Nuclear reactions

This chapter is an introduction to the physics of atomic nuclei. We will see that the sum of the masses of the constituents of a nucleus is not the same as the mass of the nucleus itself, which implies that the nucleus has enormous amounts of energy stored in it. Methods used to calculate energy released in nuclear reactions are presented.

Objectives

By the end of this chapter you should be able to:

- define the *unified mass unit*;
- state the meaning of the terms *mass defect* and *binding energy* and solve related problems;
- write *nuclear reaction equations* and balance the atomic and mass numbers;
- understand the meaning of the graph of *binding energy per nucleon* versus *mass number*;
- state the meaning of and difference between *fission* and *fusion*;
- understand that nuclear fusion takes place in the *core of the stars*;
- solve problems of *fission* and *fusion reactions*.

The unified mass unit

In nuclear physics, it is convenient to use a smaller unit of mass than the kilogram. We define a new unit called the *unified atomic mass unit*, u for short. It is defined to be $\frac{1}{12}$ of the mass of an atom of carbon-12, $^{12}_{6}C$. A mole of carbon $^{12}_{6}C$ is 12 g and the number of molecules is the Avogadro constant, therefore the carbon-12 atom has a mass M given by

$$6.0221367 \times 10^{23} \times M = 12\,\text{g}$$

$$M = \frac{12}{6.0221367 \times 10^{23}} \times 10^{-3}\,\text{kg}$$

$$= 1.992648 \times 10^{-26}\,\text{kg}$$

Hence

$$1\,\text{u} = \frac{1}{12}(1.992648 \times 10^{-26}\,\text{kg})$$

$$= 1.6605402 \times 10^{-27}\,\text{kg}$$

Example question

Q1

Find in units of u the masses of the proton, neutron and electron (use Table 3.1).

Unified mass unit	**$1.6605402 \times 10^{-27}$ kg**
Electron	$9.1093897 \times 10^{-31}$ kg
Proton	$1.6726231 \times 10^{-27}$ kg
Neutron	$1.6749286 \times 10^{-27}$ kg

Table 3.1.

Answer

From the table of the masses in kilograms (Table 3.1) we find

$m_p = 1.007276$ u
$m_n = 1.008665$ u
$m_e = 0.0005486$ u

The mass defect and binding energy

To find the mass of a particular nucleus we have to subtract the mass of the electrons in the atom from the mass of the atom. If there are Z electrons in the atom, then

$$M_{\text{nucleus}} = M_{\text{atom}} - Zm_e$$

The mass of the atom is obtained from the periodic table and m_e is given above. We can find, for example, that the mass of the nucleus of helium is

$$M_{\text{nucleus}} = 4.0026 - 2 \times 0.0005486$$
$$= 4.00156\,\text{u}$$

We now recall that the helium nucleus is made up of two protons and two neutrons. If we add up their masses we find

$$2m_p + 2m_n = 4.0320\,\text{u}$$

which is *larger* than the mass of the nucleus by 0.0304 u. This leads to the concept of **mass defect**.

▶ We can see by examining each nucleus that this is generally true: the mass of the protons plus the mass of the neutrons is larger than the mass of the nucleus. We define their difference as the mass defect δ:

$$\delta = \text{total mass of nucleons} - \text{mass of nucleus}$$

or

$$\delta = Zm_p + (A - Z)m_n - M_{\text{nucleus}}$$

(remember that $A - Z$ is the number of neutrons in the nucleus). This formula allows us to calculate the mass defect for any nucleus.

Example question

Q2

Find the mass defect of the nucleus of gold, $^{197}_{79}\text{Au}$.

Answer

From the periodic table, the mass of the *atom* of gold is 196.967 u, and since it has 79 electrons the *nuclear* mass is

$$196.967\,\text{u} - 79 \times 0.0005486\,\text{u} = 196.924\,\text{u}.$$

The nucleus has 79 protons and 118 neutrons, so

$$\delta = (79 \times 1.007276 + 118 \times 1.008665 - 196.924)\,\text{u}$$
$$= 1.67\,\text{u}$$

Einstein's mass–energy formula

Where is the missing mass? The answer is given by Einstein's theory of special relativity, which states that mass and energy are equivalent and can be converted into each other. Einstein's famous formula from 1905 reads

$$E = mc^2$$

where c stands for the speed of light. The mass defect of a nucleus has been converted into energy and is stored in the nucleus. This energy is called the **binding energy** of the nucleus, and is denoted by E_b. Thus:

▶ $E_b = \delta c^2$

The binding energy of the nucleus is the work (energy) required to completely separate the nucleons of a nucleus.

The work required to remove one nucleon from the nucleus is *very roughly* the binding energy divided by the total number of nucleons.

At a more practical level, the binding energy of a nucleus is a measure of how stable it is – the higher the binding energy, the more stable the nucleus is.

It is convenient to find out how much energy corresponds to a mass of 1 u. Then, given a nuclear mass in u, we will immediately be able to find the energy that corresponds to it. Thus, an energy of 1 u is

$$1\,\text{u} \times c^2 = 1.6605402 \times 10^{-27} \times (2.9979 \times 10^8)^2\,\text{J}$$
$$= 1.4923946316 \times 10^{-10}\,\text{J}$$

Changing this to electronvolts, using

$$1\,\text{eV} = 1.602177 \times 10^{-19}\,\text{J}$$

gives an energy equivalent to a mass of 1 u of

$$\frac{1.4923946316 \times 10^{-10}}{1.602177 \times 10^{-19}} \frac{\text{J}}{\text{J eV}^{-1}} = 931.5 \times 10^6\,\text{eV}$$
$$= 931.5\,\text{MeV}$$

(one MeV is one million electronvolts, $1\,\text{MeV} = 10^6\,\text{eV}$). So

$$1\,\text{u} \equiv 931.5\,\text{MeV}$$

Example questions

Q3

Find the energy equivalent to the mass of the proton, neutron and electron.

Answer

The masses in terms of u are $m_p = 1.0073$ u, $m_n = 1.0087$ u and $m_e = 0.0005486$ u. Hence the energy equivalents are, respectively, 938.3 MeV, 939.6 MeV and 0.511 MeV.

Q4

Find the binding energy of the nucleus of carbon-12.

Answer

The nuclear mass is

$$12.00000\,\text{u} - 6 \times 0.0005486\,\text{u} = 11.99671\,\text{u}$$

The mass defect is

$$6 \times 1.007276\,\text{u} + 6 \times 1.008665\,\text{u} - 11.99671\,\text{u}$$
$$= 0.09894\,\text{u}$$

(the nucleus has 6 protons and 6 neutrons). Hence the binding energy is

$$0.09894 \times 931.5\,\text{MeV} = 92.2\,\text{MeV}$$

The binding energy curve

We saw on the previous page that the mass defect of helium is 0.0304 u, which corresponds therefore to a binding energy of

$$0.0304 \times 931.5\,\text{MeV} = 28.32\,\text{MeV}$$

(The alpha particle has an unusually large binding energy compared with nuclei of roughly the same mass. This accounts for its exceptional stability and the fact that unstable nuclei decay by emitting alpha particles.) There are four nucleons in the helium nucleus so the binding energy per nucleon is $28.32/4 = 7.1$ MeV. For carbon, we found a binding energy of 92.159 MeV, giving a binding energy per nucleon of 7.68 MeV.

▶ We find that most nuclei have a binding energy per nucleon of approximately 8 MeV.

This is shown in Figure 3.1.

This curve is at the heart of nuclear physics. The curve has a maximum for $A = 62$ corresponding to nickel. As we shall soon see, this curve tells us that if a heavy nucleus (heavier than nickel) splits up into two lighter ones or if two light nuclei (lighter than nickel) fuse together, then energy is released as a result. This is of fundamental importance and is the basis for nuclear fission and nuclear fusion, respectively. To understand all this we must first see what happens from the energy point of view when a nucleus decays.

Energy released in a decay

Let us consider the decay of radium by alpha particle emission (see Figure 3.2):

$${}^{226}_{88}\text{Ra} \rightarrow {}^{222}_{86}\text{Rn} + {}^{4}_{2}\alpha$$

For any decay, the total energy to the left of the arrow must equal the total energy to the right of the arrow. Here total energy means the energy corresponding to each mass

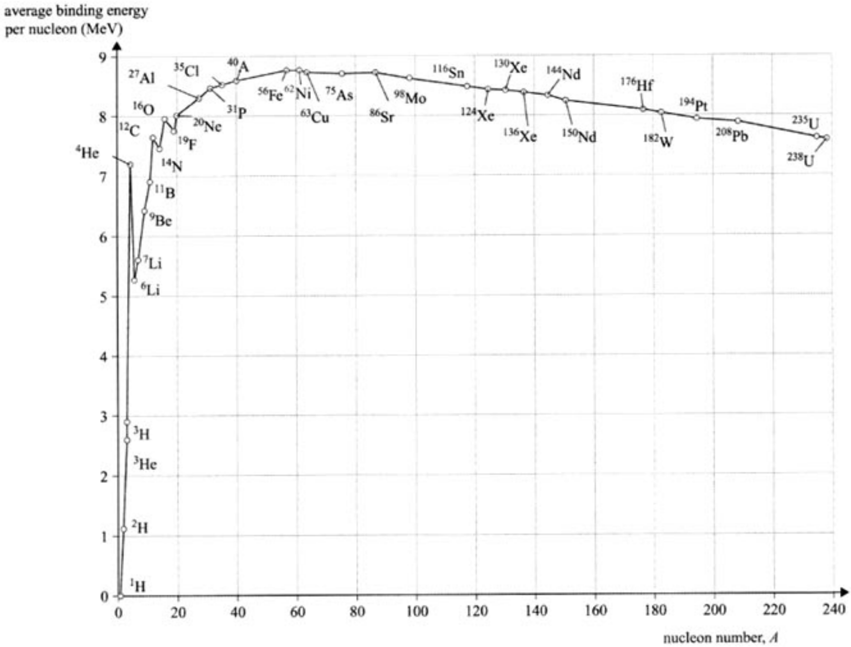

Figure 3.1 The binding energy per nucleon is almost constant for most nuclei.

according to Einstein's formula plus whatever kinetic energy each mass has. If the decaying radium nucleus is at rest, then the total energy available is simply Mc^2, where M is the mass of the nucleus of radium. To the right of the arrow, we have the energies corresponding to the masses of the radon and helium nuclei plus any possible kinetic energy: the produced nuclei are moving.

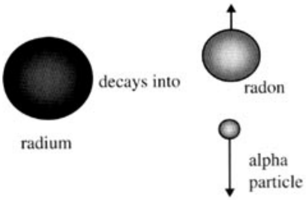

Figure 3.2 The energy released in a nuclear reaction is in the form of kinetic energy of the products.

Thus, to be at all possible, the decay must be such that at the very minimum the energy corresponding to the radium mass is larger than the energies corresponding to the radon plus alpha particle masses. Let us check if this is true. We need the *masses of the nuclei* that appear in the reaction, namely radium, radon and helium.

If we use the periodic table to find the masses, we must remember that the periodic table gives atomic masses not nuclear masses. Thus, we must subtract from each atomic mass the mass of the electrons in the atom.

However, the atomic number is conserved (i.e. it is the same before and after the decay) and equals the number of electrons in the atom. It follows that the number of electron masses that must be subtracted from the atomic mass to the left of the arrow is the same as the number of electron masses that must be subtracted from the right. Thus, as long as we are interested in *mass differences*, as we are here, it is enough to use atomic masses instead of nuclear masses.

According to the periodic table:

$$\text{mass of radium} = 226.0254\ \text{u}$$

$$\begin{array}{rr} \text{mass of radon} = & 222.0176\ \text{u} \\ + \quad \text{mass of helium} = & 4.0026\ \text{u} \\ \hline \text{sum} = & 226.0202\ \text{u} \end{array}$$

We see that the mass of radium exceeds that of radon plus helium by 0.0052 u. Thus, there is an amount of energy released in the form of kinetic energy of radon and helium of $0.0052 \times 931.5\ \text{MeV} = 4.84\ \text{MeV}$. If 50 g of radium were to decay in this way, the total energy released would be $N \times 4.84$ MeV where N is the total number of nuclei in the 50 g of radium. In 50 g of radium there are $\frac{50}{226} = 0.22$ mol and so

$$N = 0.22 \times 6 \times 10^{23} = 1.3 \times 10^{23}$$

Hence the total energy released is

$$\begin{aligned} E &= 1.3 \times 10^{23} \times 4.84\ \text{MeV} \\ &= 6.3 \times 10^{23}\ \text{MeV} \\ &\approx 10^{11}\ \text{J} \end{aligned}$$

The momenta of radon and helium are opposite in direction and equal in magnitude by the law of conservation of momentum. (We assume that the decaying radium nucleus is at rest, so its momentum is zero.) Thus

$$M_{\text{radon}} v_{\text{radon}} = M_{\text{helium}} v_{\text{helium}}$$

Therefore

$$\begin{aligned} \frac{v_{\text{helium}}}{v_{\text{radon}}} &= \frac{M_{\text{radon}}}{M_{\text{helium}}} \\ &\approx \frac{222}{4} \\ &\approx 55 \end{aligned}$$

the velocity of radon is smaller than the velocity of helium by the ratio of the masses: approximately 55. (As an exercise you can show that the ratio of kinetic energies of the helium to the radon nuclei is also 55.)

Let us now re-examine these findings in terms of the binding energy curve. For the decay to take place, the mass of the decaying nucleus has to be greater than the combined masses of the products. This means that the binding energy of the decaying nucleus must be less than the binding energies of the product nuclei. This is why radioactive decay is possible for heavy elements lying to the right of nickel in the binding energy curve.

Nuclear reactions

If a nucleus cannot decay by itself, it can still do so if energy is supplied to it. This energy can be transferred to the nucleus by a fast-moving particle that collides with it. For example, an alpha particle colliding with nitrogen produces oxygen and hydrogen (i.e. a proton):

$${}^{14}_{7}\text{N} + {}^{4}_{2}\alpha \rightarrow {}^{17}_{8}\text{O} + {}^{1}_{1}\text{p}$$

(see Figure 3.3). This is an example of a nuclear reaction. Note how the atomic and mass numbers match as they did in nuclear decays. This is a famous reaction called the transmutation of nitrogen; it was studied by Rutherford in 1909. Note that if we add up the masses to the left of the arrow we find 18.0057 u, whereas the masses to the right are 18.0070 u (i.e. larger). Thus, this reaction will only take place if the alpha particle has enough kinetic energy to make up for the imbalance in mass between the two sides.

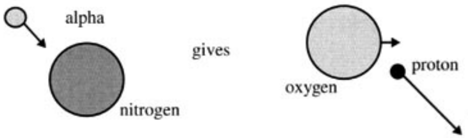

Figure 3.3 An alpha particle colliding with nitrogen produces oxygen and a proton

(Actually, the required minimum kinetic energy of the alpha particle has to be bigger than the energy equivalent of the mass difference between the two sides of the reaction. This is because the products of the reaction themselves will have kinetic energy.)

In a reaction in which four particles participate

$$A + B \rightarrow C + D$$

energy will be released if the quantity Δm given by

$$\Delta m \rightarrow (m_A + m_B) - (m_C + m_D)$$

is positive (i.e. if the total mass on the left is larger than the total mass on the right). The amount of energy released is then equal to

$$\Delta E = (\Delta m)c^2$$

There are two kinds of energy-producing nuclear reactions and we consider them separately in the following sections.

Nuclear fission

Nuclear fission is the process in which a heavy nucleus splits up into lighter nuclei. When a neutron is absorbed by a nucleus of uranium-235, uranium momentarily turns into uranium-236 according to the reaction

$${}^{1}_{0}n + {}^{235}_{92}U \rightarrow {}^{236}_{92}U$$

Uranium-236 then splits into lighter nuclei. This is the fission reaction. A number of possibilities exist as to what these nuclei are. One possibility is

$${}^{144}_{56}Ba + {}^{89}_{36}Kr + 3{}^{1}_{0}n$$

The production of neutrons is a feature of fission reactions. The produced neutrons can be used to collide with other nuclei of uranium-235 in the reactor, producing more fission, energy and neutrons. The reaction is thus self-sustaining – it is called a *chain reaction*. For the chain reaction to get going a certain minimum mass of uranium-235 must be present, otherwise the neutrons escape without causing further reactions – this is called the *critical mass*.

The energy released can be calculated as shown in Table 3.2.

mass of uranium plus neutron	= 236.0526 u
mass of products = 143.92292 u + 88.91781 u + 3 × 1.008665 u	= 235.8667250 u
mass difference	= 0.185875 u
energy released	= 0.185875 × 931.5 MeV = 173.14 MeV

Table 3.2.

This energy appears as kinetic energy of the products.

Thus, an energy of about 173 MeV per fissioning nucleus of uranium is released. This is a lot of energy! A mass of 1 kg of uranium-235 undergoing fission would produce an amount of energy that can be found as follows: 1 kg is 1000/235 mol of uranium and thus contains $(1000/235) \times 6 \times 10^{23}$ nuclei. Each nucleus produces about 173 MeV of energy and thus the total is $(1000/235) \times 6 \times 10^{23} \times 173$ MeV or about 7×10^{13} J. In a nuclear reactor, the release of energy is done in a controlled way. If the rate of neutron production is too high, too much energy is produced in a very short time. This is what happens in a nuclear bomb.

Note that the fission process begins when a neutron collides with a nucleus of uranium-235. An alpha particle cannot be used to start this process because its positive charge would be repelled by the positive charge of the uranium nucleus and so would not lead to the capture of the alpha. An electron, on the other hand, would easily be captured but its small mass would not perturb the heavy nucleus sufficiently for fission to start.

Nuclear fusion

Nuclear fusion is the joining of two light nuclei into a heavier one with the associated production of energy. An example of this reaction is:

$${}^{2}_{1}H + {}^{2}_{1}H \rightarrow {}^{3}_{2}He + {}^{1}_{0}n$$

where two deuterium nuclei (isotopes of hydrogen) produce helium-3 (an isotope of helium) and a neutron. Computing masses to the left and right of the reaction arrow as in Table 3.3 we can find the energy released.

2 × mass of deuterium	= 4.0282 u
mass of helium + neutron	= 4.0247 u
mass difference	= 0.0035 u
energy released	= 0.0035 × 931.5 MeV = 3.26 MeV

Table 3.3.

A kilogram of deuterium would thus release energy of about 10^{13} J, which is comparable to the energy produced by a kilogram of uranium in the fission process.

Example question

Q5

Another fusion reaction is $4{}^1_1\text{H} \rightarrow {}^4_2\text{He} + 2{}^0_1\text{e} + 2\nu_e + {}^0_0\gamma$, where four hydrogen nuclei fuse into a helium nucleus plus two positrons (the antiparticle of the electron – same mass, opposite charge), two electron neutrinos and a photon. Calculate the energy released in this reaction.

Answer

We must find the masses before and after the reaction.

Mass of 4 protons (hydrogen nuclei)
$= 4 \times 1.007276 \text{ u} = 4.029104 \text{ u}$

Mass on right-hand side
$= (4.0026 - 2 \times 0.0005486) \text{ u}$
$+ 2 \times 0.0005486 \text{ u} = 4.002600 \text{ u}$

Mass difference $= 0.026504$ u

This gives an energy of 24.7 MeV. The two positrons annihilate into energy by colliding with two electrons giving an additional 2 MeV (= 4 × 0.511 MeV), for a total of 26.7 MeV.

For the light nuclei to fuse, very large temperatures are required. This is so that the electrostatic repulsion between the two nuclei that fuse is overcome. The enormous temperature (recall the kinetic theory of gases) causes the nuclei to move fast enough so as to approach each other sufficiently for fusion to take place. The very hot material (over ten million kelvin) undergoing fusion is in a state called plasma (ionized atoms). Plasma, being very hot, cannot come into contact with anything else (either because it causes it to melt or because it will result in heat losses) and therefore has to be contained by unusual methods such as magnetic fields in big machines called tokamaks. There are serious unsolved problems with the prolonged confinement of plasmas and this is one reason why nuclear fusion, still, is not a commercially viable source of energy. Commercial energy from the nucleus comes now only from the fission process, which unlike fusion, however, is environmentally suspect.

Fusion in stars

The high temperatures and pressures in the interior of stars make stars ideal places for nuclear fusion. As we saw in the previous section, high temperatures are required so that the nuclei have sufficiently large kinetic energy to approach each other, overcoming the electrostatic repulsion due to their positive charges. The high pressure ensures that sufficient numbers of nuclei are found close to each other, thus increasing the probability of them coming together and fusion taking place.

The reaction $4{}^1_1\text{H} \rightarrow {}^4_2\text{He} + 2{}^0_1\text{e} + 2\nu_e + {}^0_0\gamma$ is a typical reaction that takes place in stellar cores. Nuclear fusion is the source of energy for a star; it prevents the star from collapsing under its own weight and provides the energy the star sends out in the form of light and heat, for example. Stars are, in fact, element factories, producing, for example, all the elements that our bodies are made of. More details on this can be found in Option E on Astrophysics.

Fusion and fission processes are summarized in Figure 3.4.

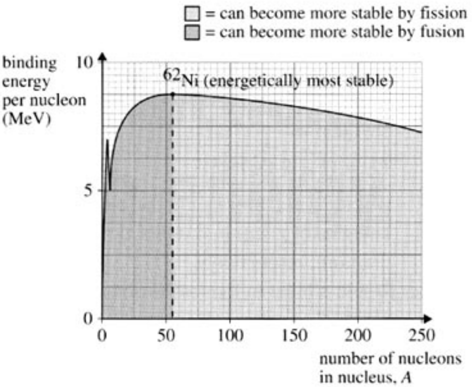

Figure 3.4 When a heavy nucleus splits up, energy is released because the produced nuclei have a higher binding energy than the original nucleus. When two light nuclei fuse, energy is produced because the products again have a higher binding energy.

Questions

1 Find the binding energy and binding energy per nucleon of the nucleus $^{62}_{28}\text{Ni}$. The atomic mass of nickel is 61.928348 u.

2 How much energy is required to remove one proton from the nucleus of $^{16}_{8}\text{O}$? A rough answer to this question is obtained by giving the binding energy per nucleon. A better answer is obtained when we write a reaction that removes a proton from the nucleus. In this case $^{16}_{8}\text{O} \rightarrow ^{1}_{1}\text{p} + ^{15}_{7}\text{N}$. Find the energy required for this reaction to take place. This is the proton separation energy. Get both values and compare them. (The atomic mass of oxygen is 15.994 u; that of nitrogen is 15.000 u.)

3 What is the energy released in the beta decay of a neutron?

4 The first excited state of the nucleus of uranium-235 is 0.051 MeV above the ground state.

(a) What is the wavelength of the photon emitted when the nucleus makes a transition to the ground state?

(b) What part of the spectrum does this photon belong to?

5 Calculate the energy released in the alpha decay $^{234}_{90}\text{Th} \rightarrow ^{230}_{88}\text{Ra} + ^{4}_{2}\text{He}$. (The atomic mass of thorium is 234.043596 u; that of radium is 230.03708 u.)

6 Assume uranium-236 splits into two nuclei of palladium-117 (Pd). (The atomic mass of uranium is 236.0455561 u; that of palladium is 116.9178 u.)

(a) Write down the reaction.

(b) What other particles must be produced?

(c) What is the energy released?

7 One possible outcome in the fission of a uranium nucleus is the reaction $^{235}_{92}\text{U} + ^{1}_{0}\text{n} \rightarrow ^{95}_{42}\text{Mo} + ^{139}_{57}\text{La} + 2^{1}_{0}\text{n} + ?$

(a) What is missing in this reaction?

(b) How much energy is released? (Atomic masses: U = 235.043922 u; Mo = 94.905841 u; La = 138.906349 u.)

8 Another fission reaction involving uranium is $^{235}_{92}\text{U} + ^{1}_{0}\text{n} \rightarrow ^{98}_{40}\text{Zr} + ^{135}_{52}\text{Te} + 3^{1}_{0}\text{n}$
Calculate the energy released. (Atomic masses: U = 235.043922 u; Zr = 97.91276 u; Te = 134.9165 u.)

9 Calculate the energy released in the fusion reaction $^{2}_{1}\text{H} + ^{3}_{1}\text{H} \rightarrow ^{4}_{2}\text{He} + ^{1}_{0}\text{n}$. (Atomic masses: $^{2}_{1}\text{H}$ = 2.014102 u; $^{3}_{1}\text{H}$ = 3.016049 u; $^{4}_{2}\text{He}$ = 4.002603 u.)

10 In the text, it was stated that the reaction $4^{1}_{1}\text{H} \rightarrow ^{4}_{2}\text{He} + 2^{0}_{1}\text{e} + 2\nu_e + ^{0}_{0}\gamma$ is the mechanism by which hydrogen in stars is converted into helium and that the reaction releases about 26.7 MeV of energy. The sun radiates energy at the rate of 3.9×10^{26} W and has a mass of about 1.99×10^{30} kg, of which 75% is hydrogen. Find out how long it will take the sun to convert 12% of its hydrogen into helium.

11 In the first nuclear reaction in a particle accelerator, hydrogen nuclei were accelerated and then allowed to hit nuclei of lithium according to the reaction $^{1}_{1}\text{H} + ^{7}_{3}\text{Li} \rightarrow ^{4}_{2}\text{He} + ^{4}_{2}\text{He}$. Find the energy released. (The atomic mass of lithium is 7.016 u.)

12 Outline the role in nuclear fusion reactions of:

(a) temperature; (b) pressure.

13 Show that an alternative formula for the mass defect is $\delta = ZM_H + (A - Z)\, m_n - M_{atom}$ where M_H is the mass of a hydrogen atom and m_n is the mass of a neutron.

14 Consider the nuclear *fusion* reaction involving the deuterium (^2_1D) and tritium (^3_1T) isotopes of hydrogen:

$$^2_1D + ^3_1T \rightarrow ^4_2He + ^1_0n$$

The energy released, Q_1, may be calculated in the usual way, using the masses of the particles involved, from the expression

$$Q_1 = (M_D + M_T - M_{He} - m_n)c^2$$

Similarly, in the *fission* reaction of uranium

$$^{235}_{92}U + ^1_0n \rightarrow ^{98}_{40}Zr + ^{135}_{52}Te + 3^1_0n$$

the energy released, Q_2, may be calculated from

$$Q_2 = (M_U - M_{Zr} - M_{Te} - 2m_n)c^2$$

(a) Show that the expression for Q_1 can be rewritten as

$$Q_1 = E_{He} - (E_D + E_T)$$

where E_{He}, E_D and E_T are the binding energies of helium, deuterium and tritium, respectively,

(b) Show that the expression for Q_2 can be rewritten as

$$Q_2 = (E_{Zr} + E_{Te}) - E_U$$

where E_{Zr}, E_{Te} and E_U are the binding energies of zirconium, tellurium and uranium, respectively.

(c) Results similar to the results obtained in (a) and (b) apply to all energy-releasing fusion and fission reactions. Use this fact and the binding energy curve in Figure 3.1 to explain carefully why energy is released in fusion and fission reactions.