in which electric current was flowing influenced a magnetic needle placed near the wire. It was thus discovered that the origin of magnetism is electrical. The magnetic field of the earth is presumably caused by moving charges in the interior of the earth and the magnetic field of an iron bar magnet is caused by the motion of electrons in the atoms of the iron. Thus, electric currents cause forces on magnets and, as we will see in this chapter, magnets cause forces on electric currents as well – a result that we might expect from Newton's third law.

## Objectives

By the end of this chapter you should be able to:

- understand the meaning of *magnetic field* and find its magnitude and direction in simple situations involving *straight-line conductors* ( $B = \frac{\mu_0 I}{2\pi r}$ ) and *solenoids* ( $B = \mu_0 \frac{NI}{l}$ ) using the right-hand rule where appropriate;
- find the *force on moving charges* ( $F = qvB \sin \theta$ ) and *currents* ( $F = BIl \sin \theta$ ) in magnetic fields and appreciate the definition of the *ampere* as a fundamental SI unit, using the right-hand rule for forces where appropriate.

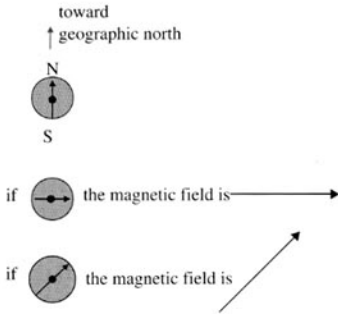
## Magnetic field

In the chapters on electricity, it was useful to introduce the concept of an electric field. A charge creates an electric field around itself and any other charge that enters this electric field will experience, as a result, an electric force. The same idea can be extended to magnetism. Both magnets and electric currents create magnetic fields around themselves and when another magnet or electric current (or moving charge in general)

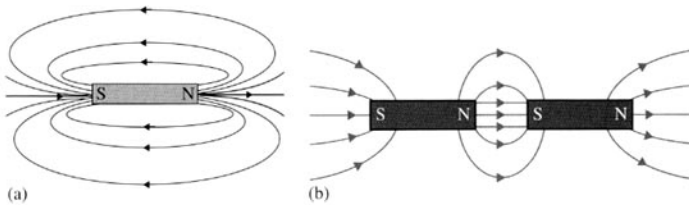
enters this magnetic field it will experience a *magnetic force*. The magnetic field is a vector quantity just like the electric field – it has magnitude and direction.

### The direction of the magnetic field

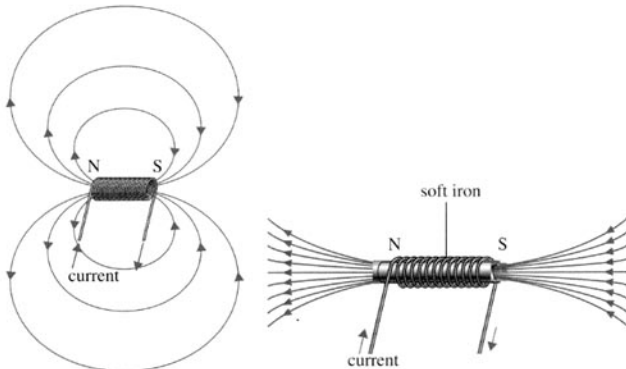
The magnetic field direction is determined by the effect it has on a compass needle (i.e. a small bar magnet), as shown in Figure 6.1. A magnetic needle aligns itself in the direction of the magnetic field vector.



**Figure 6.1** A magnetic needle is a small bar magnet whose north pole points in the direction of the geographic north pole of the earth. In the presence of another strong magnet, the needle will align itself with the magnetic field.



**Figure 6.2** (a) The magnetic field lines of a bar magnet. The field is strongest near the poles of the magnet where the lines crowd together. (b) A uniform magnetic field is obtained if two opposite poles are placed near each other.



**Figure 6.3** The magnetic field lines of a solenoid. The field is fairly uniform in the interior of the coil. Outside it resembles that of a bar magnet.

## Magnetic field lines

Just like electric field lines, magnetic field lines are defined as imaginary lines around magnets and currents, tangents to which give the direction of the magnetic field. The magnetic field lines of permanent magnets and the field created by a solenoid (a coil of wire in which electric current flows) are shown in Figures 6.2 and 6.3.

In Figure 6.3, current is flowing in a solenoid and a magnetic field is created inside and outside the solenoid. The current is flowing in the clockwise direction if we look along the axis of the solenoid from right to left.

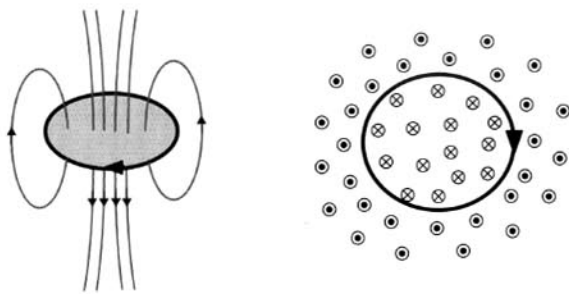
The magnetic field of a single loop of wire in which current flows is somewhat more

complicated and is shown in Figure 6.4. In the right-hand diagram, we are looking at the loop 'from above'; the crosses indicate that the magnetic field is directed into the page while the dots indicate a magnetic field coming out of the page.

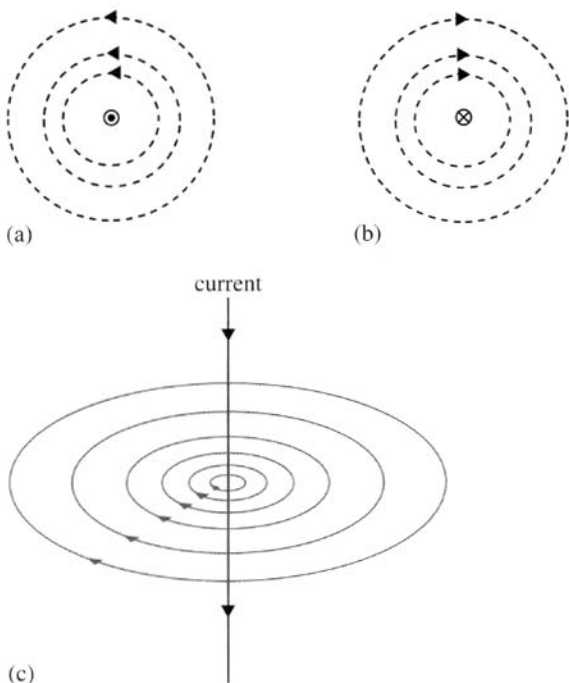
Figure 6.5 shows the magnetic field lines of a long straight wire. In Figure 6.5a, the current is coming out of the page. The magnetic field lines are circles centred at the wire. In Figure 6.5b, the current goes into the page. Remember that the magnetic field direction is tangent to the magnetic field lines and the arrows on the lines tell us which tangent to take.

The magnetic field of the earth resembles that of a bar magnet except that the bar magnet does not coincide with the line through the geographic north and south poles of the earth.

The direction of the magnetic field caused by a given current (a few examples of which we have seen in this section) is given by a right-hand rule, which we will describe later.



**Figure 6.4** The magnetic field lines of a single turn of wire. In the plane of the loop the magnetic field is going into the page inside the loop and out of the page outside the loop. The current in the loop is flowing in the clockwise direction.



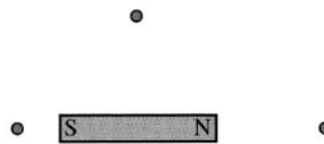
**Figure 6.5** Magnetic field lines for straight current wires. The magnetic field magnitude is largest near the wire.

**Example question**

**Q1**

A magnetic monopole is a particle that is a pure north or pure south magnetic pole. (These are predicted to exist by modern theories of elementary particle physics but none have been found.) Suppose that a *south* magnetic monopole is placed at various positions in the vicinity of a bar magnet, as shown in Figure 6.6. Draw the

force experienced by the monopole at the positions shown.



**Figure 6.6.**

**Answer**

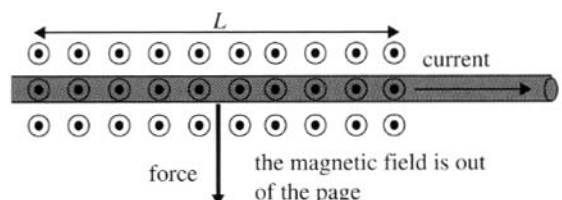
The force on a north monopole would be in the same direction as the magnetic field direction at the position of the monopole. The force on a south monopole would be opposite to the direction of the magnetic field. Thus, the forces on the south monopole are as shown in Figure 6.7.



**Figure 6.7.**

**The magnetic force on a current**

If a current is placed in a region of magnetic field, it will experience a magnetic force. In Figure 6.8 a magnetic field is established out of the page and a wire carries a current from left to right, perpendicular to the magnetic field. The magnitude of the force is proportional to the current  $I$ , the magnetic field magnitude  $B$  and the length  $L$  of the wire that is in the magnetic field.



**Figure 6.8** A current in a magnetic field experiences a magnetic force. The force is on that part of the wire that is in the magnetic field.

Mathematically

$$F \propto BIL$$

$$\Rightarrow F = kBIL$$

where  $k$  is a constant of proportionality. This constant can be made to equal one by proper choice of the unit of magnetic field. We can make  $k = 1$  by saying that when the force on 1 m of wire carrying a current of 1 A is 1 N, then the magnitude of the magnetic field is defined to be 1 tesla (so  $1 \text{ T} = 1 \text{ N A}^{-1} \text{ m}^{-1}$ ). So the force on the current-carrying wire is

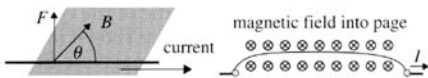
$$F = BIL$$

Remember, though, that the magnetic field was at right angles to the wire. If there is an angle between them then:

► The force on a length  $L$  of the wire is given by

$$F = BIL \sin \theta$$

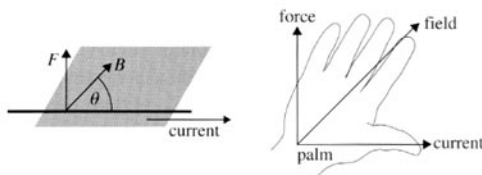
where  $\theta$  is the angle between the current and the direction of the magnetic field



**Figure 6.9** The force on a current-carrying wire in a magnetic field is normal to the plane containing the field and the current. If the ends of the wire are kept fixed, the wire will bend.

The formula above gives the magnitude of the force on the wire. The direction of the magnetic force is always normal to both the current and the magnetic field: that is, it is normal to the plane containing the current and the magnetic field vectors (see Figure 6.9). To find this direction we use a right-hand rule for force which says:

► Using the right hand place the thumb in the direction of the current and the fingers in the direction of the magnetic field. The direction away from the palm is the direction of the magnetic force. (See Figure 6.10.)



**Figure 6.10** The magnetic force on a current is normal to both the magnetic field and the current direction. Its direction is given by a right-hand rule for force.

### Supplementary material

Those familiar with the vector product of two vectors may recognize that the equation for the magnetic force is

$$\vec{F} = I\vec{L} \times \vec{B}$$

This equation correctly gives the magnitude as well as the direction of the force.

## The magnetic force on a moving charge

An electric current that is in a magnetic field will experience a force as we just saw. But an electric current is just moving charges, so a moving charge will experience a magnetic force as well.

Consider a positive charge  $q$  that moves with speed  $v$  to the right. In time  $\Delta t$  the charge will move a distance  $L = v \Delta t$ . The current created by this charge is  $I = \frac{q}{\Delta t}$ , so the force on this current is

$$F = BIL \sin \theta$$

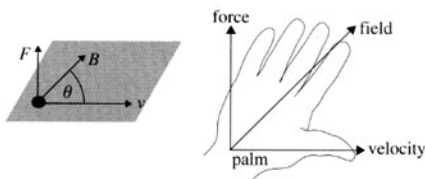
$$= B \frac{q}{\Delta t} v \Delta t \sin \theta$$

$$= qvB \sin \theta$$

► A charge  $q$  moving with speed  $v$  in a magnetic field of magnitude  $B$  will experience a force  $F$  given by

$$F = qvB \sin \theta$$

where  $\theta$  is the angle between the direction of the velocity and the magnetic field. (See Figure 6.11.)



**Figure 6.11** An electric charge moving in a magnetic field experiences a magnetic force. The charge shown is positive. The right hand is placed palm up on the page with the thumb pointing in the direction of the velocity and the fingers pointing in the direction of the magnetic field. The direction away from the palm (i.e. out of the page) is the direction of the force on a positive charge.

This implies that the magnetic force is zero if the charge moves parallel or antiparallel to the magnetic field. There is also no magnetic force if the charge is not moving. This is to be contrasted with the electric force on a charge, which is always non-zero irrespective of whether the charge moves or not. The magnetic force on particles that are electrically neutral ( $q = 0$ ) is, of course, zero.

### Supplementary material

Those familiar with the vector product of two vectors may recognize that the equation for the magnetic force is

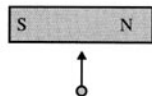
$$\vec{F} = q\vec{v} \times \vec{B}$$

This equation correctly gives the magnitude as well as the direction of the force.

### Example question

Q2

An electron approaches a bar magnet as shown in Figure 6.12. What is the direction of the force on the electron?



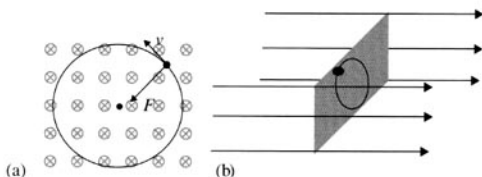
**Figure 6.12.**

### Answer

The magnetic field at the position of the electron is to the left. Placing the right hand such that the thumb points up the page (velocity direction) and the fingers to the left (field direction), the palm is pointing out of the page. But the charge is negative and so the force is into the page.

### Motion of charges in magnetic fields

The fact that the magnetic force on a charge is always normal to the velocity means that the path of a charge in a magnetic field must be a circle, as shown in Figure 6.13 (the path can also be helical – see question 21 at the end of the chapter).



**Figure 6.13** A charge in a magnetic field moves in a circle, as shown in (a). The plane of the circle is normal to the magnetic field, as shown in (b). (The charge here is positive.) The magnetic field is into the page in (a).

Consider a charge  $q$  moving with speed  $v$  in a magnetic field  $B$ . Assume that the charge's velocity is normal to the magnetic field, then the force on the charge is  $F = qvB$  and so by Newton's second law

$$qvB = m\frac{v^2}{R}$$

where  $R$  is the radius of the circle the charge will move on. Therefore

$$R = \frac{mv}{qB}$$

Very massive or very fast charges will move on large circles; large charges and large magnetic fields will result in small circles. The time to make one full revolution in a magnetic field is

found from

$$\begin{aligned} T &= \frac{2\pi R}{v} \\ &= \frac{2\pi mv}{v qB} \\ &= \frac{2\pi m}{qB} \end{aligned}$$

and is thus independent of the speed. This is an important result in experimental particle physics and forms the basis for an accelerator called the cyclotron.

### Work done and magnetic forces

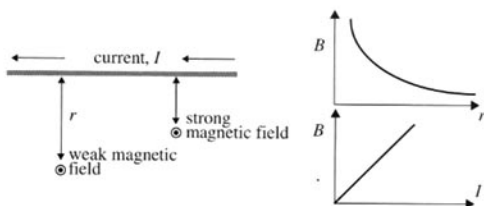
Since the magnetic force is always normal to the velocity of the charge, it follows that it cannot do any work. The big magnets in particle accelerators are used only to deflect particles not to increase the particles' kinetic energy (this job is done by electric fields).

## Ørsted's discovery

A current in a straight long wire produces a magnetic field around it. The Danish scientist H. C. Ørsted found that:

▶ The magnitude of the magnetic field  $B$  created by the current in a wire varies linearly with the current in the wire and inversely with the perpendicular distance from the wire (see Figure 6.14). In equation form

$$B = \frac{\mu_0 I}{2\pi r}$$



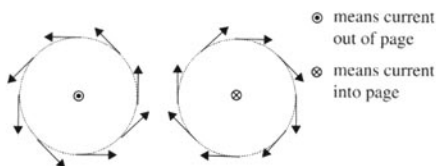
**Figure 6.14** The magnitude of the magnetic field vector is inversely proportional to the distance from the wire. The magnetic field is directly proportional to the current.

The constant of proportionality involves the new physical constant  $\mu_0$ , which is called the *magnetic permeability of vacuum*. If the wire is surrounded by something other than a vacuum, the appropriate permeability of that medium must be used in the formula above. The value of the magnetic permeability of the vacuum is (exactly)

$$\mu_0 = 4\pi \times 10^{-7} \text{ NA}^{-2}$$

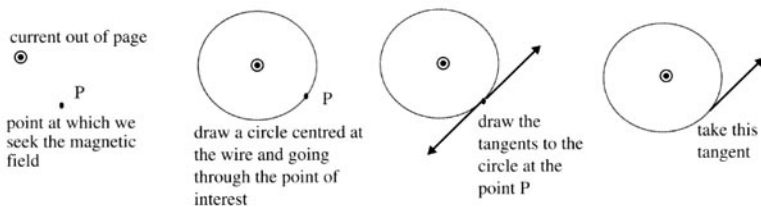
It is the analogue in magnetism of the electric permittivity  $\epsilon$  in electricity. The unit of the magnetic field is the tesla (T). The tesla is a big unit. The magnetic field of the earth is about  $10^{-4}$  T on the earth's surface. A wire carrying a current of about 2000 A (as in some high-voltage transmission lines) produces a magnetic field of  $8 \times 10^{-5}$  T at a distance of 5 m from the wire.

Whereas the magnitude of the magnetic field is straightforward to investigate, its direction is less so. Let us consider a wire that carries a current normal to the page. The direction of the magnetic field at a given point in space is found by placing magnetic needles around the wire and seeing how they align themselves. Figure 6.15 shows the result of this simple experiment for various points around the wire.



**Figure 6.15** The direction of the magnetic field at various points around straight wires carrying current out of the page (left) and into the page (right).

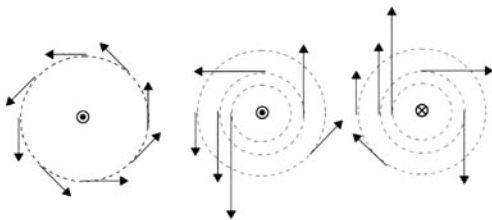
The structure of the magnetic field direction is thus vectors that are tangent to a circle centred on the wire and 'flow' around the circle in the counter-clockwise sense (as looked at from above) if the current comes out of the page (shown by the full circle) and clockwise if the current goes into the page (shown by the cross in the circle).



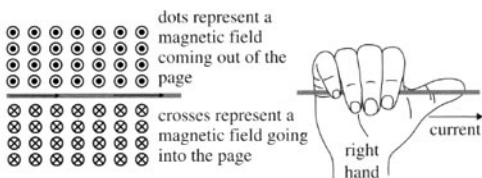
**Figure 6.16** To find the magnetic field at a point near a straight wire, draw the imaginary circle centred at the wire, and going through the point of interest. Grip the wire with the fingers of the right hand with the thumb pointing in the direction of the current. Draw the tangent to the circle at the point of interest so that the vector drawn follows the curl of the fingers.

We can formalize this finding into a 'right-hand rule'.

► Grip the wire with the fingers of the right hand in such a way that the thumb points in the direction of the current. Then the direction in which the fingers curl is the direction of the 'flow' of the magnetic field vectors. (See Figures 6.16–6.18.)



**Figure 6.17** The magnetic field around a straight wire at various distances from the wire. Note that as the distance gets bigger the length of the arrow representing the magnetic field gets smaller.

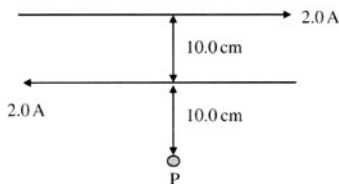


**Figure 6.18** The magnetic field of a straight current-carrying wire looked at from a different point of view.

### Example question

#### Q3

Find the magnetic field at point P in Figure 6.19.



**Figure 6.19.**

#### Answer

The top wire produces a magnetic field into the page of magnitude

$$B_1 = 4\pi \times 10^{-7} \frac{2.00}{2\pi \times 0.200} \text{ T} \\ = 2.00 \times 10^{-6} \text{ T}$$

and the second wire produces a magnetic field out of the page of magnitude

$$B_2 = 4\pi \times 10^{-7} \frac{2.00}{2\pi \times 0.100} \text{ T} \\ = 4.00 \times 10^{-6} \text{ T}$$

resulting in a net magnetic field of  $2.00 \times 10^{-6} \text{ T}$  out of the page.

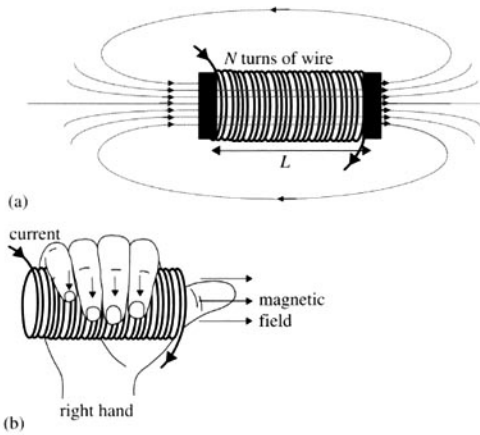
### The single current loop

The magnetic field of a single current loop was shown in Figure 6.4 on page 338. The magnetic field strength  $B$  at the centre of a circular loop of radius  $R$  carrying current  $I$  is

$$B = \frac{\mu_0 I}{2R}$$

### The solenoid

In various applications it is necessary to have a uniform magnetic field – one that has the same magnitude and direction in a region of space. A



**Figure 6.20** (a) A solenoid. If it has an iron core, a much stronger magnetic field results. (b) The second right-hand rule giving the direction of the solenoid magnetic field.

way of achieving such a field is through a solenoid, which is a wire wound tightly many times around an axis (see Figure 6.20a).

► In the *interior* of the solenoid the magnetic field is uniform in magnitude and direction and is given by

$$B = \mu_0 \frac{NI}{L}$$

where  $N$  is the number of turns,  $l$  the length of the solenoid and  $I$  the current through it.

A much stronger magnetic field can be obtained if the solenoid has an iron core.

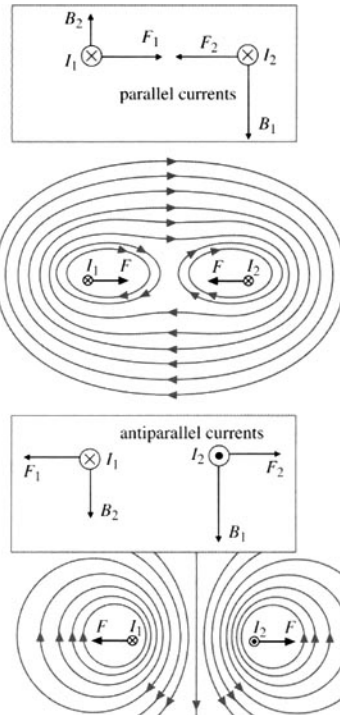
The direction of the magnetic field of a solenoid is found by a second right-hand rule (see Figure 6.20b).

► Hold the solenoid with the right hand so that the fingers curl in the direction of the current in the coils of the solenoid. Then the thumb points in the direction of the magnetic field.

The solenoid magnetic field outside the solenoid resembles that of a bar magnet.

## The force between two current-carrying wires

Consider now two long, straight, parallel wires each carrying current, say  $I_1$  and  $I_2$ . The first wire (wire 1) creates a magnetic field in space, and in particular at the position of the second wire (wire 2). Thus, wire 2 will experience a magnetic force. Similarly, wire 2 will produce a magnetic field at the position of wire 1, so that wire 1 will also experience a magnetic force. By Newton's third law, the forces experienced by the two wires are equal and opposite (see Figure 6.21). If the currents are parallel, the forces are attractive and if they are antiparallel, the forces are repulsive.



**Figure 6.21** The forces on two parallel currents are equal and opposite.

Let us look at the problem of the forces between the two wires in more detail. Consider two long, straight, parallel wires



each carrying electric current, say  $I_1$  and  $I_2$ . The first wire (wire 1) creates a magnetic field  $B_1$  and the second wire a magnetic field  $B_2$ . This means that wire 2 is in the magnetic field of wire 1 ( $B_1$ ), and so will experience a force. Similarly, wire 1 is in the magnetic field of wire 2 ( $B_2$ ), and so it too will experience a force. If the two parallel wires are separated by a distance  $r$ , then

$$B_1 = \mu_0 \frac{I_1}{2\pi r}$$

$$B_2 = \mu_0 \frac{I_2}{2\pi r}$$

Note that since the currents are different, the magnetic fields are different too. Now, the force on a length  $L$  of wire 2 is

$$F_2 = B_1 I_2 L$$

$$\Rightarrow F_2 = \mu_0 \frac{I_1}{2\pi r} I_2 L$$

and similarly the force on an equal length of wire 1 is

$$F_1 = B_2 I_1 L$$

$$\Rightarrow F_1 = \mu_0 \frac{I_2}{2\pi r} I_1 L$$

so, the two forces are equal in magnitude even though the magnetic fields are different. The equality of the forces is expected. The force that wire 1 exerts on wire 2 must be accompanied (Newton's third law) by an equal and opposite force. Let us now use the right-hand rule to find the directions of these forces. Assume first that the currents are flowing into the page. Then the magnetic fields are as shown and the forces are therefore attractive. If wire 1 carries current into the page and wire 2 carries current out of the page, the forces are repulsive. In both cases, we are consistent with Newton's third law. This is how it should be.

This fact is used to define the ampere, the unit of electric current. The ampere equals a coulomb divided by a second but it is no longer defined this way.

► The ampere is defined through the magnetic force between two parallel wires. If the force on a 1 m length of two wires that are 1 m apart and carrying equal currents is  $2 \times 10^{-7}$  N, then the current in each wire is defined to be 1 A.

The coulomb is defined in terms of the ampere as the amount of charge that flows past a certain point in a wire when a current of 1 A flows for 1 s.

### Questions

- 1 Draw the magnetic field lines for two parallel wires carrying equal currents into the page. Repeat for antiparallel currents.
- 2 Find the direction of the missing quantity from  $B$ ,  $v$  and  $F$  in each of the cases shown in Figure 6.22. The circle represents a positive charge.

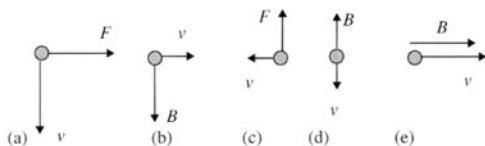


Figure 6.22 For question 2.

- 3 Two long, straight wires lie on the page and carry currents of 3.0 A and 4.0 A as shown in Figure 6.23. Find the magnetic field at point P.

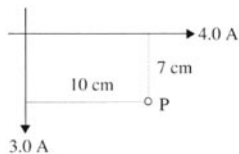


Figure 6.23 For question 3.

- 4 Find the magnetic field at points P, Q and R in Figure 6.24. The currents are parallel and each carry 5.00 A. Point Q is equidistant from the wires. (The three points lie on the same plane as the wires.)

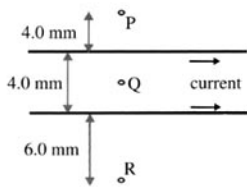


Figure 6.24 For question 4.

- 5 Draw the magnetic field lines that result when the magnetic field of a long straight wire carrying current into the page is superimposed on a uniform magnetic field pointing to the right that lies on the page. (See Figure 6.25.)

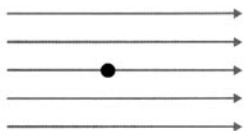


Figure 6.25 For question 5.

- 6 A long straight wire carries current as shown in Figure 6.26. Two electrons move with velocities that are parallel and perpendicular to the current. Find the direction of the magnetic force experienced by each electron.

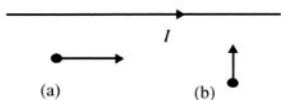


Figure 6.26 For question 6.

- 7 A proton moves past a bar magnet as shown in Figure 6.27. Find the direction of the force it experiences in each case.

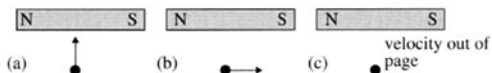


Figure 6.27 For question 7.

- 8 An electron is shot along the axis of a solenoid that carries current. Will it experience a magnetic force?

- 9 What is the direction of a magnetic field in each of the four cases in Figure 6.28 that results in a force on the current as shown?

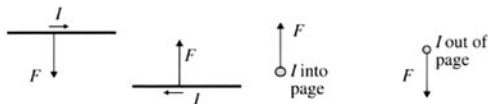


Figure 6.28 For question 9.

- 10 A rectangular loop of wire of size 5 cm  $\times$  15 cm is placed near a long straight wire with side CD at a distance of 5 cm from it, as shown in Figure 6.29. What is the net force exerted on the loop (magnitude and direction)? How does your answer change if the current in the loop is reversed?

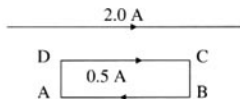


Figure 6.29 For question 10.

- 11 A rectangular coil of size 20 cm  $\times$  10 cm is placed in a horizontal uniform magnetic field of magnitude 0.050 T, as shown in Figure 6.30. A current of 2.0 A flows in the coil in a counter-clockwise direction as shown.
- Find the force on sections AB, BC, CD and DA.
  - What is the net force on the coil?

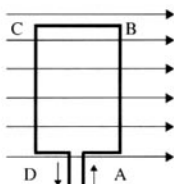


Figure 6.30 For question 11.

- 12 A tightly wound solenoid of length 30 cm is to produce a magnetic field of  $2.26 \times 10^{-3}$  T along its axis when a current of 15.0 A flows in it. If the radius of the solenoid is 12.0 cm, what length of wire is required to make the solenoid?

- 13 What is the direction of the magnetic field at points P and Q in the plane of a circular loop carrying a counter-clockwise current, as shown in Figure 6.31?

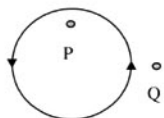


Figure 6.31 For question 13.

- 14 Two parallel wires a distance of 20.0 cm apart carry currents of 2.0 A and 3.0 A as shown in Figure 6.32.

- (a) At which points is the magnetic field zero?  
 (b) How would your answer change if the direction of the 3.0 A current were reversed?

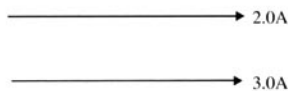


Figure 6.32 For question 14.

- 15 Figure 6.33 shows two parallel plates with a potential difference of 120 V a distance 5.0 cm apart. The top plate is at the higher potential and the shaded region is a region of magnetic field normal to the page.

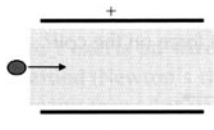


Figure 6.33 For question 15.

- (a) What should the magnetic field magnitude and direction be such that an electron experiences zero net force when shot through the plates with a speed of  $2 \times 10^5 \text{ m s}^{-1}$ .  
 (b) Would a proton shot with the same speed through the plates experience zero net force?  
 (c) If the electron's speed were doubled, would it still be undeflected if the magnetic field took the value you found in (a)?

- 16 A bar magnet is placed in a uniform magnetic field as shown in Figure 6.34.

- (a) Is there a net force on the bar magnet?  
 (b) Will it move? If so, how?

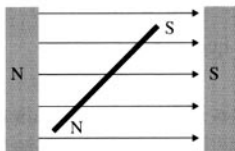


Figure 6.34 For question 16.

- 17 A high-tension electricity wire running along a north-south line carries a current of 3000.0 A. If the magnetic field of the earth at the position of the wire has a magnitude of  $5.00 \times 10^{-5} \text{ T}$  and makes an angle of  $30^\circ$  below the horizontal, what is the force experienced by a length of 30.0 m of the wire?

### HL only

- 18 Two circular loops of wire have their planes parallel and one is directly below the other, as shown in Figure 6.35. Current flows in a counter-clockwise direction (when looked at from above the loops) in both loops. Will there be a force between the loops? If yes, what will its direction be. If not, why is the force zero?

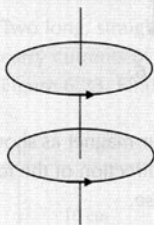


Figure 6.35 For question 18.

- 19 Figure 6.36 shows two parallel conductors carrying current out of the page. Conductor 1 carries double the current of conductor 2. Draw to scale the magnetic fields created by each conductor at the position of the other and the forces on each conductor.

1, 2/



2. /

Figure 6.36 For question 19.

- 20 An electron of speed  $v$  enters a region of magnetic field  $B$  directed normally to its velocity and is deflected into a circular path. Find an expression for the number of revolutions per second the electron will make. If the electron is replaced by a proton, how does your answer change?

**HL only**

- 21 A proton of velocity  $1.5 \times 10^6 \text{ m s}^{-1}$  enters a region of uniform magnetic field  $B = 0.50 \text{ T}$ . The magnetic field is directed vertically up (along the positive  $z$  direction) and the proton's velocity is initially on the  $z$ - $x$  plane making an angle of  $30^\circ$  with the positive  $x$  axis. (See Figure 6.37.)

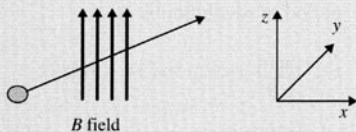


Figure 6.37 For question 21.

- Show that the proton will follow a helical path around the magnetic field lines.
  - What is the radius of the helix?
  - How many revolutions per second does the proton make?
  - How fast is the proton moving along the field lines?
  - What is the vertical separation of the coils of the helix?
- 22 An electron enters a region of uniform magnetic field  $B = 0.50 \text{ T}$ , its velocity being normal to the magnetic field direction. The electron is deflected into a circular path and leaves the region of magnetic field after being deflected by an angle of  $30^\circ$  with respect to its original direction. How long was the electron in the region of magnetic field?

- 23 Find the magnetic field at point P due to three currents as shown in Figure 6.38.

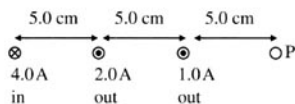


Figure 6.38 For question 23.

**HL only**

- 24 Find the magnetic field at point P due to the currents shown in Figure 6.39.

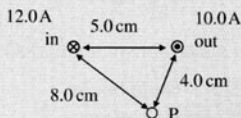


Figure 6.39 For question 24.

- 25 Three parallel wires carry currents as shown in Figure 6.40. Find the force per unit length that wires 1 and 3 exert on wire 2.

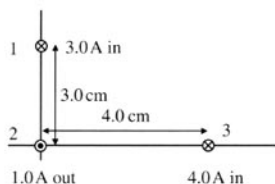


Figure 6.40 For question 25.

- 26 The magnetic field at the centre of a circular loop of wire of radius  $r$  carrying current  $I$  is given by the formula

$$B = \mu_0 \frac{I}{2r}$$

Use this expression to find the magnetic field created by an electron as it rotates with speed  $v$  in a circular orbit of radius  $r$  around a nucleus.

## HL only

- 27 A tightly wound solenoid is to be made with wire from a fixed quantity (mass) of copper. It will then be connected to a source of fixed potential difference. How should the solenoid be made in order to produce the largest magnetic field?
- 28 Two parallel wires separated by a distance  $d$  carry the same current  $I$ , as shown in Figure 6.41.

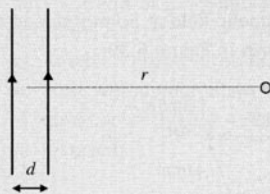


Figure 6.41 For question 28.

- (a) Calculate the magnitude of the magnetic field at a point in the plane of the wires a distance  $r$  from the middle of the wires.
- (b) By using the binomial expansion when  $r$  is much larger than  $d$ , show that the leading term in the expansion is  $B = \mu_0 \frac{2I}{2\pi r}$ . Why is this so?
- (c) Repeat for the case where the currents are antiparallel. This time show that the leading term is  $B = \mu_0 \frac{Id}{2\pi r^2}$ . Why are the two expansions so different?
- 29 In a particle accelerator called the cyclotron, a charged particle is accelerated by an electric field and bent into a circular path by a magnetic field. The magnetic field is assumed uniform and the north and south poles are separated by a small gap. The particles to be accelerated originate from a source at the centre of the bottom magnet pole (the south pole in Figure 6.42) and begin to move outward in a circular path. The bottom magnet pole is split into two pieces called dees. An alternating potential difference is set up between the two dees

so that every time the particle crosses from one dee to the other it increases its kinetic energy and thus moves on a circle of larger radius.

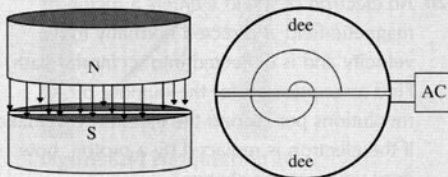


Figure 6.42 For question 29.

- (a) Explain why the particle follows a spiral path.
- (b) Show that the operation of the cyclotron depends on the frequency of the alternating voltage source, being equal to the frequency of revolution of the particle to be accelerated.
- (c) If the mass and charge of the particle are  $m$  and  $q$ , respectively, show that the period of revolution is

$$T = \frac{2\pi m}{qB}$$

- where  $B$  is the magnetic field, and is thus independent of the speed of the particle.
- (d) Find the frequency (i.e. the number of revolutions per second) of an electron assuming that the magnetic field has a value of 0.50 T.
- (e) If the potential difference between the dees at the instant the electrons cross is 120 kV, what would the kinetic energy of the electrons be after 100 revolutions?

- 30 A uniform magnetic field is established in the plane of the paper and has magnitude 0.3 mT. Two parallel wires separated by 5.0 cm carry currents of 200 A and 100 A into the plane of the paper as shown in Figure 6.43. Find the magnetic force per unit length on the 100 A wire.

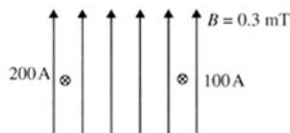


Figure 6.43 For question 30.

- 31 A uniform magnetic field is established in the plane of the paper as shown in Figure 6.44. Two wires carry *parallel* currents of equal magnitudes normally to the plane of the paper at P and Q. Point R is on the line joining P to Q and closer to Q. The magnetic field at position R is zero.

- (a) Are the currents going into the paper or out of the paper?  
 (b) If the current is increased slightly, will the point where the magnetic field is zero move to the right or to the left of R?

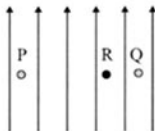


Figure 6.44 For question 31.

- 32 Two identical charged particles move in circular paths at right angles to a uniform magnetic field as shown in Figure 6.45. The radius of particle 2 is twice that of particle 1.

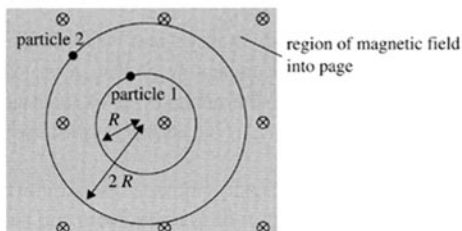


Figure 6.45 For question 32.

Determine the following ratios:

- (a)  $\frac{\text{period of particle 2}}{\text{period of particle 1}}$ ;  
 (b)  $\frac{E_k \text{ of particle 2}}{E_k \text{ of particle 1}}$ .